

Universe, a Singular Quantum State and It's Expansion

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Abstract Quantum cosmology has a quantum approach toward understanding of the universe. Physicists have proposed various approach toward understanding Universe as a quantum state. In this paper the intention is to introduce the Big Bang as a simplified “singular quantum state” with internal operators. This singular quantum state by virtue of quantum mechanical operators, produce the first physical “spacetime particle,” being physical universe at the moment of its inception. Energy and “duplication” operators energize and multiply spacetime particles and hence expansion of the universe. The “charge-mass’ operators convert the energized spacetime particles into massive fermions in different epochs. Spacetime particles are the building block of not only the universe, but also contain masses and radiation within the universe in the form of fundamental particles. The charge-mass operator mixing will lead to the existence of quark’s partner and anti-partner. It will also lead to the existence of not-matter particles. According to this theory, there arise four versions of matter, namely, Matter (particle, particle partner, anti-particle and anti-particle partner) accompanied with the analog of these particles in the Not-matter version. In this paper, not a rigorous mathematical, rather, a simple intuitive approach is used toward understanding the nature of the universe and the fundamental particles generated.

Keywords: *Big Bang is introduced as a simple quantum state. Internal space and time operators change this state into spacetime particle. Energy and multiplication operators, energies and expand the spacetime particle. Charge-mass operators convert some of the internal energy of the spacetime particles into massive fermions. The charge-mass operators lead to the existence of massive fermion particles, particle partners, anti-particles and anti-particle partners and not-massive version of these. There is a proposal that quantum state of the universe prior to expansion is a six dimensional space-time. The spacetime particle quantization is estimated by Planck’s natural constants*

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1. Introduction

The Universe we understand so far is predominantly a classical one, however, S.W. Hawking [1] was one of physicists who suggested a quantum approach toward understanding of it. In his paper, quantum state of the universe is determined by metric and matter fields that are summed over in path integral. In a mini-super-space model an asymptotic boundary condition will lead to the wave function related to the observed universe. Considering Quantum Cosmology, Don N. Page [2] uses FRW-scalar model to enhance the no-boundary (Hawking’s approach) wave function of the universe while incorporating zero-loop approximation for the action. Also, James B. Hartle [3] investigate the central question of quantum cosmology while suggesting two necessary condition approach toward understanding the universe, namely Hawking’s state of universe and superstring theory.

Classically, however, universe at the moment of Big Bang is assumed to be a singularity with zero volume and infinite energy density [4]. It is rather possible that in the beginning universe was a “singular quantum state” with

“internal operators” being activated. One can actually assign a planewave-like state for the beginning of the universe while operators being activated on this state. The activation of the energy and “duplication operators” will lead to the energization and multiplication of the so called “spacetime particles,” and hence the expansion of the universe. Following this, “Charge-mass” operators will tunnel-in the internal energy of the spacetime particles into charge and mass eigenvalues, thereby generating various fundamental fermion particles at different epochs. The charge-mass mixing will lead to the understanding that a u quark, and anti-u quark each have partners that we have neglected. Namely, a u (color, +2/3) quark will also have a partner u’ (color, -2/3). This will lead to the postulation of existence of particle, anti-particle, particle partner and anti-particle partner. Further, charge-mass mixing will lead to the existence of not-matter particles that we shall explore their nature in the following. A photon is simply an energized spacetime particle that cannot remain localized. Due to spacetime particle’s overlapping wave functions, the energy from one spacetime particle will be transferred to the next and thereby “photon vibration” will be propagated. A massive particle is a localized spacetime particle in which the charge-mass operators have converted the photonic energy

into “mass and charge eigen values,” where they are different form of vibrational energies. As a consequence of charge-mass mixing, universe contain four types of matter particles; matter (particle; P , anti-particle; $\bar{P}$, particle partner; p' , anti-particle partner; $\bar{P}'$) and also the not-matter analog of these which we shall intent to investigate in detail.

2. Concept of Classical Quantization of Space and Time

Nearly every constant in nature is an aperture to see fascinating mysteries of the universe and therefore, further enhancement of the knowledge. The three constants c , G , h , namely, the propagation of light in space, gravitational constant and Planck's constant are fundamental constants in physics which together hold a key to the further understanding of physics of the universe.

It is not coincidental that Planck's length, $L_{Pl} = \left(\frac{G\hbar}{C^3}\right)^{1/2} \cong 1.61 \times 10^{-35}m$ [5], is where the concept of length could have a physical meaning. One also could consider the Planck's time, $T_{Pl} = \left(\frac{G\hbar}{C^5}\right)^{1/2} \cong 5.39 \times 10^{-44}s$, in which a time increment smaller than this will become a non-physical concept. It is also not coincidental that the ratio of these two constants, $\frac{L_{Pl}}{T_{Pl}} = C$, is the speed of light! A photon traveling in space (and time) with speed c , will propagate through increments of space and time.

We can also recall Planck's mass, $M_{Pl} = \left(\frac{C\hbar}{G}\right)^{1/2} \cong 2.1 \times 10^{-8}kg$ and the energy associated with it as some fundamental form of energy that we shall precisely elaborate on that. We should also notice that the Planck's length and time relation,

$$L_{Pl} = CT_{Pl} \quad (1a)$$

are bind by c . Therefore, one can consider both Planck's length and time as natural quantization of space and time. If indeed, we consider space and time quantization, Planck's natural constants can be expressed as, $\sim 10^{-35}m$ / space quantization ,

and $\sim 10^{-44}s$ / time quantization . In other words

$$L_{Pl} \sim 10^{-35}m / q_s \quad , \quad T_{Pl} \sim 10^{-44}s / q_t \quad (1b)$$

and the reciprocal

$$\sim 10^{35} q_s/m \quad \sim 10^{44} q_t/s \quad (1c)$$

There are approximately 10^{35} space quantization per meter and 10^{44} time quantization per second. We should notice that Planck's natural constants are not relativistic, rather space quantization is the average length of a three-dimensional space. Nature reveals itself in a rather simple, non-relativistic manner through Planck's natural constants.

We shall now proceed by the uncertainty principle and the theory of relativity (relativistic interval) to further understand and solidify this classical quantum concept of space and time. Let us recall Heisenberg uncertainty relation for both momentum and energy, respectively

$$\Delta x \Delta p \sim \hbar \quad \text{and} \quad \Delta t \Delta E \sim \hbar. \quad [6] \quad (2a)$$

Dividing the two approximate equations above, will give

$$\frac{\Delta x}{\Delta t} \sim c \quad (2b)$$

At the very core of the quantum mechanics, namely uncertainty principle, nature reveals that the ratio in uncertainty in displacement to the uncertainty in time is related by the relativistic constant of nature, c . In other words, nature's fundamental uncertainty behavior will lead to the constraint of Eqn.(2b). Planck's relation in space and time in Eqn.(1a), and the uncertainty relation in space and time, Eq.(2b), express same physics but from two different point of views.

$$\frac{L_{Pl}}{T_{Pl}} = C \quad \frac{\Delta x}{\Delta t} \sim c \quad (3)$$

Lastly, let us recall relativistic interval in a light-like (null) situation

$$c^2 dt^2 - dl^2 = 0 \quad \text{or} \quad \frac{dl}{dt} = c. \quad (4)$$

The relativistic consequence of spacetime relation although expresses similar physics as the two other cases, it has a subtle point which we shall elaborate in detail later on. However, briefly, what it means is that the speed of propagation of photon (non-massive) particle in an empty space is constant, c . In a light-like situation, the non-massive particle travels on the boarder of the relativistic conic shape, where we refer to as “fabric of spacetime, FST.” Incorporating equations in (3) and (4), namely, consequence of Planck's natural constants, uncertainty relation and special relativity, will give us more insight and better understanding of $[L_{Pl}, T_{Pl}; c]$.

Space and time are quantized, and in their quantization, they are bind together by constant, c . This quantization and its bind by c , happens in what we call FST, where mass does not exist. Fabric of spacetime can be characterized as universe right before expansion with a null (empty) relativistic interval, where only energy propagation occurs. On the other hand, the time-like relativistic interval is the spacetime where mass will propagate (expanded universe). We shall investigate all these in much greater detail as we go along.

3. Universe Prior to Relativistic 4-d Spacetime

Let us recall relativistic four-position and the interval square

$$dx^\mu = \left(c dt, \sum_{i=1}^3 d\vec{x}^i\right) \quad \text{and} \quad ds^2 = c^2 dt^2 - dl^2. \quad [7,13] \quad (5)$$

Notice that there are three components of space and one

component of time. Why shouldn't universe be symmetrical in space and time. That is to say, what if universe actually started symmetrical, then it evolved into a non-symmetrical spacetime. According to Eqns.(5), the four-velocity will be

$$\frac{dx^\mu}{d\tau} = \gamma \frac{d}{dt} (c dt, \sum_{i=1}^3 d\bar{x}^i) = \gamma(c, \bar{v}^i) \quad (6a)$$

and the invariant 4-velocity square

$$u^\mu u_\mu = \gamma^2 (c^2 - v^2) = c^2 \quad (6b)$$

Now, assuming an equal footing for a symmetrical relativistic spacetime position and interval square, then we will have

$$dx^\mu = \sum_{i=1}^3 (c dt^i, d\bar{x}^i), \quad ds^2 = [c^2 (dt_1^2 + dt_2^2 + dt_3^2) - (d\bar{x}_1^2 + d\bar{x}_2^2 + d\bar{x}_3^2)] = c^2 dt^2 - d\bar{l}^2 \quad (7)$$

We realize that the symmetrical and non-symmetrical spacetime produce similar relativistic intervals. However, the symmetrical relativistic velocity will be

$$\frac{dx^\mu}{d\tau} = \sum_{i=1}^3 \gamma_i (c, \bar{v}^i) \quad (8)$$

Eqn.(8) generates three distinct relativistic factors by the virtue of having equal footing for both space and time. On the other hand, Eqn.(6a) incorporates all relativistic factors into one γ

$$\sum_{i=1}^3 \gamma_i \rightarrow \gamma \quad (9)$$

In a symmetrical spacetime, all three components of spacetime (3 x space & 3 x time) are independent from one another (orthogonal) in which, will make it nonrealistic. However, fortunately in the non-symmetrical spacetime (our observing universe) the space and time components bind together (through c) by having a mutual gamma which in essence is the incorporation of all gammas in the symmetrical spacetime. This could happen only if an observer measures one common t , with three orthogonal spaces in which it can simultaneously measure the velocity in each orthogonal direction. In other words, it is the consequence of an observer that a symmetrical spacetime will collapse into a non-symmetrical Einstein's spacetime. We have to notice that even in the case of theory of special relativity, the γ factor is a consequence of an observer's measurement! Therefore,

$$\sum_{i=1}^3 \gamma_i \xrightarrow{C, \text{Observer}} \gamma. \quad (10)$$

In this case the observer factor itself as we shall see, is the consequence of expansion and energization of "spacetime particle." We thus, suggest that universe prior to its expansion (a singular quantum state) had a symmetrical spacetime, where after expansion (duplication and energization of the spacetime particle) it would have collapsed into a non-symmetrical expanded "observing" spacetime.

4. Singular Quantum State of the Universe

In the beginning of the creation of the universe, let us propose universe instead of being a singularity with zero

volume and infinite energy density [8], there was a "singular quantum state." That is to say, universe at the moment of Big Bang, instead of being physical while mathematically non-realistic, being a mathematical quantum state.

As we recall, energy and momentum become real observables when operators act on the eigen function. In other words, from an abstract quantum mathematical realm we can generate real physicals that can be measured in the universe. Universe by the same analogy can be a mathematical quantum state (singular quantum state) in which after being operated on, will have physical characteristics. We suggest in the beginning the universe was a singular quantum state with eigen function $\psi(\mathbb{H}, \mathbb{P})$, such that

$$\hat{TS} |\psi(\mathbb{H}, \mathbb{P})\rangle = (T, S)_{6D} |\psi(\mathbb{H}, \mathbb{P})\rangle = |\psi(\mathbb{H}, \mathbb{P}); (T, S)_{6D}\rangle. \quad (11)$$

Where TS is the spacetime operator, $(T, S)_{6D}$ is the spacetime eigen value and $|\psi(\mathbb{H}, \mathbb{P}); (T, S)_{6D}\rangle$ is the physical observable in this quantum mechanical operation that we refer to as 6-d spacetime particle. It is interesting to note that this is the physical universe right before expansion. It is not the Einstein's non-symmetrical spacetime which is the universe after expansion, rather it is the symmetrical spacetime with three orthogonal spacetime dimensions as we discussed in section (3). Hypothetically, we can assume some "primitive" form of Hamiltonian and Momentum being part of this eigen-function that will generate space and time respectively. Having space and time operators operating on this singular quantum state

$$i\partial_{\mathbb{H}} e^{-i\mathbb{H}T} = T e^{-i\mathbb{H}T} \text{ and } -i\partial_{\mathbb{P}} e^{i\mathbb{P}S} = S e^{i\mathbb{P}S} \quad (12)$$

That is to say, the singular quantum state $|\psi(\mathbb{H}, \mathbb{P})\rangle$, should already have contained $(\mathbb{H}, \mathbb{P})$. The two operators, $i\partial_{\mathbb{H}}$ and $-i\partial_{\mathbb{P}}$ are time and space operators respectively. So far, we have universe as a singular quantum state, being operated on by time and space operators to generate first 6-d spacetime particle. Afterward, we will have energy operator $\left(\hat{E}\right)$ for energization followed by duplication operator $\left(\hat{D}\right)$ to multiply this energized spacetime particle

$$(\hat{D})(\hat{E}) |\psi(\mathbb{H}, \mathbb{P}); (T, S)_{6D}\rangle = 2 \, h\nu |\psi(\mathbb{H}, \mathbb{P}); (T, S)_{4D}\rangle = 2 |\psi(\mathbb{H}, \mathbb{P}); (T, S)_{4D}, h\nu\rangle. \quad (13)$$

We should notice that $|\psi(\mathbb{H}, \mathbb{P}); (T, S)_{4D}, h\nu\rangle = |\psi(\mathbb{H}, \mathbb{P})_{4D}; h\nu\rangle$ is the first moment where our universe is in its physical state, with energy $h\nu$ being manifested while Einstein's relativity governs the spacetime characteristics. Another interesting point is that the duplication operator keeps constantly being recalled to generate more energized spacetime particles. That is the reason universe keeps expanding geometrically. Since duplication propagates as, 2, 4, 8, 16, etc, we conclude

that universe expansion would be in the form of 2^n power series. However, after ending the energization era, the universe expansion will not be duplication, rather, random with much less expanding factor magnitude than 2^n .

The spacetime particle when energized, contains an intrinsic energy that can be estimated by Planck's energy constant [5]. Also, the space and time quantization as discussed in section (1), do apply to spacetime particles after expansion. Each spacetime particle has a classical space and time characteristic, $L_{Pl} = CT_{Pl}$, bind by c . The equivalent length of a 3-d spacetime particle is approximately $10^{-35\infty} m / \text{spacequantization}$.

Here we understand that quantization is actually quantized spacetime particle, after being energized and multiplied. Prior to this, spacetime particle is a non-physical, rather, purely mathematical entity.

Universe expansion is geometrical and accelerating as 2^n power series up to the point where energization ends. During energization period, every spacetime particle contains an intrinsic energy (Planck's energy) of approximately, $\left(\frac{c^5 \hbar}{G}\right)^{1/2} \sim 10^{19} \text{ ev}$ [2]. This is a tremendous

amount of energy contained in one spacetime particle, but it is what it will suffice for this vast universe with the amount of mass and energy that it contains. The quantum mechanical machinery (energy operator) that was the cause of unleashing this amount of energy was active for a short period of time. Although, all spacetime particles are absolutely identical, it is the operators that bring forth additional characteristics as we shall see in detail in later sections. It is also interesting, for example to note that two electrons are absolutely identical because the same operators have acted on spacetime particle(s) leading to the creation of electrons.

5. Photon and its propagation

At this point we are able to talk about photon and its propagation. Mathematically, a photon [9] can be considered as an energized spacetime particle

$$|\psi(\mathbb{H}, \mathbb{P}); (T, S)_{4D}, hv\rangle = |\psi(\mathbb{H}, \mathbb{P})_{4D}; hv\rangle \quad (14)$$

Eqn.(14) describes a spacetime particle (photon) having energy hv , while propagating in 4-d spacetime. In essence such energy is contained in the vibrational modes of a spacetime particle. It is not the photon that propagates through space, rather it is the vibrational modes of a spacetime particle that are effectively transformed from one spacetime particle to another. This transfer of energy from one spacetime particle to another is propagated with speed c . That means for photon travelling a distance of one meter, according to Eqn.(1b), photon operators should pass through 10^{35} space quantization, while scanning 10^{44} time quantization.

Another subtle point to consider is that spacetime quantization according to Planck can be written as

$$\frac{1}{T_{Pl}} L_{Pl} = C \quad (15a)$$

In a way this equation tells us that the propagation of quantum operations from one spacetime particle to the next should inherently be periodic and although the quantum space and quantum time are constant, the periodicity is related to

$$\nu \lambda = C \quad (15b)$$

where λ and ν are wavelength and frequency respectively, and it is also an indication that photons (energy propagations) will propagate with different periodicity in the context of energy transfer between different spacetime particles.

In essence every particle propagation is the same, even considering a massive particle. The only difference, as we shall see is that for a massive particle it is the "charge-mass" operators that are being repeated from one spacetime particle to another, and speed of propagation will be less than speed of light. Electromagnetic propagation is due to non-massive operation within spacetime particle, in what we referred to as propagation in "fabric of spacetime." A massive particle by virtue of having been operated by other operation than simply energy operator, do not propagate in fabric of spacetime. That is the reason light has null characteristic where as a massive particle has time-like characteristic. It should also be pointed out that space-like characteristic has its own physics that has to be investigated thoroughly within this domain where it may lead to the existence of a parallel universe!

A spacetime particle is a "point-like" particle (4-d spacetime, Planck's volume, or $(c\Delta t, \Delta x^i)_{4D}$) and further,

spacetime particles are quantized, i.e., $[L_{Pl}, T_{Pl}; c]$, where there are overlapping between adjacent wave-functions. The overlapping is due to vibrational frequency in which makes the propagation of energy possible. In this context, there are two possibilities; one is an energized spacetime particle in the form of a photon with energy hv propagating through spacetime and the other would be a non-energized spacetime particle with residual quantum energy. In essence spacetime particles in the universe will be in interaction with each other due to minimal non-zero quantum energy that they contain, similar to a harmonic oscillator.

6. Fermions Being Generated from Spacetime Particles

Once the duplication operator is activated, universe will expand indefinitely and therefore, it will enter into GUT era [8,10]. Accordingly, gravity will distinguish itself from the three other forces. Gravity has its foundation in spacetime curvature and that cannot happen without spacetime particle expansion and its vibration. We shall see that spacetime curvature in effect is an unsymmetrical form of vibration in spacetime particle.

Electro-weak era is followed by expansion of the universe where the building blocks of fermions are being prepared and manufactured. One can suggest foundation of electro-weak machinery being in charge-mass operator

$\hat{QM}$. Among the properties of a fundamental particles, charge and mass are very distinguished, important and real. In classical physics charge and mass are analog of each other in terms fields and potentials. We propose at a fundamental level, charge-mass operator be the cause of energy conversion into a particle having mass and charge, or statistically being void of charge. A massless particle such as a photon is simply a spacetime particle having energy with no charge-mass operator acting on it.

In an analogy to charge-mass operator we recall 4-d spacetime that is neither space nor time. It will, however, reduce to 3-d space and 1-d time once we approach lower relative speed. Similarly, charge-mass operator is neither charge, nor mass, however, it will eventually generate eigen values that we can identify as charge and mass, as we shall see. One can suggest charge-mass operator leading to a mixed state of charge-mass as shown below,

$$\hat{QM} |\psi(\mathbb{H}, \mathbb{P})_{4D}; h\nu\rangle = |\psi(\mathbb{H}, \mathbb{P})_{4D}; (Q, M, \bar{Q}, \bar{M})\rangle, \quad (16)$$

where, we get eigen values that are a mixture of charge, mass, anti-charge and “not-mass”. Due to symmetry between charge and mass in classical physics, we have introduced “not-mass,” which as we shall see later on it will lead to interesting consequences.

It is important to note that mass and not-mass are not particles and anti-particles. The energized spacetime particle, $|\psi(\mathbb{H}, \mathbb{P}); h\nu\rangle$ activates the charge-mass

operator $\hat{QM}$ to produce electro-weak machinery. The charge-mass operator will generate distinct mixtures of charge and mass that is the foundation for new fundamental fermion particles. The energy already stored in spacetime particle as radiation, will eventually be converted to mass and associated charge through process of charge-mass operation within the spacetime particle

$$\gamma \rightarrow (Q(M), \bar{Q}(\bar{M})) \quad (17)$$

where γ is the radiation energy in the form of photon. Table (1) shows the mixing of charge, mass, anti-charge and not-mass.

Table 1. General charge-mass mixing

	Q	$\bar{Q}$
M	MQ	$M\bar{Q}$
$\bar{M}$	$\bar{M}Q$	$\bar{M}\bar{Q}$

In order to generate the four fundamental fermion particles (8), (11), we would need four options as shown in Table (1). We also should not confuse the concept of not-mass with that of anti-particle. In this situation, MQ and $M\bar{Q}$ are matter particle and matter anti-particle, whereas, $\bar{M}Q$ and $\bar{M}\bar{Q}$ are not-matter particle and not-matter anti-particle. As we can see there will be mixing in charge-mass eigen values. Since fermions are made up of two

fundamental flavor groups, i.e., quarks and leptons, namely,

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \quad (18)$$

we have to modify the mixing

$$(\mathcal{Q}(M), \bar{\mathcal{Q}}(\bar{M})) \rightarrow \sum_{i=1}^2 (\mathcal{Q}(M)_i, \bar{\mathcal{Q}}(\bar{M})_i) \quad (19)$$

The mixing would be as shown in Table (2).

Table 2. Fermions generated from charge-mass mixing

	Q_1	$\bar{Q}_1$	Q_2	$\bar{Q}_2$
m_1	e^-	e^+	d	$\bar{d}$
$\bar{m}_1$	$\tilde{e}^-$	$\tilde{e}^+$	$\tilde{d}$	$\tilde{\bar{d}}$
m_2	ν_e	$\bar{\nu}_e$	$\bar{U}$	U
$\bar{m}_2$	$\tilde{\nu}_e$	$\tilde{\bar{\nu}}_e$	$\tilde{\bar{U}}$	$\tilde{U}$

The mixing will generate the fundamental fermions; matter (particles/anti-particle): $M(Q, \bar{Q})$, and not-matter (particle/anti-particle): $\bar{M}(Q, \bar{Q})$.

The Table (2) shown above depicts fundamental fermions we know now, however, this will not be the situation in actual mixing. We clarify this in great detail later on, although, a quick touch on the subject will be useful now. We know in nature the charge and color designations for up and down quarks namely, $u(c, 2/3)$ and $d(c, -1/3)$ and their associated anti-particles. Why is that a down quark comes with negative $1/3$ coulomb charge and color, rather than anti-color? This based on symmetry should also give us other combinations with mixing. For example why shouldn't nature not include $u(c, -2/3)$ and $d(c, 1/3)$? We know why nature prefers to have $u(c, 2/3)$ and $d(c, -1/3)$, because it can produce proton (uud) and neutron (udd) with $+1$ and zero charges respectively, while not utilizing any anti-quarks. But what happen to other options, how can nature just disregard them? There is a possibility that all the options are there, such that,

$$1: d'(c, 1/3) \quad 2: u(c, 2/3) \quad 3: d(c, -1/3) \quad u'(c, -2/3), \quad (20)$$

where prime is an indication of a partner quark. However, option 2 and 3 will produce proton with $+1$ coulomb charge and color, and neutron with zero coulomb charge and color. Options, 1 and 4 on the other hand will produce negative coulomb charge proton and neutron with anti-color! Options 1 and 3 are “opposite partners” (having same color but opposite coulomb charges) and so are options 2 and 4. We will also have anti-particle of these options. Therefore, one can be tempted and say that nature had to produce all mixing but we have only seen and investigated positive proton with color, rather than also seeing negative proton with color. Nature also picked and have chosen between positive and negative coulomb charges mainly to produce proton and neutron from the combination of up and down quarks, otherwise all particles would have been one of one quark flavor. If we did not have positive and negative coulomb charges with

the same color, then fundamental quarks would have been completely different!

It is interesting to note that since an (uud) proton has e-association (to generate an atom for example), a proton partner u'u'd', would also require an associated electron e+' which require the existence of new quantum charge number which we refer to as "shade charge, s" for fermions. We shall investigate this very soon.

Following, will be the mixing as a consequence of electro-weak era where operators are not yet acting to generate individual fermion flavored particles

$$\text{Leptons: } M(Q_1, \bar{Q}_1), \text{ Not-matter Leptons: } \bar{M}(Q_1, \bar{Q}_1) \quad (20a)$$

$$\text{Quarks: } M(Q_2, \bar{Q}_2), \text{ Not-matter Quarks: } \bar{M}(Q_2, \bar{Q}_2). \quad (20b)$$

As we know matter fermion/anti-fermion are

$$\left[\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}; \begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix} \right] \quad (21a)$$

where, these according to the new mixing are coupled with not-matter (fermion/anti-fermion)

$$\text{Add symbol } (\sim). \quad (21b)$$

Each matter fermion particle would have specific operator (from the mixing $\sum_{i=1}^2 (Q(M)_i, \bar{Q}(\bar{M})_i)$) acting on the energized spacetime particle(s) $|\psi(\mathbb{H}, \mathbb{P}); h\nu\rangle$ as follows

$$\widehat{Q_i M_j} |\psi(\mathbb{H}, \mathbb{P}); h\nu_{ij}\rangle = |\psi(\mathbb{H}, \mathbb{P}); q, m, c, s\rangle, \quad (22a)$$

$$\begin{aligned} \widehat{Q_1 M_1} |\psi(\mathbb{H}, \mathbb{P}); h\nu_{11}\rangle &= |e^-\rangle, \quad \widehat{Q_1 M_1} |\psi(\mathbb{H}, \mathbb{P}); h\nu_{11}\rangle = |e^+\rangle \quad (22b) \\ \widehat{Q_2 M_1} |\psi(\mathbb{H}, \mathbb{P}); h\nu_{21}\rangle &= |d\rangle, \quad \widehat{Q_2 M_1} |\psi(\mathbb{H}, \mathbb{P}); h\nu_{21}\rangle = |\bar{d}\rangle \\ \widehat{Q_1 M_2} |\psi(\mathbb{H}, \mathbb{P}); h\nu_{12}\rangle &= |\nu_e\rangle, \quad \widehat{Q_1 M_2} |\psi(\mathbb{H}, \mathbb{P}); h\nu_{12}\rangle = |\bar{\nu}_e\rangle \\ \widehat{Q_2 M_2} |\psi(\mathbb{H}, \mathbb{P}); h\nu_{22}\rangle &= |\bar{u}\rangle, \quad \widehat{Q_2 M_2} |\psi(\mathbb{H}, \mathbb{P}); h\nu_{22}\rangle = |u\rangle, \end{aligned}$$

where c and s are color and "shade" charges respectively. Each particle in (22) will have its own partner. We will elaborate on shade quantum charge as we go along.

$\widehat{Q_i M_j}$ are charge-mass operators acting on energized eigen functions with specific energy $h\nu_{ij}$, with vibrational frequency ν_{ij} . We should also include 8 more of not-matter (particles/ anti-particles) that will couple to Eqns.(22). Those will have charge/anti-charge mixing with not-mass operators. A fundamental fermion particle based on its mass value, will require a superposition of several spacetime particles. Due to various energy requirements, fermions will be generated at different epochs in the evolution of the universe.

The universe is homogenous and isotropic (H&I) [12]. As duplication operator processes continue, in every epoch there will be similar (symmetrical) operational processes that lead to (H&I). Therefore, due to identical processes in spacetime particles, inflation period will not be required.

Charge-mass mixing have to be further expanded because of different types of charges, namely coulomb, color and "shade." Therefore,

$$(Q(M), \bar{Q}(\bar{M})) \rightarrow ((q, m): (c, s), (\bar{q}, \bar{m}): (\bar{c}, \bar{s})). \quad (23)$$

Where $(q, m): (c, s)$, is an indication that if coulomb charge is fractional, then the mixing of charge -mass will

also generate color charge. One can express all charges as follows (not including s),

Table 3. Coulomb and color charge mixing

$\bar{q}, q$	-1	-2/3	-1/3	0	+1/3	+2/3	+1
$\bar{c}, c$	$\bar{c}$			0	c		

where $c = (r, g, b)$. Table (3) is the general deduction of all charge possibilities. However, for fundamental leptons and quarks in nature, charge assignments are as shown below.

Table 4. Table showing fractional coulomb charge generating color

	e^-	ν_e	$\bar{u}$	d
q	-1	0	-2/3	-1/3
c	c = 0	c = 0	$\bar{c}$	c
Leptons		Quarks		

From Tables (3) and (4), we realize that lepton's coulomb charge (whole charge) is prohibited to be mixed with color. Whereas for quarks the fractional coulomb charges mixed with color charges are allowed. If we introduce in analogy to iso-spin, charge states such as charge-isospin for $Q = (q, c, s)$, we will have

$$\begin{aligned} q = 1, & \quad q_i = -1, 0, 1 \\ q = 2/3 & \quad q_i = -\frac{2}{3}, -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3} \\ c & \quad c_i = \bar{c}, o, c \\ s & \quad s_i = \bar{s}, 0, s, \end{aligned} \quad (24a)$$

where s (shade charge) is a quantum charge label associated with fermions which have whole charge mixing with mass (see Eqns.(25)). Unlike color charge $c = (r, g, b)$, that is a consequence of fractional coulomb charge mixing with mass, s charge is not a vector and its major function is to serve as a balance between the number of quarks and leptons. What is shown in (24a) can be reorganized in (24b) as shown below.

$$\begin{aligned} & +1 \\ -\frac{2}{3}, & -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}. \end{aligned} \quad (24b)$$

The coulomb whole and fractional charges mix with mass will generate "shade (s)" and color (c) charges, and anti of these respectively. For example, fractional charges, $+2/3, -2/3$, mixed with mass will generate u quark,

$$u = \left(+\frac{2}{3}, m_u, c \right), \text{ anti-u quark, } \bar{u} = \left(-\frac{2}{3}, m_u, \bar{c} \right), \text{ partner of}$$

$$u \text{ quark, } u' = \left(-\frac{2}{3}, m_u, c \right) \text{ and partner of anti-u quark,}$$

$$\bar{u}' = \left(\frac{2}{3}, m_u, \bar{c} \right) \text{ Also, whole charges, } -1, +1, \text{ mixed with}$$

$$\text{mass will generate shade and anti-shade quantum charges in the form of electron, } e^- = (-1, m_e, s) \text{ and anti-electron,}$$

$$e^+ = (+1, m_e, \bar{s}) \text{ and their two partners as shown in}$$

(Eqns.25a,b). The total charge-mass mixing will be leptons and anti-leptons with their shade charges s and anti-s

$$\begin{aligned}
(Q_1 m_1) &\rightarrow (-1, m_e, s) = e^- & (\bar{Q}_1 m_1) &\rightarrow (+1, m_e, \bar{s}) = e^+ \\
(Q_1 m_2) &\rightarrow (0, m_{\nu_e}, s) = \nu_e & (\bar{Q}_1 m_2) &\rightarrow (0, m_{\nu_e}, \bar{s}) = \bar{\nu}_e, \\
(Q_1 m_1) &\rightarrow (+1, m_e, \bar{s}) = e^{+'} & (\bar{Q}_1 m_1) &\rightarrow (-1, m_e, s) = e^{-'} \\
(Q_1 m_2) &\rightarrow (0, m_{\nu_e}, \bar{s}) = \nu_e' & (\bar{Q}_1 m_2) &\rightarrow (0, m_{\nu_e}, s) = \bar{\nu}_e'.
\end{aligned} \quad (25a)$$

Notice that in Eqn.(25a) the muon and anti-muon are distinguished from one another by virtue of quantum number s . Bellow, there are also quarks and anti-quarks with their associated color charges as a consequence of fractional charge mixing with mass

$$\begin{aligned}
(\bar{Q}_2 m_2) &\rightarrow \left(\frac{2}{3}, m_u, c\right) = U & (Q_2, m_2) &\rightarrow \left(-\frac{2}{3}, m_u, \bar{c}\right) = \bar{U} \\
(Q_2 m_1) &\rightarrow \left(-\frac{1}{3}, m_d, c\right) = d & (\bar{Q}_2 m_1) &\rightarrow \left(+\frac{1}{3}, m_d, \bar{c}\right) = \bar{d},
\end{aligned} \quad (25c)$$

and the (quark and anti-quark) partners

$$\begin{aligned}
(\bar{Q}_2 m_2) &\rightarrow \left(-\frac{2}{3}, m_u, c\right) = U' & (Q_2, m_2) &\rightarrow \left(\frac{2}{3}, m_u, \bar{c}\right) = \bar{U}' \\
(Q_2 m_1) &\rightarrow \left(\frac{1}{3}, m_d, c\right) = d' & (\bar{Q}_2 m_1) &\rightarrow \left(-\frac{1}{3}, m_d, \bar{c}\right) = \bar{d}'.
\end{aligned} \quad (25d)$$

There also will be 16 “not-mass particles” analog to the particles above, as $m \rightarrow \bar{m}$. As an example, consider Eqn.(25a) in which we show the not-mass particles version of the leptons and anti-leptons

$$\begin{aligned}
(Q_1 \bar{m}_1) &\rightarrow (-1, \bar{m}_e, s) \equiv \bar{e}^- & (\bar{Q}_1 \bar{m}_1) &\rightarrow (+1, \bar{m}_e, \bar{s}) \equiv \bar{e}^+ \\
(Q_1 \bar{m}_2) &\rightarrow (0, \bar{m}_{\nu_e}, s) \equiv \bar{\nu}_e & (\bar{Q}_1 \bar{m}_2) &\rightarrow (0, \bar{m}_{\nu_e}, \bar{s}) \equiv \bar{\nu}_e'.
\end{aligned} \quad (26)$$

Notice that in Eqns.(25a,b) there are leptons, anti-leptons, lepton partner and anti-lepton partners. The same applies for quarks family.

Consider a proton that is made from (uud) and anti-proton that is $\left(\bar{u}\bar{u}\bar{d}\right)$. We can also take a proton partner (u'u'd') which is a negative coulomb proton with the same color charge, and anti-proton partner $\left(\bar{u}'\bar{u}'\bar{d}'\right)$ which is

negative coulomb anti-proton with the same color charge. Consequently, we can consider four types of hydrogen atoms; hydrogen, and anti-hydrogen made up of positron and anti-proton, hydrogen partner made up of electron partner (positive electron with same shade charge, s) and proton partner (negative proton with same color charge) and finally, anti-hydrogen partner, as follows;

Hydrogen: $P + e^-$ positive proton and negative electron
 Anti-hydrogen: $\bar{p} + e^+$ anti-proton and positron
 Hydrogen partner: $P' + e^{-'}$ negative proton and positive electron
 Anti-hydrogen partner: $\bar{p}' + e^{+'}$ negative anti-proton and negative positron

We should clarify here, that for anti-particles both coulomb and color charges are opposite, whereas for partner particles coulombs are opposite whereas colors are the same. The fact that nature choses u quark with charges $(2/3, c)$ and d quark with charges $(-1/3, c)$ from different baskets is that it can allow u and d to generate whole electric charge without mixing quarks and anti-quarks. A good example is proton and neutron which are made of (uud) and (udd) respectively. Without utilizing any anti-quarks, we have generated particles with $q = +1$ and $q = 0$. If we had a situation where d quark was specified as $(1/3, c)$ then we could not have used a three mixture of up and down quarks to generate proton. A scenario where if d quark was $(1/3, c)$, would have worked, if up quark was of the form $(-2/3, c)$, which is indeed u quark partner. So, we

can see based on Table (3) mixing, all options can be valid as seen in Eqns.(25). It is possible that in the beginning all these various forms of leptons and quarks were created in the universe and they have been separated without any interaction.

7. Charge-mass

Recalling that the (fermionic) operator acting on spacetime particle was charge-mass operator. We have spoken about charge which has the characteristic of charge-isospin, having quantized values for both electric and color and shade. Consider an electron which is a point-like fundamental particle. The charge-mass operator acting on spacetime particle by the virtue of mixing, will generate characteristics that is what we identify as electron, $(q = -1, m_e, s) = e^-$.

Color charge eigen value is the consequence of mixing of fractional electric charge with that of mass, namely, $\widehat{Q_2 M_2} |\psi(\mathbb{H}, \mathbb{P}); h\nu_{22}\rangle = |\psi(\mathbb{H}, \mathbb{P}); q = -\frac{2}{3}, m_u, \bar{c}\rangle = |\bar{U}\rangle$, with mass eigen value, $m = m_u$, for anti-up quark. The energy associated with this mass is what has been tunneled from stored energy in the spacetime particle(s) as a consequence of mixing.

The question now is what actually is mass? Although according to $E = mc^2$, mass and energy are equivalent, they are not the same. Pure energy ($E = h\nu$) cannot be localized in a spacetime particle (similar to a photon). Energy is the spacetime vibration that propagates from one spacetime particle to the next by propagation speed c , in what we call as propagation in fabric of spacetime. Mass on the other hand, $m = c^{-2}E$, is a form of energy (with different vibrational modes) that is localized in spacetime particle and, therefore cannot propagate from one spacetime particle to the next, unless if there is a driving “force.” We should note that the energy already existing in a spacetime particle $h\nu$, is the one that is converted into what we call mass (and charge). This mass and charge become quantum properties of spacetime particle.

Let us again consider an electron. The localized energy in spacetime particle as a consequence of charge-mass operator mixing, manifests itself as m_e , with charge $q = -1$ and shade charge, s . An electron by virtue of having the characteristic of a local mass m_e (localized vibrational energy) can be moved from one spacetime particle to the next. It is not a particular electron that moves in space and time, rather, it is the quantum operator that transfers electron property from one spacetime particle to the next. The cause of this operational transfer is the force that acts on spacetime particle. Similar to a photon propagation, an electron travelling through a distance will also go into non-existence and existence as it transfers through spacetime particles. The difference between photon and an electron spacetime particle is that one is unlocalized energy propagation (pure energy) and the other is localized energy (energy is converted to mass while having a specific charge property).

So, a classical Newton force, here, can be interpreted as the impetus (vibrational energy) that causes electron

operator ceasing to exist from one spacetime particle while starting the exact operation in the adjacent spacetime particle. If a force being applied on a massive particle, that particle will propagate forever in spacetime (leaping will continue forever), unless the energy associated with impetus, somehow transfers to some other spacetime particles. So, in actuality, it is again energy (localized vibration) that is the core of force for moving a massive particle. This energy is the cause for operators to be activated from one spacetime particle to the next one. This energy either moves the particle indefinitely, or gets transferred to another spacetime particle or, it gets stored in the form of potential energy. When stored as a potential energy, it means the energy is not being transferred (operators are not being activated in the next spacetime particles), where if there will be a suitable condition, then potential energy will change into kinetic. We should note that physical properties such as energy, mass and charge are vibrational excitations with specific modes in spacetime particles.

It is also interesting to notice that matter neutrino and matter anti-neutrino do not have the same state due to shade and anti-shade quantum scalars, although according to Eqns. (25,a,b), they have the same state as their partners,

$$\begin{aligned} (Q_1 m_2) &\rightarrow (0, m_{\nu_e}, s) = \nu_e \\ (\overline{Q_1 m_2}) &\rightarrow (0, m_{\nu_e}, \bar{s}) = \bar{\nu}_e \end{aligned} \quad (27)$$

similarly, not-matter neutrino and not-matter anti-neutrino have different states. In general, eight kinds of matter-fermions are being created

Matter particle: P	Matter anti-particle: $\bar{P}$
Matter particle partner: p'	Matter anti-particle partner: $\bar{P}'$
.....	
Not-matter particle: $\bar{P}$	Not-matter anti-particle: $\bar{\bar{P}}$
Not-matter particle partner: $\bar{P}'$	Not-matter anti-particle partner: $\bar{\bar{P}}'$

(28)

Once again, we emphasize that quarks color charges are a consequence of fractional coulomb charge and mass, mixing. We should notice that charge-mass mixing produce masses in quadruple form $(P, \bar{P}, p', \bar{P}')$ and also the not-matter analog of these.

All same species of matter and not-matter (particle, anti-particle, particle partner and anti-particle partner) have same amount of masse. We don't know the nature of not-matter ($\bar{M}$) in comparison with that of matter (M). The process of operators leading to the creation of matter particles and matter anti-particles should have been the same as not-matter particles and not-matter anti-particles. We know a photon " γ_m " (gamma mediating between matter/anti-matter particles and associated particle partners), lead to the creation of both matter and anti-matter [13]. The matter and anti-matter as we know have the same mass (m) but opposite charges, hence a photon is the boson in this interaction. For a not-matter particle and not-matter anti-particle, while both particles have identical not-mass ($\bar{m}$), they have opposite charges, analogous to their matter partners. There should be a not-matter boson gamma particle γ_n , being involved in this interaction. There also can be a spectrum of not-matter photons that act the same way as matter photons do (acting as visible electromagnetic spectrum (light) for not-matter particles!). Matter photons most probably cannot interact with not-

matter particles, so not-matter particles remain dark with γ_m

$$\begin{aligned} 2\gamma_m &= P + \bar{P}, \quad 2\gamma_m = P' + \bar{P}' \\ 2\gamma_n &= \bar{P} + \bar{\bar{P}}, \quad 2\gamma_n = \bar{P}' + \bar{\bar{P}}' \end{aligned} \quad (29)$$

It is also not clear how a particle and its particle partner can react.

8. Quantum Interactions within Universe

In the beginning of the creation of the universe instead of a singularity with zero volume and infinite energy density, we proposed in section (3), universe being a single quantum state. Intrinsic operations within this singular quantum state led to the energization, expansion and creation of particles in different epochs. Expansion of universe was due to duplication operators acting on energized spacetime particles. Universe's expansion in the beginning is of the form 2^n because of duplication operator.

Four-dimension spacetime in actuality are spacetime particles stacked together, as they are being created and therefore, expanding universe. Each spacetime particle has intrinsically a 4-d spacetime characteristic. Spacetime particles having wave-function characteristics, that overlap and therefore, communicate with each other while having zero energy vibration. Every massive and non-massive particle in the universe is actually a spacetime particle with that particular particle properties due to internal operations.

The natural constant c , is what binds all spacetime and the activities within it together. The spacetime, spacetime particle, energy, mass, propagation, etc, are all bind together by constant c . For example, energy within a spacetime particle either propagates in spacetime with speed c , or it can be converted into charge and mass via charge-mass operation. C is the key to many physical characteristics and activities in the universe.

What we refer to as a distance between two objects is nothing but 4-d spacetime particles stacked together. A moving physical object either massive or non-massive, will travel this distance through each spacetime particle while appearing and disappearing. The energy associated with force is the impetus that causes these internal operations (for massive particles) to turn on, and remain on indefinitely (Newton's first law). As we indicated, although spacetime particles are quantized and are bind by c , they vibrate and overlap with neighbors (overlapping wave-functions). Through this approach one can explain concepts such as distance, speed, acceleration, energy, momentum, etc. Within this context, mass can be explained as a localized energy within spacetime particle. Photon is the escaping vibrational energy (due to overlapping between spacetime particles wave-functions) within spacetime particle, whereas a localized energy within spacetime particle does not scape and will remain as an intrinsic property of the spacetime particle. A spacetime particle also has a zero-quantum vibrational energy in the absence of other energies. Therefore, universe is not only expanding, it is also vibrating!

One can explain the attraction between two massive particles by overlapping between the two spacetime particles wave-functions. Normally, for an empty space,

overlapping would be residual due to zero quantum energy vibration, but spacetime particles having localized energies (mass) can interact and effect each other in a reciprocal manner. That is to say, overlapping will cause spacetime particles wavefunction to “bend.” Therefore, gravity is spacetime curvature due to spacetime particles quantum interactions. A massive spacetime particle by the virtue of its specific modes of vibrations will cause the adjacent non-energized (and energized) spacetime particles to curve (interaction between wave-functions will cause deflection). We can also deduce the charge behavior, as something analog to mass. Afterall, charge-mass operator is what that leads to charge and mass eigen values.

If energy is vibration, then not only mass is a form of vibration but also charge, due to spacetime curvature. Charge and mass are always coupled together in a particle. A particle cannot be void of charge and mass eigen values; they come together. Charge eigen values are neither “escaping” nor localized energies. What we can say is that charge eigen values are a form of quantized energy that are vibrational (due to attraction and repulsion) but not similar to mass vibration. It is possible that although charge and mass are coupled eigen values, they do have distinct modal vibrations which are orthogonal and have different characteristics. Due to symmetry between charge and mass, it is possible that there are masses that repel one another similar to charges.

A particle and anti-particle will annihilate each other because they have same mass eigen value but opposite charges. Consider d quark and anti-quark,

$$\begin{aligned} d + \bar{d} &= 2 \gamma_m & (30) \\ \left(-\frac{1}{3}, m_d, c\right) + \left(+\frac{1}{3}, m_d, \bar{c}\right) &\leftrightarrow 2 |\psi(\mathbb{H}, \mathbb{P}); h\nu\rangle = \\ &2 \gamma_m |\psi(\mathbb{H}, \mathbb{P})\rangle \end{aligned}$$

where this interaction involves gamma associated with matter (particle/anti-particle, γ_m). The way quantum mechanical operation in spacetime particles is set, is that, if two identical spacetime particles with opposite coulomb charges overlap, the charge mass operation will collapse and energy associated with that will become an escaping vibrational energy in the form of two photons. One can wonder what would happen in the interaction below, between two not-matter particles overlapping, while having opposite charges, namely

$$\begin{aligned} \tilde{d} + \bar{\tilde{d}} &\overset{?}{\leftrightarrow} 2 \gamma_n & (31) \\ \left(-\frac{1}{3}, \overline{m_d}, c\right) + \left(\frac{1}{3}, \overline{m_d}, \bar{c}\right) &\overset{?}{\leftrightarrow} 2 \gamma_n. \end{aligned}$$

Lastly, QFT and QCD are the physics associated with the quantum behavior of spacetime particles’ operators and their interactions. As we indicated, charge and mass are localized forms of vibrational energies. In QFT and QCD, quantum vibrational overlapping is mediated by virtual particles. The real particles are spacetime particles and vibrational interactions are the minute energy overlapping (virtual particles) between spacetime particles. The virtual particles existence is constraint by uncertainty in energy and time, where it can be borrowed by nothing other than spacetime particles’ quantum residual vibrational energy. Quantum gravity is also

physics associated with vibrational behavior associated with local energy (mass) between spacetime particles where they bend.

9. Conclusion

Universe at the moment of Big Bang being a singularity, is an undefined mathematical object. As quantum mechanics rules the physics of the universe, it also did at the very moment of universe’s inception. Universe began as a singular quantum state with its internal operators governing its energization, expansion, and consequently creation of particles. Not only particles within universe behave quantum mechanically, the universe itself by virtue of being collection of spacetime particles and their associated operators are completely quantum mechanical. Photon is an energized spacetime particle, in which this energy propagates sequentially from one spacetime particle to the next with propagation speed c . The mixing of charge-mass eigen values lead to the existence of quarks, i.e., $u, \bar{u}, u', \bar{u}'$, and similarly d quarks. Such flavors of quarks will also lead to the existence of shade, s quantum number for leptons. Our universe is made up of 4 flavor type of particles, namely, matter (particle; P , anti-particle; $\bar{P}$, particle partner; p' , anti-particle partner; $\bar{P}'$) and also the not-matter analog of these. As matter photon interacts between matter particles, so does not-matter photon interact between not-matter particles. The kinds of flavor of matter and not-matter may be a key to further understanding of the universe.

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