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The Magnetic and Convective Effect Through a Vertical Wavy Cone in Nano-fluid

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Abstract The novelty of this paper investigates the influence of magnetic field on a nanofluid flow over curvy vertical cone in presence of natural convection with viscosity inversely proportional to linear temperature function. Use the nano-particle Cu and the based fluid is water. The fundamental equations are transformed into non-dimensional partial differential equations by using the dimensionless form, and the curvy surface impact will be eliminated from the boundary conditions applying this non-dimensional form. The transformed equations will be solved numerically by finite difference (fully implicit method). The numerical results will be obtained for magnetic parameter, Prandtl number, surface amplitude, cone half-angle and viscosity variation parameter and their effect on velocity, temperature and Nusselt number. The effect of increasing the magnetic parameter reduces the velocity and increases the temperature. Redundancy of Prandtl number increases the temperature, while the increase of Cone half-angle (CHA) increases the velocity profile. The influence of increasing the viscosity parameter tends to increase the Nusselt number.

Keywords: *Magnetic Field, Nano-fluid, Vertical Wavy Cone, Natural Convection, Fully implicit method*

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1. Introduction

The propose of this research is to study the magnetic field over vertical wavy cone in presence of natural convection flow immersed in incompressible viscous nano-fluid, the viscosity of Nano-fluid is inversely proportional to linear temperature.

Nano-fluid is a mixture of fluid and nano-particles, which it's diameter between 1and 100 nanometers (nm) and it is made of metal or non-metal. The importance of using a nano-particles in fluid to improving the behavior of the thermal systems by enhancing heat transfer rate. There are many applications of nano-fluid mention some of them as in electronic equipment to reduce the thermal resistance, coolant in automobiles, ceramic coating for solar cell and the strain-repellent fabrics. Choi [1] is the first scientist study the nano-fluids and discover that the nano-particles enhanced thermal conductivity.

It is so important to study the impact of magnetic field on the fluid's velocity which is subject to Lorentz force law. The force of Lorentz increases when field of Magnetic intensified, whereas the magnetic field resists the motion of fluid and any transitional process.

There are several applications for the magnetic field, including health, water treatment, medicine, home appliance, and other. In the field of health and water treatment, as people around the world suffer from the scarcity of available water and the increase in population, especially projects that use water permanently and repeatedly, such as agriculture, poultry and livestock farms, hotels and others. Since the scarcity of potable water has dire consequences, to the point that harmful chemicals such as chlorine, fluoride and Aluminum salts were added to obtain clean and potable water. Despite the effectiveness of this method, it does more harm than good because it causes many diseases and causes an imbalance in the environment. The harmful effects of these materials on the health of humans, animals and plants have been proven, leading to the formation of what is called dead water, which is water that has lost many of its vital properties. In addition, the chlorine used in sterilization reacts with organic materials, forming carcinogenic materials. Which led to the search for modern methods and techniques for water treatment, including magnetic water treatment. Exposure to the magnetic field changes the chemical properties and physical properties of water molecules, resulting in unique properties. Magnetized water has shown different properties with its use, such as enhancing soil, plant growth, improving crop productivity, saving water, and treating waste water.

Magnetic treatment of water reconstructs the water molecules into ultra-fine molecules, facilitating their movement through passages in animal and plant cell membranes. In addition, toxic substances cannot enter the structure of magnetically treated water.

Many studies have proven that magnetic water treatment does not affect the acidity of the water. Magnetic treatment increases the percentage of dissolved oxygen, which kills germs. It also enhances the levels of carbon dioxide and hydrogen, so soil fertilizers are not required to be added in the soil. These features make magnetic water treatment a friendly compound for plant and animal cells. Therefore, magnetic water treatment has become the ideal solution for obtaining clean water.

Magnetic processing devices consist of the permanent magnet or electromagnet to produce the field of magnetic within a channel through which the fluid passes.

There are several applies of magnetic fields in the field of medicine, including the use of magnetic fields to treat many diseases or relieve pain, such as arthritis, cancer, circulatory disorders, weak immune system, inflammation, insomnia, muscle pain, sciatica, and stress. There are a lot of types of magnetic therapy, including what we can wear, such as a magnetic bracelet or a bandage containing a magnet, or the magnet can be worn as a shoe insole or a sleeping pillow and some of it is done through acupuncture.

There is another very important application, the latest magnetic field applications in the field of medicine, which is interventional radiology.

Interventional radiology (IR) is a group of techniques that rely on the use of radiological images, which are X-ray imaging, ultra sound imaging, tomography computation, magnetic resonance imaging, and magnetic resonance therapy. Many diseases can be treated using interventional radiology, including diseases of the blood vessels, digestive system, liver, respiratory system, central nervous system, tumor treatment, uterine fibroids treatment, prostatic artery embolization treatment, and relief of symptoms of prostate enlargement. The most important reasons for interventional radiology treatment are reducing risks to the patient, improving health outcomes, reducing pain, and less recovery time in opposition to open surgery.

Rasha Adel [2] studied the consequence of the field of magnetic on nano-fluid through stretching or shrinking plate. Hoda M. Mobarak and Rasha Adel [3] investigate the consequence of the field of magnetic on nano-fluid over stretching plate. Magnetic field and thermal flux influence on nano-fluid through stretching or shrinking plate presented by Rasha Adel and M. Abdelhakem et. al. [4]. The impact of magnetic field and heat transfer of nano-fluid investigated by Emad M. Abo-Eldahab, Rasha Adel and et. al [5]. Mehmet Gurdal et.al. [6] show that the consequence of the field of magnetic on a nanofluid in presence of convective heat transfer. The influence of the field of magnetic on a nano-fluids has been examined by Mahmoud M. Selim et. al. [7]. Hamid Shafiee and others [8] investigate that the consequence of the field of magnetic and transfer of heat on nano-fluid. Imene Rahmoune and Saadi Bougoul [9] studied the consequence of the field of magnetic and heat transmission on nano-fluid over curved mini-channel. Abdulhassan A. Karamallah et. al. [10] presents the influence of the field of magnetic and heat transfer on nanofluid through horizontal pipe. Guannan Wang, et. al. [11] shows that the influence of the field of magnetic and heat transfer of nano-fluid by applied electric field. The impact of magnetic field on nano-fluid, temperature and volume fractions of nanoparticles have been examined by A. Malekzadeh and other authors [12]. Mahdi Alibeigi, Somayeh Davoodabadi Farahani, and Hamed Hossienabadi Farahani [13] studied the consequence of the field of magnetic and transfer of heat of nano-fluid through flat tube. The consequence of field of magnetic and heat transfer on nano-fluid based on molecular dynamics have been investigate by Xilong Zhang et. al. [14]. Chakravarthula S. K. Raju, Ilyas Khan, Suresh Kumar Raju S., Mamatha S. Upadhya [15] presents the impact of magnetic field of nano-fluid convection. Amir Fattahi, Maziar Dehghan, and Mohammad Sadegh Valipour [16] present the field of magnetic consequence on nanofluid and analyze the transfer of heat enhance in a three-dimensional micro-channel. Nano-fluid flow and impact of magnetic force are investigated by Elzbieta Fornalik-Wajs et. al. [17]. Sekan Doganay, Rahime Alsangur and Alpaslan Turgut [18] show the impact of magnetic field and thermal conductivity of magnetic nano-fluid.

Heat transfer is an essential process involved in many industries and daily life. It is the thermal energy movement between two bodies at different temperature. Energy transfer can occur through three main mechanisms: conduction, convection, and radiation. Understanding how heat is transferred is essential to designing heating and cooling systems and optimizing energy consumption.

Conduction is the transfer of heat through a matter without any movement of the matter itself. It happens when there is a temperature gradient within a fluid or solid. Energy is transferred from a section of higher temperature to a section of less temperature by collisions between neighboring molecules.

Convection is the transfer of heat by the motion of a fluid. It occurs in fluids that are not at rest due to temperature differences. The hot fluid becomes less dense and rises, while the cold fluid descends. This circulation pattern creates convection flow that carries heat from a hotter area to a cooler area. Convection can be natural, such as air in a room, or forced, such as water flowing through a radiator.

Radiation is the transfer of heat by waves of electromagnetic. a medium is not essential and can occur through a vacuum. All objects emit radiation, but the amount and wavelength depend on temperature of object and the properties of its surface. As the objects temperature increases the more radiation it emits, the wavelength of the electromagnetic wave gets shorter. Radiation can be absorbed, transmitted, or reflected by other objects. For example, the sun emits radiation that is absorbed by atmosphere of the Earth, causing the planet to be warmer. Convection has many practical applications in daily life. For example, convection is used in heating systems and cooling systems, where fluids are circulated to transfer heat and from the air.

Moreover, convection is used in many industrial processes, like reactions of chemical, chemical processing, manufacturing, likewise in the process of cooking, sterilizing and preserving food.

Nano-fluids are a relatively new development in heat transfer technology, they provide highly efficient heat transfer and are used in a various of applications such as electronics cooling, automotive cooling, and industrial processes. Yan Cao, Ibrahim B. Mansir et.al. [19] studied the consequence of natural convection on nano-fluid for cooling a hot plat in a square corner. The nano-particle's shape impacts are very exceptional on the thermal conductivity of suspension has been examined by Umair Rashid, Dianchen Lu, Quaid Iqbal [20]. Emad M. Abo-Eldahab, Rasha Adel, Yasmin M. Saad [21] presents the consequence of the natural convection and the thermal radiation on nano-fluid through a stretching plate. The study of natural convection in a differentially-heated enclosure filled with a CuO-EG- water nano-fluid investigated by Eiyad Abu-Nad, and Ali J. Chamkha [22]. Abdellatif Dayf and the other authors [23] shows the impact of natural convection in three-dimensional cubic enclosure of nano-fluid. The influence of natural convection of nanofluid with non-uniform heating of bottom wall has been studied by Nader Ben-Cheikh, Ali J. Chamkha, Brahim Ben-Beya, Taieb Lili [24]. Nandy Putra, Wilfried Roetzel, Sarit Kumar Das [25] studies the impact of natural convection of nano-fluids inside horizontal cylinder heated from one end and cooled from the other. The natural convection and its impact of nano-fluid in square enclosure is presents by C. J. Ho, M. W. Chen, and Z.W. Li [26]. Minh Tuan Nguyen, Abdelraheem M. Aly and Sang-Wook Lee [27] show the consequence of natural convection of a nano-fluid over porous media. Nepal Chandra Roy [28] illustrate the natural convection of a nanofluid among a square enclosure and another geometrical shapes. The influence of natural convection with CuO-water nano-fluid and subjected to a uniform magnetic field is investigated by Ali Chamkha, Muneer Ismael, Abbas Kasaeipoor and Taher Armaghani [29]. Innocent Nkurikiyimfura, Yanmin Wang, Zhidong Pan and Dawie Hu [30] are investigates the enhancement of thermal conductivity of engine oil-based magnetite nano-fluids.

The fundamental partial differential equations are transformed into non-dimensional partial differential equations with the aid of non-dimensional form, the transformed equations will be solved numerically by applying the finite difference (fully implicit method [31]). The results are obtained for magnetic field parameter, wavy surface amplitude, cone half-angle, Prandtl number and viscosity parameter variation and their consequence on velocity, temperature and Nusselt number. Ress, Pop [32], and Moulic, Yao [33] used numerical finite difference method to solve the transformed equations about natural convection in Newtonian fluids.

2. Mathematical Formulation

Consider the flow is steady and viscous incompressible in a two- dimensional laminar of nano-fluid with the impact of both magnetic field and natural convection over vertical wavy cones present in figure (1).

The curvy sheet cone can be illustrated by equation (1):

$$\overset{*}{y} = \sigma(\overset{*}{x}) = \overset{*}{a}\sin(\pi x) \tag{1}$$

Suppose that $\sigma(\overset{*}{x})$ being the arbitrary function, where $\overset{*}{a}$ is curvy plate amplitude of the cone, let T_w is the cone temperature at surface (constant temperature) and T_∞ is temperature placed at the ambient fluid, where $T_w \succ T_\infty$. The viscosity of Nano-fluid depends on its temperature.

Assume there is no body force acting on the system, no Joule heat effect, no heat generations and no viscous dissipation.

The fundamental equations are; equation of continuity, equation of momentum and equation of energy can be described as [21]:

$$\overset{*}{r}\overset{*}{u}_x + \overset{*}{r}\overset{*}{v}_y = 0 \tag{2}$$

$$\overset{*}{u}\overset{*}{u}_x + \overset{*}{v}\overset{*}{u}_y = -\frac{1}{\rho_{nf}}\overset{*}{p}_x + \frac{1}{\rho_{nf}}\nabla\cdot(\mu_{nf}\nabla\overset{*}{u}) - \frac{\sigma_{nf}B_\circ^2}{\rho_{nf}}\overset{*}{u} + g\beta(\overset{*}{T}-T_\infty)COS\phi \tag{3}$$

$$\overset{*}{u}\overset{*}{v}_x + \overset{*}{v}\overset{*}{v}_y = -\frac{1}{\rho_{nf}}\overset{*}{p}_y + \frac{1}{\rho_{nf}}\nabla\cdot\left(\mu_{nf}\nabla\overset{*}{v}\right) - g\beta\left(\overset{*}{T}-T_\infty\right)\sin\phi \tag{4}$$

$$\overset{*}{u}\overset{*}{T}_x + \overset{*}{v}\overset{*}{T}_y = \alpha_{nf}\nabla^2\overset{*}{T} \tag{5}$$

Where x, y are Cartesian axes, x is the horizontal axis and y is axis which perpendicular to the x-axis, $(\overset{*}{u}, \overset{*}{v})$ are the dimensional components of velocity in directions of (x, y) respectively, the subscripts $\overset{*}{u}_x$, $\overset{*}{u}_y$ represents the partial derivative with reference to x and y, g the acceleration of the gravity, ρ_{nf} the density of the Non-fluid, $\overset{*}{p}$ the dimensional pressure, β the coefficient of thermal expansion, α_{nf} the thermal diffusivity of the Non-fluid, μ_{nf} the viscosity of the Non-fluid depended on the temperature, σ_{nf}, B_o are the conductivity of Nano-fluid electrical and magnetic conduction respectively, ϕ is the cone half-angle, the subscript x, y means the derivative with reference to x and y. and $\overset{*}{r}$ is the local radius of the flat surface of the cone which is prescribed:

$$\overset{*}{r} = x\sin\phi \tag{6}$$

There are some relationships between fluid, Nano-fluid and solid that have been studied which are:

$$\alpha_{nf} = \frac{k_{nf}}{\left(\rho c_p\right)_{nf}}$$

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s$$

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\varphi)^{2.5}}$$

$$\left(\rho c_p\right)_{nf} = (1-\varphi)\left(\rho c_p\right)_f + \varphi\left(\rho c_p\right)_s$$

Where φ is the Nano-particle volume fraction, $\left(\rho c_p\right)_{nf}$ is capacity of heat of Nano-fluid. The subscript (nf) means of Nano-fluid, (f) means of fluid and (s) means of solid.

The boundary conditions of fundamental equations (2) – (5) are:

$$\left.\begin{array}{l} y = \overset{*}{\sigma}(x)\ ,\ \overset{*}{u} = 0\ ,\ \overset{*}{v} = 0\ ,\ \overset{*}{T} = T_w \\ y \to \infty\ ,\ \overset{*}{u} = 0\ ,\ \overset{*}{T} = T_\infty\ ,\ \overset{*}{p} = p_\infty \\ x = 0\ ,\ \overset{*}{u} = U_\infty\ ,\ \overset{*}{T} = T_\infty \end{array}\right\} \quad (7)$$

The non- dimensional form of boundary layer is:

$$\tilde{x} = \frac{x}{L},\ \tilde{y} = \frac{y-\tilde{\sigma}}{L}Gr^{1/4}\ ,\ \tilde{r} = \frac{\overset{*}{r}}{L}\ ,\ \tilde{p} = \frac{L^2}{\rho_{nf}\upsilon_\infty^2}Gr^{-1}\overset{*}{p}\ ,$$

$$\tilde{u} = \frac{\rho_{nf}L}{\mu_\infty}Gr^{-1/2}\overset{*}{u}\ ,\ \tilde{v} = \frac{\rho_{nf}L}{\mu_\infty}Gr^{-1/4}\left(\overset{*}{v} - \tilde{\sigma}_x\overset{*}{u}\right)\ ,\ \theta = \frac{\overset{*}{T}-T_\infty}{T_w - T_\infty}\ ,\ \tilde{a} = \frac{\overset{*}{a}}{L}$$

$$,\ \tilde{\sigma}(\tilde{x}) = \frac{\overset{*}{\sigma}(x)}{L}\ ,\ \tilde{\sigma}_x = \frac{d\overset{*}{\sigma}}{dx} = \frac{d\tilde{\sigma}}{d\tilde{x}}\ ,\ Gr = \frac{g\beta(T_w - T_\infty)\cos\phi L^3}{\upsilon_\infty^2} \quad (8)$$

Where L is the distinctive length related with the curvy surface of cone, Gr the number of Grashof, $\upsilon_\infty = (\mu_\infty / \rho_{nf})$ is the kinematics viscosity, θ is the non-dimensional temperature $\tilde{x}$, $\tilde{y}$ are non-dimensional horizontal axes, and $\tilde{y}$ is axis which perpendicular to the $\tilde{x}$ -axis $\tilde{u}$, $\tilde{v}$ are the non-dimensional velocities along $\tilde{x}$, $\tilde{y}$ axes respectively, and μ_∞ is the viscosity of ambient Nano-fluid. By applying the non-dimensional transformation (8) into equations (2)-(5) and ignoring the terms of small order in Grashof number Gr. The basic equations turn into:

$$\tilde{r}\tilde{u}_{\tilde{x}} + \tilde{r}\tilde{v}_{\tilde{y}} = 0 \quad (9)$$

$$\tilde{u}\tilde{u}_{\tilde{x}} + \tilde{v}\tilde{u}_{\tilde{y}} = -\tilde{p}_{\tilde{x}} + Gr^{1/4}\tilde{\sigma}_x\tilde{p}_{\tilde{y}} + \frac{(1+\tilde{\sigma}_x{}^2)}{1+\varepsilon\theta}\tilde{u}_{\tilde{y}\tilde{y}} - \frac{\varepsilon(1+\tilde{\sigma}_x{}^2)}{(1+\varepsilon\theta)^2}\tilde{u}_{\tilde{y}}\theta_{\tilde{y}} + \theta - M\tilde{u} \quad (10)$$

$$\tilde{\sigma}_x(\tilde{u}\frac{\partial\tilde{u}}{\partial\tilde{x}} + \tilde{v}\frac{\partial\tilde{u}}{\partial\tilde{y}}) + \tilde{\sigma}_{xx}\tilde{u}^2 = -Gr^{1/4}\tilde{p}_{\tilde{y}} + \frac{\tilde{\sigma}_x(1+\tilde{\sigma}_x{}^2)}{1+\varepsilon\theta}\tilde{u}_{\tilde{y}\tilde{y}} - \frac{\varepsilon\tilde{\sigma}_x(1+\tilde{\sigma}_x{}^2)}{(1+\varepsilon\theta)^2}\tilde{u}_{\tilde{y}}\theta_{\tilde{y}} - \theta\tan\phi \quad (11)$$

$$\tilde{u}\theta_{\tilde{x}} + \tilde{v}\theta_{\tilde{y}} = \frac{1}{\Pr}(1+\tilde{\sigma}_x^2)\theta_{\tilde{y}\tilde{y}} \quad (12)$$

Assume that

$$\mu = \frac{\mu_\infty}{1+\gamma(\overset{*}{T} - T_\infty)} \quad (13)$$

Where γ a constant, Pr is Prandtl number $\Pr = \frac{\upsilon_\infty}{\alpha_{nf}}$ and $M = \frac{\sigma_{nf}B_o{}^2L^2}{\rho_{nf}\upsilon_\infty Gr^{1/2}}$ is the parameter of magnetic field.

Consider $\varepsilon = \gamma(T_w - T_\infty)$ (14)

Where ε is the parameter of viscosity variation. From equation (13) It is obtained that the non-dimensional viscosity μ_{nf}/μ_∞ lies between $1/(1+\varepsilon)$ and 1, its value decreases by increasing the temperature when $\varepsilon \succ 0$. Equation (10) refers to the gradient of pressure along $\tilde{y}$ direction is $O(Gr^{1/4})$ which implies that the lowest order pressure gradient toward $\tilde{x}$ direction can be resolved from the solution of inviscid flow. Where the gradient of pressure is equal to zero while there is no external induced free stream. By eliminating $\tilde{p}_{\tilde{y}}$ from both equations (10) and (11) then we have:

$$\tilde{u}\tilde{u}_{\tilde{x}} + \tilde{v}\tilde{u}_{\tilde{y}} = \frac{(1+\tilde{\sigma}_x{}^2)}{1+\varepsilon\theta}\tilde{u}_{\tilde{y}\tilde{y}} - \frac{\varepsilon(1+\tilde{\sigma}_x{}^2)}{(1+\varepsilon\theta)^2}\tilde{u}_{\tilde{y}}\theta_{\tilde{y}} + \left(\frac{1-\tilde{\sigma}_x \tan\phi}{1+\tilde{\sigma}_x{}^2}\right)\theta - \left(\frac{M}{1+\tilde{\sigma}_x{}^2}\right)\tilde{u} - \frac{\tilde{\sigma}_{xx}\tilde{\sigma}_x}{1+\tilde{\sigma}_x{}^2}\tilde{u}^2 \tag{15}$$

The basic equations boundary layer will be:

$$\tilde{r}\tilde{u}_{\tilde{x}} + \tilde{r}\tilde{v}_{\tilde{y}} = 0 \tag{16}$$

$$\tilde{u}\tilde{u}_{\tilde{x}} + \tilde{v}\tilde{u}_{\tilde{y}} = \frac{(1+\tilde{\sigma}_x{}^2)}{1+\varepsilon\theta}\tilde{u}_{\tilde{y}\tilde{y}} - \frac{\varepsilon(1+\tilde{\sigma}_x{}^2)}{(1+\varepsilon\theta)^2}\tilde{u}_{\tilde{y}}\theta_{\tilde{y}} + \left(\frac{1-\tilde{\sigma}_x \tan\phi}{1+\tilde{\sigma}_x{}^2}\right)\theta - \left(\frac{M}{1+\tilde{\sigma}_x{}^2}\right)\tilde{u} - \frac{\tilde{\sigma}_{xx}\tilde{\sigma}_x}{1+\tilde{\sigma}_x{}^2}\tilde{u}^2 \tag{17}$$

$$\tilde{u}\theta_{\tilde{x}} + \tilde{v}\theta_{\tilde{y}} = \frac{1}{\Pr}(1+\tilde{\sigma}_x^2)\theta_{\tilde{y}\tilde{y}} \tag{18}$$

Applying the non-dimensional transformation (8) into the boundary conditions (7) then it will be in the form:

$$\tilde{y}=0\ ,\ \tilde{u}=0\ ,\ \tilde{v}=0\ ,\ \theta=1$$

$$\tilde{y}\to\infty\ ,\ \tilde{u}=0\ ,\ \theta=0 \tag{19}$$

$$\tilde{x}=0\ ,\ \tilde{u}=U_\infty\ ,\ \theta=0$$

The physical quantity interest is the number of local Nusselt Nu, which is in this form:

$$Nu\left(\frac{Gr}{\tilde{x}}\right)^{-1/4} = -(1+\tilde{\sigma}_x)^{1/2}\theta_{\tilde{y}} \tag{20}$$

3. Numerical Method

The fundamental equations (16) -(18), the boundary conditions (19) and the number of local Nusselt (20) were solved numerically by finite difference method (fully implicit method) [31].

In this procedure, the derivative with reference to $\tilde{x}$, $\tilde{y}$ is approximated by using central difference. where θ is the factor of weighting. suppose that the value of θ = 0.9, H is the difference of the temperature (in this part).

The explicit method is used for boundary layers in this research. Considering two-dimensional laminar incompressible flow with transfer of heat. The basic equations in partial form are given as:

For example, the momentum equation of u is

$$\frac{\theta u^{n+1}{}_{i,j}\left(u^{n+1}{}_{i+1,j} - u^{n+1}{}_{i-1,j}\right) + (1-\theta)u^{n}{}_{i,j}\left(u^{n}{}_{i+1,j} - u^{n}{}_{i-1,j}\right)}{2\Delta\tilde{x}}$$

$$+\frac{\theta v^{n}{}_{i,j}\left(u^{n+1}{}_{i,j+1} - u^{n+1}{}_{i,j-1}\right) + (1-\theta)v^{n}{}_{i,j}\left(u^{n}{}_{i,j+1} - u^{n}{}_{i,j-1}\right)}{2\Delta\tilde{y}}$$

$$= \frac{1}{\Delta\tilde{y}^2}\left[\begin{array}{l} \theta\left[\frac{(\sigma^{*}{}_x{}^2+1)}{1+\zeta H^n{}_{i,j}}\left(u^{n+1}{}_{i,j+1} - u^{n+1}{}_{i,j}\right) - \frac{(\sigma^{*}{}_x{}^2+1)}{1+\zeta H^n{}_{i,j}}\left(u^{n+1}{}_{i,j} - u^{n+1}{}_{i,j-1}\right)\right] + \\ (1-\theta)\left[\frac{(\sigma^{*}{}_x{}^2+1)}{1+\zeta H^n{}_{i,j}}\left(u^{n}{}_{i,j+1} - u^{n}{}_{i,j}\right) - \frac{(\sigma^{*}{}_x{}^2+1)}{1+\zeta H^n{}_{i,j}}\left(u^{n}{}_{i,j} - u^{n}{}_{i,j-1}\right)\right] \end{array}\right]$$

$$-\frac{\sigma^{*}{}_x\sigma^{*}{}_{xx}}{1+\sigma^{*}{}_x{}^2}\left(u^n{}_{i,j}\right)^2 - \frac{M^{\otimes}}{1+\sigma^{*}{}_x{}^2}\left(u^n{}_{i,j}\right) + \left(\frac{1-\sigma^{*}{}_x\tan\varphi}{1+\sigma^{*}{}_x{}^2}\right)H^n{}_{i,j}$$

$$-\zeta\frac{\left(1+\sigma^{*2}{}_x\right)}{\left(1+\zeta H^n{}_{i,j}\right)^2}\frac{\left(H^n{}_{i,j+1} - H^n{}_{i,j-1}\right)}{\Delta\tilde{y}}\left[\begin{array}{l}\frac{\theta\left(u^{n+1}{}_{i,j+1} - u^{n+1}{}_{i,j-1}\right)}{2\Delta\tilde{y}} + \\ \frac{(1-\theta)\left(u^{n}{}_{i,j+1} - u^{n}{}_{i,j-1}\right)}{2\Delta\tilde{y}}\end{array}\right]$$

If θ > 0 indicates that the scheme becomes implicit and inherently stable if θ ≥ ½. values of θ between ½ and 1 have been used successfully. This equation will be written in form:

$$A_{i,j}u^{n+1}{}_{i-1,j} + B_{i,j}u^{n+1}{}_{i,j-1} + C_{i,j}u^{n+1}{}_{i,j} + D_{i,j}u^{n+1}{}_{i+1,j} + E_{i,j}u^{n+1}{}_{i,j+1}$$

$$= F_{i,j}u^{n}{}_{i-1,j} + G_{i,j}u^{n}{}_{i,j-1} + K_{i,j}u^{n}{}_{i,j} + Q_{i,j}u^{n}{}_{i+1,j} + R_{i,j}u^{n}{}_{i,j+1} + O_{i,j}$$

Where

$$A_{i,j}=\frac{-\theta u^{n+1}{}_{i,j}}{2(\Delta\tilde{x})}$$

$$B_{i,j}=\frac{-\theta v^{n}{}_{i,j}}{2(\Delta\tilde{y})}-\frac{\theta\left(\sigma^{*}{}_{x}{}^{2}+1\right)}{\left(1+\zeta H^{n}{}_{i,j}\right)\Delta\tilde{y}^{2}}$$
$$-\frac{\theta\zeta\left(1+\sigma^{*}{}_{x}{}^{2}\right)H^{n}{}_{i,j+1}}{2(\Delta\tilde{y})^{2}\left(1+\zeta H^{n}{}_{i,j}\right)^{2}}$$

$$+\frac{\theta\zeta\left(1+\sigma^{*}{}_{x}{}^{2}\right)H^{n}{}_{i,j-1}}{2(\Delta\tilde{y})^{2}\left(1+\zeta H^{n}{}_{i,j}\right)^{2}}$$

$$C_{i,j}=\frac{2\theta\left(\sigma^{*}{}_{x}{}^{2}+1\right)}{\left(1+\zeta H^{n}{}_{i,j}\right)(\Delta\tilde{y})^{2}}$$

$$D_{i,j}=\frac{\theta u^{n+1}{}_{i,j}}{2(\Delta\tilde{x})}$$

$$E_{i,j}=\frac{\theta v^{n}{}_{i,j}}{2(\Delta\tilde{y})}-\frac{\theta\left(\sigma^{*}{}_{x}{}^{2}+1\right)}{\left(1+\zeta H^{n}{}_{i,j}\right)(\Delta\tilde{y})^{2}}$$
$$+\frac{\theta\zeta\left(1+\sigma^{*}{}_{x}{}^{2}\right)H^{n}{}_{i,j+1}}{2(\Delta\tilde{y})^{2}\left(1+\zeta H^{n}{}_{i,j}\right)^{2}}$$

$$-\frac{\theta\zeta\left(1+\sigma^{*}{}_{x}{}^{2}\right)H^{n}{}_{i,j-1}}{2(\Delta\tilde{y})^{2}\left(1+\zeta H^{n}{}_{i,j}\right)^{2}}$$

$$F_{i,j}=\frac{(1-\theta)u^{n}{}_{i,j}}{2(\Delta\tilde{x})}$$

$$G_{i,j}=\frac{(1-\theta)v^{n}{}_{i,j}}{2(\Delta\tilde{y})}+\frac{(1-\theta)\left(\sigma^{*}{}_{x}{}^{2}+1\right)}{\left(1+\zeta H^{n}{}_{i,j}\right)(\Delta\tilde{y})^{2}}$$
$$+\frac{(1-\theta)\zeta\left(\sigma^{*}{}_{x}{}^{2}+1\right)H^{n}{}_{i,j+1}}{\left(1+\zeta H^{n}{}_{i,j}\right)^{2}2(\Delta\tilde{y})^{2}}$$

$$+\frac{(\theta-1)\zeta\left(\sigma^{*}{}_{x}{}^{2}+1\right)H^{n}{}_{i,j-1}}{\left(1+\zeta H^{n}{}_{i,j}\right)^{2}2(\Delta\tilde{y})^{2}}$$

$$K_{i,j}=\frac{2(\theta-1)\left(\sigma^{*}{}_{x}{}^{2}+1\right)}{\left(1+\zeta H^{n}{}_{i,j}\right)(\Delta\tilde{y})^{2}}$$
$$-\frac{\left(\sigma^{*}{}_{x}\right)\left(\sigma^{*}{}_{xx}\right)}{1+\sigma^{*}{}_{x}{}^{2}}u^{n}{}_{i,j}-\frac{M}{1+\sigma^{*}{}_{x}{}^{2}}$$

$$Q_{i,j}=\frac{(\theta-1)u^{n}{}_{i,j}}{2(\Delta\tilde{x})}$$

$$R_{i,j}=\frac{(\theta-1)v^{n}{}_{i,j}}{2(\Delta\tilde{y})}+\frac{(1-\theta)\left(\sigma^{*}{}_{x}{}^{2}+1\right)}{\left(1+\zeta H^{n}{}_{i,j}\right)(\Delta\tilde{y})^{2}}+\frac{(\theta-1)\zeta\left(\sigma^{*}{}_{x}{}^{2}+1\right)H^{n}{}_{i,j+1}}{\left(1+\zeta H^{n}{}_{i,j}\right)^{2}2(\Delta\tilde{y})^{2}}+$$

$$+\frac{(1-\theta)\zeta\left(\sigma^{*}{}_{x}{}^{2}+1\right)H^{n}{}_{i,j-1}}{\left(1+\zeta H^{n}{}_{i,j}\right)^{2}2(\Delta\tilde{y})^{2}}$$

$$O_{i,j}=\frac{1-\sigma^{*}{}_{x}\tan\varphi}{\left(1+\sigma^{*}{}_{x}{}^{2}\right)}H^{n}{}_{i,j}$$

4. Result and Discussion

The fundamental equations (16)-(18) and the boundary conditions (19) were solved numerically by finite difference fully implicit method [31]. The derivatives of velocities over the reference $\tilde{x}$ and $\tilde{y}$ are matched by the central difference method. Many numerical results were obtained throughout the course of this research and representative set of graphical results for the profile of velocity u^{*} and profile of temperature θ, in addition, Nusselt number Nu is presented. Results are given for the parameter of variable viscosity ε =0 ,0.5 ,1.0 and 2.0; the amplitude parameter a^{*} = 0, 0.2 ,0.4 ,0.5 and 1.0; Prandtl number Pr = 0.1 ,0.7 ,1.0 and 7.0; cone half angle ϕ = $0^{\circ}, 30^{\circ}$ and 45°, and the parameter of Magnetic M = 0 , 0.5, 1.0, 2.0 and 5.0 . Figure (2) demonstrates that the influence of parameter of magnetic M on the velocity viewing the other aspects while the parameter of magnetic M increases, the velocity reduces depending on Lorentz force. Figure (3) illustrates the profile of temperature with magnetic field parameter M, the increase of the magnetic field, tends to increase temperature, the increased temperature is due to the generation of joule heating, accordingly the interaction between the nano-fluid and the magnetic field. Figures (4) explain that Prandtl number influence on the temperature profile, changing the amount of Pr will not affect the fluid velocity (so, there is no figure showing the profiles of velocity) from this Figure, we can deduce that the increasing of number of Prandtl Pr tends to decrease the temperature profiles due to the reduced thermal diffusivity and a thinner thermal boundary layer. Figure (5) illustrates that the profile of velocity increases with the rise of cone half-angle ϕ . Figure (6) presents the influence of cone half-angle on profiles of temperature; obviously that as ϕ intensity temperature is reduced. Figure (7) demonstrates that the profile of velocity increases when the wave amplitude a increases. Figure (8) introduces the temperature profile which decreases owing to the rise of the amplitude of wave a , as a result of the enhanced mixing and convection it induces. This results in a more uniform distribution of heat Figure (9) establishes that the profile of velocity increases leads to the throughout the fluid. increase of the viscosity parameter ε as a result of the shear thinning fluid behavior and the reduction in resistance to flow. Figure (10) state the influence of temperature profiles with the viscosity parameter, the rise of the parameter of viscosity ε tends to reduce the temperature. Figure (11) states that the influence of cone half-angle ϕ on the number of local Nusselt, obviously that as ϕ increases the number of local Nusselt increases. Figure (12) illustrates that the influence of parameter of viscosity ε on the number of local Nusselt and it is obvious that Nusselt number rise as ε increases. As expected, the effect is more pronounced for larger full cone angles. Figure (13) demonstrates that raise of the field of magnetic leads to the reduce of local Nusselt number. Figure (14) shows the relation between the influence of wave amplitude on the number of local Nusselt, the increase of wave amplitude a tends to the increase of Nusselt number Nu.

5. Conclusions

This research demonstrates the importance of using the field of magnetic on a conical wavy surface in two of most important fields related to human life, namely the field of medicine, where is interventional radiology and the treatment of cancerous tumors and various diseases are concerned, and the health field and agricultural land reclamation, which is water treatment.

The fundamental equations are turned into non-dimensional equations by applying dimensionless form, these transforming equations are solved numerically by implicit finite-difference (fully implicit) method. The dimensionless form transformed the complex geometry into a simple shape. Finally, we will mention crucial results obtained. Firstly, it has been observed that the raising of parameter of magnetic M increases the temperature, decreases the velocity and decreases the Nusselt number. Secondly, it was also noticed that the viscosity parameter raises ε due to the rise of the velocity, number of Nusselt and the reduction of the temperature. Thirdly, the detailed results for the profile of velocity and the profile of temperature are presented and also the results for Nusselt number. These results are given for distinguishable values of the curvy surface amplitude a , parameter of magnetic, Prandtl number Pr, cone half-angle ϕ and viscosity parameter ε.

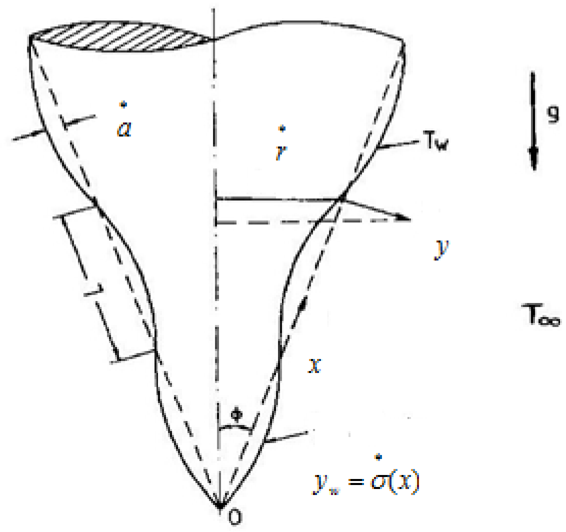

Figure 1. The Physical form and reference frame.

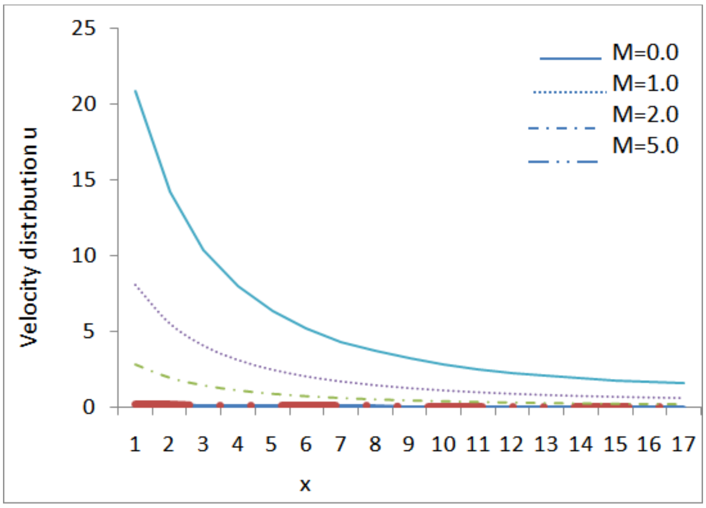

Figure 2. velocity u for distinct value of magnetic parameter

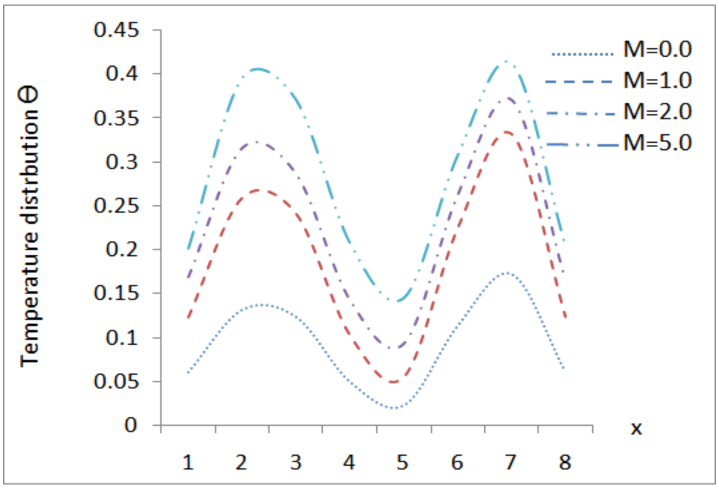

Figure 3. Temperature θ for distinct value of magnetic parameter

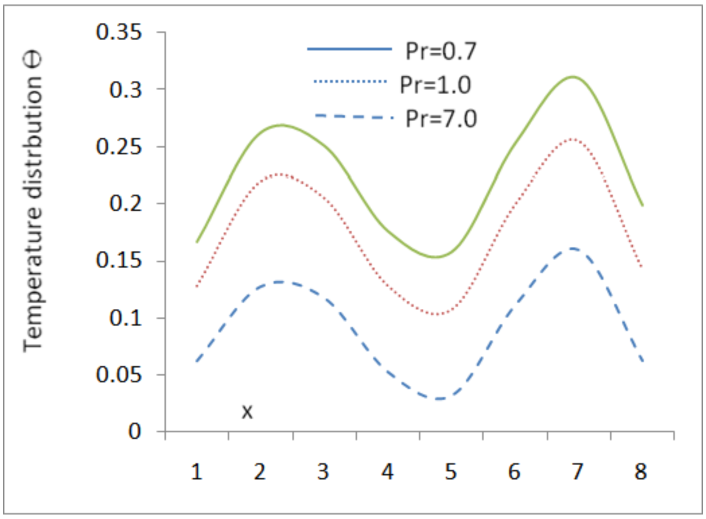

Figure 4. Temperature θ for distinct value of Prandtl number

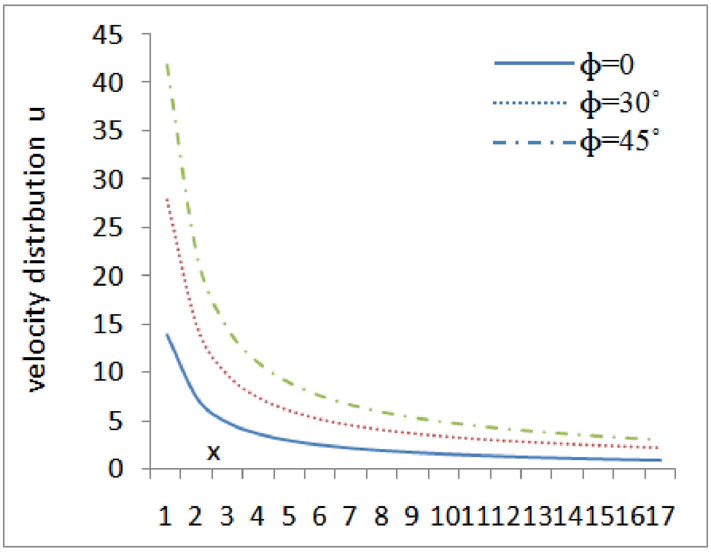

Figure 5. velocity u for distinct value of cone half –angle

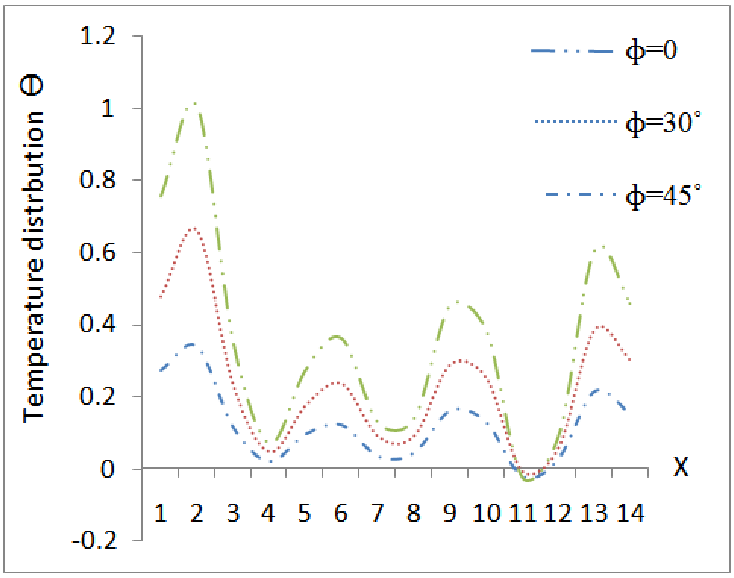

Figure 6. Temperature θ for distinct value of cone half –angle

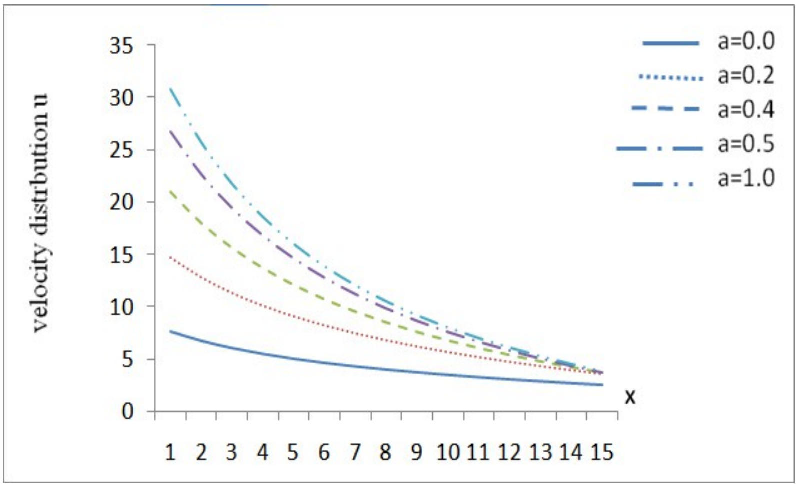

Figure 7. velocity u for distinct wavy surface amplitudes

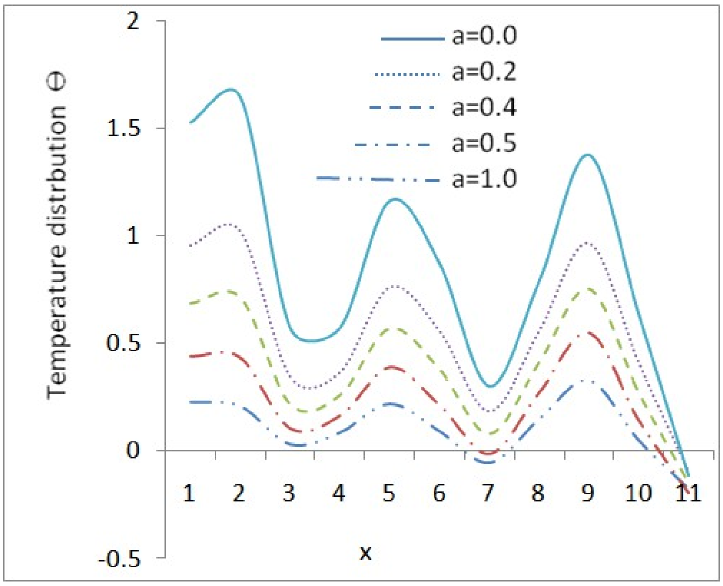

Figure 8. Temperature θ for distinct wavy surface amplitudes

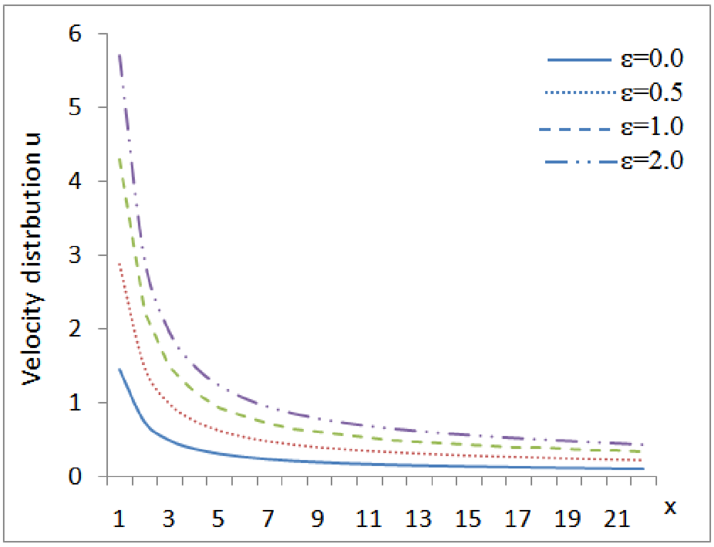

Figure 9. velocity u for distinct value of viscosity

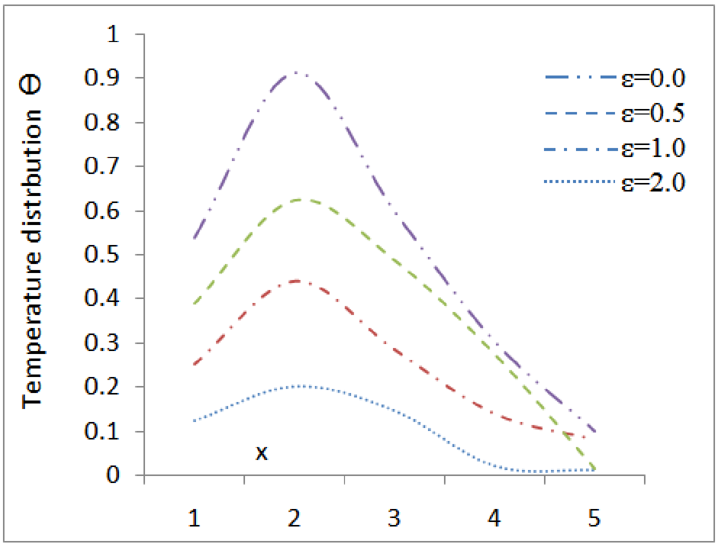

Figure 10. Temperature θ for distinct value of viscosity

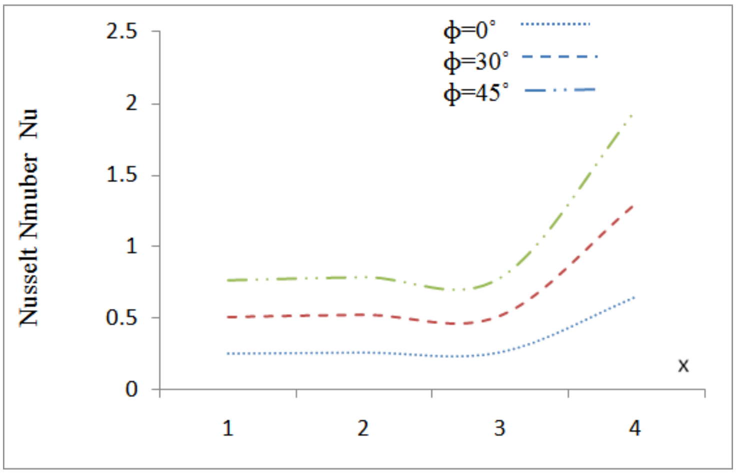

Figure 11. Nusselt number Nu for distinct value of cone half –angle

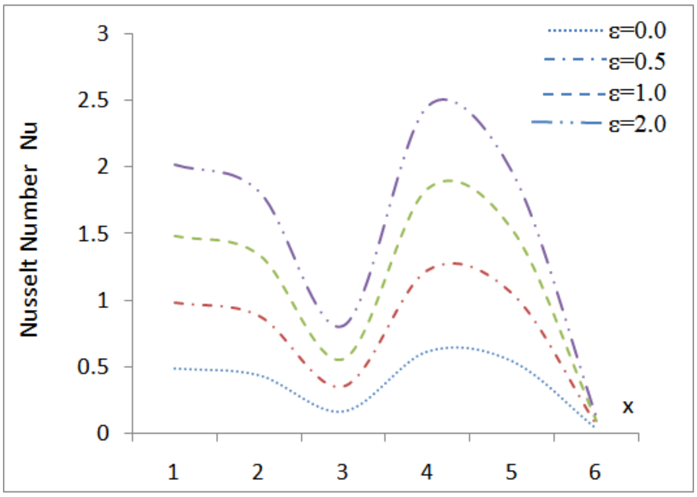

Figure 12. Nusselt number Nu for distinct value of viscosity

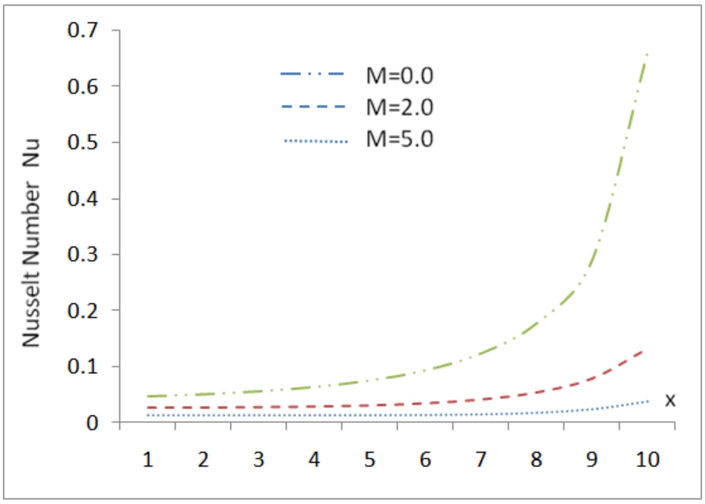

Figure 13. Nusselt number Nu for distinct value of magnetic field

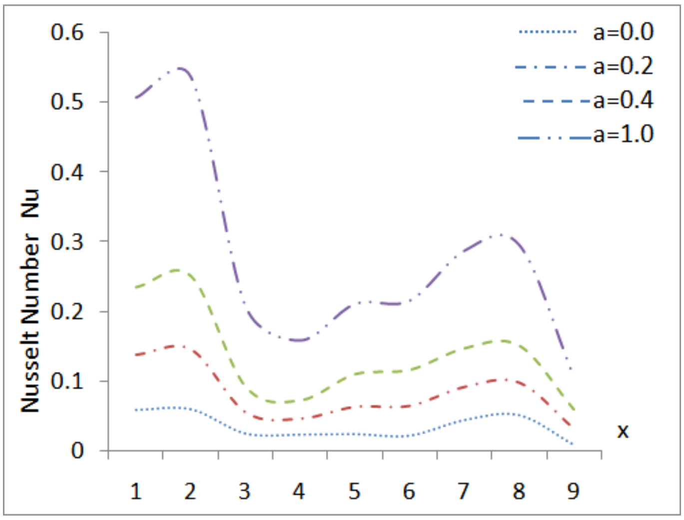

Figure 14. The number of Nusselt toward the various wavy surface amplitude

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