

Discussion on the Correction of Classical Electromagnetic Wave Theory Through Transmission Lines

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Received December 01, 2023; Revised January 03, 2024; Accepted January 11, 2024

Abstract The author has made revisions to the classical electromagnetic theory. The solution of Maxwell's equation has been reinterpreted. It is believed that advanced waves exist, radiation does not overflow the universe, and the Poynting vector of electromagnetic waves is reactive power. The electric and magnetic fields in the far field have a 90 degree phase difference. Transmission line problems are often solved using circuit methods. The voltage and current are in phase. The results of calculating the electric and magnetic fields according to Maxwell's equations are the same. According to the author's revised electromagnetic theory, there are two types of electric fields on transmission lines: electrostatic field and induced electric field. The electrostatic field is in phase with the current on the transmission line. The induced electric field lags 90 degrees behind the current. This is consistent with a simple circuit. Therefore, for transmission lines, the author's theory contradicts the conclusions of existing textbooks. Transmission lines are a good example of distinguishing between two theories. The author also proposed a method for experimental verification.

Keywords: Maxwell's equation, Poynting, retarded wave, advanced wave, retarded potential, advanced potential, antenna, transformer, electromagnetic wave, electromagnetic field, energy flow, quasi-static state, transmission line

Cite This Article: Shuang-ren Zhao, "Discussion on the Correction of Classical Electromagnetic Wave Theory Through Transmission Lines." *International Journal of Physics*, vol. 12, no. 1 (2024): 1-18. doi: 10.12691/ijp-12-1-1.

1. Introduction

1.1. Maxwell's Electromagnetic Theory

The author found that Maxwell's electric field theory is a theory of retarded potential, and its key is that the vector potential in the frequency domain is,

$$\mathbf{A}(x, \omega) \equiv \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}(x')}{r} dV \quad (1)$$

$$\mathbf{A}^{(r)}(x, \omega) \equiv \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}(x') \exp(-jkr)}{r} dV \quad (2)$$

The superscript (r) indicates retardation. $\mathbf{A}^{(r)}(x, \omega)$ is a vector potential with retardation, while $\mathbf{A}(x, \omega)$ is a vector potential without considering retardation. Electric and magnetic fields are defined as,

$$\mathbf{E} = -\nabla \phi^{(r)} - \frac{\partial}{\partial t} \mathbf{A}^{(r)} \quad (3)$$

$$\mathbf{B} = \nabla \times \mathbf{A}^{(r)} \quad (4)$$

The above $\phi^{(r)}$ is determined by the Lorenz gauge condition.

$$\nabla \cdot \mathbf{A}^{(r)} = -j\omega\mu_0\epsilon_0\phi^{(r)} \quad (5)$$

Here we assume that all quantities have a time factor $\exp(j\omega t)$. We omitted this factor. The key to this theory is to ensure that the following generalizations have been made when transitioning from quasi-static electromagnetic field theory or magnetic quasi-static electromagnetic field theory to radiation electromagnetic field theory considering retardation,

$$\mathbf{A}(x, \omega) \rightarrow \mathbf{A}^{(r)}(x, \omega) \quad (6)$$

This promotion ensures that

$$\lim_{kr \rightarrow 0} \mathbf{A}^{(r)}(x, \omega) = \mathbf{A}(x, \omega) \quad (7)$$

$$kr \rightarrow 0 \quad (8)$$

i.e.,

$$\frac{2\pi}{\lambda} r \rightarrow 0 \quad (9)$$

$$r \ll \lambda \quad (10)$$

$$r = \|x - x'\| \quad (11)$$

is the distance from the field point to the origin. If this distance is much smaller than the wavelength λ , the retarded potential $A^{(r)}$ degenerates into a non retarded vector potential A . The Maxwell equation is completely equivalent to the retarded potential theory above. The theory of retarded potential was first completed by Lorenz in 1867 [1], and the theory of Lorenz retarded potential was only 5 years later than the Maxwell equation (completed in 1861-2). Maxwell did not solve the Maxwell equation, but Lorenz's theory is not only equivalent to Maxwell's equation, but also provides a general form of solution of Maxwell's equation.

The author found that this condition is too simplistic for electromagnetic theory. There are other conditions that should be considered when transitioning from retarded potential theory to non retarded electromagnetic theory.

1.2. The Author's Electromagnetic Theory

The author's electromagnetic theory has made revisions to Maxwell's classical electromagnetic field theory. This correction is based on the following considerations:

1. Not only the retarded potential (wave), but also the advanced potential (wave) should be considered.

2. Radiation does not overflow the universe, where Γ is a sphere with an infinite radius, assuming that the source of the electromagnetic field is the current J near the center of the sphere.

$$\int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E} \times \mathbf{H}) \cdot \hat{n} d\Gamma = 0 \quad (12)$$

No matter should overflow the universe, including electromagnetic waves. It is wrong for classical electromagnetic theory to allow electromagnetic waves to overflow the universe.

3. Ensure that the law of conservation of energy holds

$$\sum_{i=1}^N \sum_{j=i, j \neq i}^N \int_{t=-\infty}^{\infty} dt \int_V (\mathbf{E}_i \cdot \mathbf{J}_j) dV = 0 \quad (13)$$

This law tells us that if there are only two currents J_1 , J_2 . Current J_1 give a portion of the energy to the current J_2 . So current J_1 is a primary coil, current J_2 is a secondary coil. The energy loss of J_1 is exactly equal to the energy obtained by the current J_2 . The algebraic sum of the energy lost by the two coils together is 0. This indicates the conservation of energy. We cannot derive this law of energy conservation from our classical electromagnetic theory today. We can only take a step back and prove this self proclaimed energy law as the energy theorem or reciprocity theorem.

4. The current generates half retarded and half advanced waves. Therefore, when the retarded wave and advanced wave degenerate into a non retarded and non advanced electromagnetic field, this electromagnetic field must be superimposed, that is, the electric field is superimposed, and the magnetic field is also superimposed. They cannot cancel each other out at the

surface of the current. Our classical electromagnetic theory today does not meet this requirement.

5. For separate systems, such as systems composed of primary and secondary transformers, the energy of the system composed of transmitting and receiving antennas is transmitted by mutual energy flow.

The first of the five above is the characteristic of the author's electromagnetic field theory. This theory is an electromagnetic field theory that includes advanced waves. The theory of advanced waves, also known as the absorber theory, was first proposed by Wheeler Feynman in 1945 [2,3]. This theory also applies the principle of action at a distance [4,5,6]. In addition, Stephenson also conducted in-depth research on the theory of advanced waves around 1980 [7]. Cramer further developed this theory into a transactional interpretation of quantum mechanics in 1986 [8,9]. Welch proposed the electromagnetic field reciprocity theorem including advanced waves in 1960 [10].

Classical electromagnetic field theory cannot be directly applied to form a theory that includes advanced waves, which can be obtained by advanced potentials

$$\mathbf{A}^{(a)}(x, \omega) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}(x') \exp(+jkr)}{r} dV \quad (14)$$

$$\mathbf{E} = -\nabla\phi - \frac{\partial}{\partial t} \mathbf{A}^{(a)} \quad (15)$$

$$\mathbf{B} = \nabla \times \mathbf{A}^{(a)} \quad (16)$$

The electromagnetic field obtained in this way cannot be directly used, as the original Maxwell electric field theory is difficult to reconcile with after the introduction of advanced waves. The conservation of energy theorem is the first challenge encountered. Because once there is an advanced wave, the retarded wave radiates energy, and the advanced wave also radiates energy (this energy is negative). However, the author found that there is also an interaction between the retarded wave and the advanced wave at this time. This interaction also transfers energy. The energy transferred by this interaction is exactly the energy from the source to the sink. Here, the source refers to the source of electromagnetic waves, including transmitting antennas, primary coils of transformers, light sources, etc. A sink is a device that receives electromagnetic waves, including a receiving antenna, a secondary coil of a transformer, a screen for receiving light, and so on.

Since the energy flow of interaction, also known as the mutual energy flow, can transfer energy from the source to the sink. So the energy transmitted by the advanced and retarded waves themselves are redundant. Wheeler and Feynman also attempted to solve this contradiction in the absorber theory. Their viewpoint is that the current radiates half of the retarded wave and half of the advanced wave. The retarded wave carries positive radiation energy, while the advanced wave radiates negative energy. Therefore, current does not radiate energy in average. The author agrees with Wheeler Feynman's viewpoint of half retarded wave and half advanced wave, but does not agree that the energy of the retarded and advanced waves emitted by the current can be offset. The author proposed the concept of time reversal wave in 2017 [11]. Time reversal waves can cause both retarded and advanced

waves to collapse in the opposite direction. The reverse collapse cancels out the energy flow of the original retarded wave and advanced wave. This leaves only the mutual energy flow to transfer energy. The law of conservation of energy is guaranteed. That is to say, item 2 has been guaranteed.

But time reversal waves require adding new electromagnetic fields to the Maxwell's equations, which makes the problem more complex. And it cannot solve all the problems. Since 2017, the author has attempted to apply the theory proposed by the author in many examples. These examples include electromagnetic waves of double infinite plane currents. The energy flow problem from the primary coil to secondary coil of the transformer. The energy flow problem from the dipole transmitting antenna to the dipole receiving antenna. In these specific problems, the author found that if Maxwell's electromagnetic theory is used to calculate the electric and magnetic fields, and then the mutual energy flow is calculated, the mutual energy flow is often an imaginary number, so it transfers reactive power, that is, it does not transfer energy in time average. Therefore, time reversal waves cannot truly solve the problem and complicate it.

The author proposes the concept that both retarded wave and advanced wave are reactive power waves, and reactive power waves are waves that maintain a 90 degree phase difference between the electric and magnetic fields. On average, the energy transmitted by this wave is 0. Therefore, it is no longer transmitting energy. If both the retarded wave and the advanced wave are reactive power waves, they themselves will not transmit energy. Time reversal waves are no longer needed. The reactive power wave can completely replace the original retarded wave and advanced wave, and also includes the time reversal wave of retarded wave and advanced wave. The author found that if the retarded wave and advanced wave are reactive power, the mutual energy flow formed by them happens to be active power. The 1-5 items mentioned earlier have been perfectly implemented.

According to Maxwell's electromagnetic field theory, the far-field electric and magnetic fields of the antenna remain in phase, therefore they are not reactive power waves. However, the author found that Maxwell's electromagnetic field theory is actually a retarded potential theory, which can ensure that the electric field is a retarded field, but cannot guarantee that the magnetic field is also a retarded field. Therefore, the author made corrections to the magnetic field obtained from Maxwell's classical electromagnetic theory [12,13,14] [15,16,17,18] [19,20,21,22,23,24] [28,29]

Here we summarize the conclusions from these literature as follows. Assuming we calculate the magnetic field $\mathbf{H}_{Maxwell}$ according to retarded potential of the electromagnetic field theory, the magnetic field corrected by the author is

$$\mathbf{E}_i^{(r)} = \mathbf{E}_{iMaxwell}^{(r)} \quad (17)$$

$$\mathbf{H}^{(r)} = (-j) \mathbf{H}_{Maxwell}^{(r)} \quad (18)$$

$\mathbf{H}$ is the true magnetic field of the system. The above correction is for the electromagnetic wave part, that is, for the far-field correction. No correction is needed for near

field. The above correction factor $(-j)$ holds for the retarded wave. If it is a advanced wave, the factor is (j) .

$$\mathbf{E}_i^{(a)} = \mathbf{E}_{iMaxwell}^{(a)} \quad (19)$$

$$\mathbf{H}^{(a)} = (j) \mathbf{H}_{Maxwell}^{(a)} \quad (20)$$

Above, we are correcting the magnetic field. Sometimes, the correct magnetic field is already known, and it is necessary to correct the induced electric field. At this time, we also add a phase factor to the above correction formula, such as adding (j) to the retarded wave

$$\mathbf{E}_i^{(r)} = j \mathbf{E}_{iMaxwell}^{(r)} \quad (21)$$

$$\mathbf{H}^{(r)} = j(-j) \mathbf{H}_{Maxwell}^{(r)} = \mathbf{H}_{Maxwell}^{(r)} \quad (22)$$

For advanced waves, the correction factor is $(-j)$,

$$\mathbf{E}_i^{(a)} = -j \mathbf{E}_{iMaxwell}^{(a)} \quad (23)$$

$$\mathbf{H}^{(a)} = -j(j) \mathbf{H}_{Maxwell}^{(a)} = \mathbf{H}_{Maxwell}^{(a)} \quad (24)$$

Note that the superscript (r) means retarded wave, the superscript (a) means advanced wave, and the subscript i means induced electric field. The subscript $Maxwell$ refers to the meaning calculated according to Maxwell's equation.

The author's theory has achieved the following results:

1. For independent systems, such as simple circuit, transmit energy through the self energy flow corresponding to the Poynting vector. For the separate systems, such as the primary and secondary coils of transformers, transmitting and receiving antennas, and the light source to the light sink, the energy flows from source to sink are mutual energy flows.

2. The law of conservation of energy and the theorem of mutual energy flow hold:

$$-\int_{V_2} \mathbf{E}_1 \cdot \mathbf{J}_2^* dV = (\xi_1, \xi_2) = \int_{V_1} \mathbf{E}_2^* \cdot \mathbf{J}_1 dV \quad (25)$$

$$(\xi_1, \xi_2) = \oint_{\Gamma} (\mathbf{E}_1 \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_1) \cdot \hat{n} d\Gamma \quad (26)$$

The author has verified the above laws for systems with dual planar transformers, a system with planar transmitting antenna and a planar receiving antenna, and a system with a dipole transmitting antenna and a dipole receiving antenna. Note that the above theorem is actually the law of conservation of energy, and it is a localized law of conservation of energy. (Please note that the above law of conservation of energy is not valid in Maxwell's electromagnetic theory. As mentioned earlier, the mutual energy flow calculated based on Maxwell's equations for electric and magnetic fields is often a pure imaginary number and therefore does not transfer energy. The above mutual energy flow theorem is valid in the author's revised electromagnetic theory.)

3. The mutual energy flow is generated at the source and annihilated at the sink, thus possessing the properties of photons. Therefore, the author believes that photons are mutual energy flows. The mutual energy flow is

composed of retarded waves and advanced waves, which have the properties of waves. The mutual energy flow has the properties of particles, especially at the light source and light sink, where it gathers into a point and carries energy and momentum. Therefore, like particles. The mutual energy flow itself is both a retarded wave and an advanced wave, therefore it is also a wave. This explains the problem of wave particle duality.

4. The author's electromagnetic field theory further supports Cramer's quantum mechanical transactional interpretation [8,9]. Therefore, the interpretation of quantum mechanics through transactional interpretation is also applied to the author's electromagnetic theory.

1.3. The Focus of this Article Is on Transmission Lines

This article studies the transmission line problem. The transmission line problem is a very simple problem. The electromagnetic waves inside the transmission line are TEM waves, which should be easily solved by Maxwell's classical electromagnetic theory. However, the author have discovered a strange phenomenon. Few people use Maxwell's equations to solve transmission line problems, usually using circuit methods. The author reviews circuit methods for solving transmission line problems. Review the plane wave problem. Review simple electromagnetic field transmission problems in transmission lines. The dilemma of using Maxwell's classical electromagnetic theory to solve transmission line problems is studied. Then, using the electromagnetic field theory proposed by the author to calculate the transmission line problem, the author found that the results are consistent with the circuit method.

The author's revision of Maxwell's electromagnetic theory has not been widely accepted, although several papers have been published. If the transmission line problem can be perfectly solved according to the author's theory, but there are difficulties in solving the classical electromagnetic theory using Maxwell's equations, this itself supports the author's electromagnetic field theory. On the other hand, any measurement within the transmission line is very easy to perform. If experimental verification is required, it can also be done very easily on the transmission line.

2. Plane Waves

2.1. Solving Plane Waves According to Maxwell's Equation

Solving plane waves according to Maxwell's equation

$$\nabla \cdot \mathbf{E} = 0 \quad (27)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (28)$$

$$\nabla \times \mathbf{E} = -j\omega\mu_0 \mathbf{H} \quad (29)$$

$$\nabla \times \mathbf{H} = j\omega\epsilon_0 \mathbf{E} \quad (30)$$

The above formula is the Maxwell' equations without sources.

2.2. Wave Equation

$$\nabla \times \nabla \times \mathbf{E} = -j\omega\mu_0 \nabla \times \mathbf{H} = -j\omega\mu_0 (j\omega\epsilon_0 \mathbf{E}) \quad (31)$$

$$\nabla \times \nabla \times \mathbf{E} = \omega^2 \mu_0 \epsilon_0 \mathbf{E} \quad (32)$$

$$\nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \omega^2 \mu_0 \epsilon_0 \mathbf{E} \quad (33)$$

Considering $\nabla \cdot \mathbf{E} = 0$,

$$\nabla^2 \mathbf{E} + \omega^2 \mu_0 \epsilon_0 \mathbf{E} = 0 \quad (34)$$

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0 \quad (35)$$

wherein

$$k^2 = \omega^2 \mu_0 \epsilon_0 \quad (36)$$

2.3. Wave Equation Solving

$$\mathbf{E} = \mathbf{E}_0 \exp(-jk \cdot \mathbf{x}) \quad (37)$$

Consider $\nabla \cdot \mathbf{E} = 0$, there is,

$$-jk \cdot \mathbf{E} = 0 \quad (38)$$

or

$$\mathbf{k} \cdot \mathbf{E} = 0 \quad (39)$$

therefore,

$$\mathbf{E} \perp \mathbf{k} \quad (40)$$

To simplify the problem, we can assume that,

$$\mathbf{k} = k \hat{x} \quad (41)$$

This indicates that the wave propagates along the x -axis direction, so there is,

$$\mathbf{E} = \mathbf{E}_0 \exp(-jkx) \quad (42)$$

wherein

$$\mathbf{E}_0 = E_0 \begin{pmatrix} -\hat{z} \end{pmatrix} \quad (43)$$

$$\mathbf{E} = E_0 \exp(-jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \quad (44)$$

We calculate the magnetic field using Faraday's law (29)

$$\begin{aligned} \mathbf{H} &= \frac{1}{-j\omega\mu_0} \nabla \times \mathbf{E} \\ &= \frac{1}{-j\omega\mu_0} \nabla \times \left(E_0 \exp(-jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \right) \\ &= \frac{1}{-j\omega\mu_0} \begin{pmatrix} -jk \hat{x} \end{pmatrix} \times \left(E_0 \exp(-jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \right) \\ &= \frac{k}{\omega\mu_0} \hat{x} \times \left(E_0 \exp(-jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \right) \\ &= \frac{k}{\omega\mu_0} E_0 \exp(-jkx) \hat{y} \\ &= \frac{1}{\eta_0} E_0 \exp(-jkx) \hat{y} \end{aligned} \quad (45)$$

$$\eta_0 = \frac{\omega\mu_0}{k} = \frac{\mu_0}{\omega\sqrt{\mu_0\epsilon_0}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \quad (46)$$

In this way, we obtain a plane invariant solution, which is the solution of the retarded wave. Therefore, we add a superscript (r) to them,

$$\mathbf{E}^{(r)} = E_0 \exp(-jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \quad (47)$$

$$\mathbf{H}^{(r)} = \frac{1}{\eta_0} E_0 \exp(-jkx) \hat{y} \quad (48)$$

Based on Maxwell's theory, the electric and magnetic fields of retarded waves were obtained.

2.4. Advanced Wave

The above is the retarded wave. Considering the electric field of the advanced wave,

$$\mathbf{E} = E_0 \exp(+jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \quad (49)$$

$$\begin{aligned} \mathbf{H} &= \frac{1}{-j\omega\mu_0} \nabla \times \mathbf{E} \\ &= \frac{1}{-j\omega\mu_0} \nabla \times \left(E_0 \exp(+jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \right) \\ &= \frac{1}{-j\omega\mu_0} jk \hat{x} \times \left(E_0 \exp(+jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \right) \\ &= -\frac{1}{\omega\mu_0} k \hat{x} \times \left(E_0 \exp(+jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \right) \\ &= -\frac{1}{\eta_0} E_0 \exp(+jkx) \hat{y} \end{aligned} \quad (50)$$

Therefore, we obtain the advanced wave solution,

$$\mathbf{E}^{(a)} = E_0 \exp(+jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \quad (51)$$

$$\mathbf{H}^{(a)} = -\frac{1}{\eta_0} E_0 \exp(+jkx) \hat{y} \quad (52)$$

We obtain the electric and magnetic fields of the advanced wave based on Maxwell's electromagnetic theory.

2.5. Sources of Plane Waves

Assuming there is a constant AC current, this current is on an infinite flat plate. See Figure 1. This current is

$$\mathbf{J}(t) = J_0 \exp(j\omega t) \hat{z} = J(t) \hat{z} \quad (53)$$

Considering the retarded vector potential,

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}}{r} \exp(-jkr) d\sigma \hat{z} \quad (54)$$

Consider that the flat plate current is a circle with a radius of R .

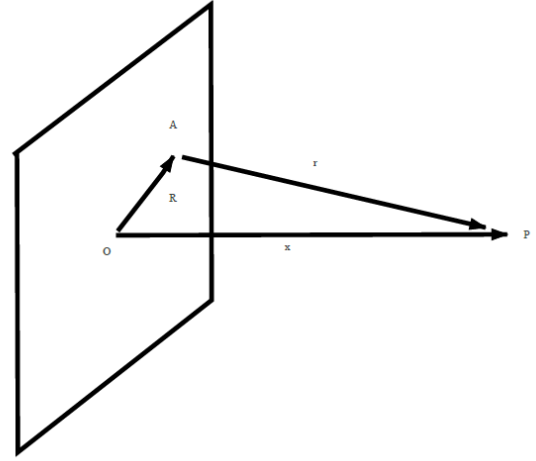


Figure 1. Plane electromagnetic waves are generated by an infinite flat plate current

$$R = \sqrt{y^2 + z^2} \quad (55)$$

Consider,

$$r = \sqrt{x^2 + R^2} \quad (56)$$

$$\begin{aligned} A(t, x) &= \frac{\mu_0}{4\pi} J \int_0^{2\pi} d\theta \int_0^R \frac{1}{\sqrt{x^2 + R^2}} \\ &\quad \exp(-jk\sqrt{x^2 + R^2}) R dR \end{aligned} \quad (57)$$

$$= \frac{\mu_0}{4\pi} 2\pi J \int_0^R \frac{1}{\sqrt{x^2 + R^2}} \exp(-jk\sqrt{x^2 + R^2}) \frac{1}{2} dR^2 \quad (58)$$

Consider,

$$d\sqrt{x^2 + R^2} = (x^2 + R^2)^{-\frac{1}{2}} \frac{1}{2} dR^2 \quad (59)$$

$$\begin{aligned} A(t, x) &= \frac{\mu_0}{4\pi} 2\pi J \int_0^R \exp(-jk\sqrt{x^2 + R^2}) d\sqrt{x^2 + R^2} \\ &= \frac{\mu_0}{2} \frac{J}{-jk} \int_0^R d \exp(-jk\sqrt{x^2 + R^2}) \\ &= \frac{\mu_0}{2} \frac{J}{-jk} \exp(-jk\sqrt{x^2 + R^2}) \Big|_0^R \\ &= \frac{\mu_0}{2} \frac{J}{-jk} (\exp(-jk\sqrt{x^2 + R^2}) - \exp(-jk\sqrt{x^2})) \\ &= \frac{1}{jk} \frac{\mu_0 J}{2} (\exp(-jkx) - \exp(-jk\sqrt{x^2 + R^2})) \end{aligned} \quad (60)$$

From this, the electric field can be calculated

$$\begin{aligned} \mathbf{E} &= -\frac{\partial}{\partial t} \mathbf{A} = -j\omega \mathbf{A} \\ &= -j\omega \frac{1}{jk} \frac{\mu_0 J}{2} (\exp(-jkx) - \exp(-jk\sqrt{x^2 + R^2})) \hat{z} \\ &= -\frac{\omega\mu_0}{k} \frac{J}{2} (\exp(-jkx) - \exp(-jk\sqrt{x^2 + R^2})) \hat{z} \end{aligned} \quad (61)$$

$$\frac{\omega\mu_0}{k} = \frac{\omega\mu_0}{\omega\sqrt{\mu_0\epsilon_0}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0 \quad (62)$$

therefore

$$\mathbf{E} = -\eta_0 \frac{J}{2} \left(\exp(-jkx) - \exp\left(-jk\sqrt{x^2 + R^2}\right) \right) \hat{z} \quad (63)$$

Consider,

$$\begin{aligned} \mathbf{E}^\infty &= \lim_{R \rightarrow \infty} \mathbf{E} \\ \mathbf{E}^\infty &= \lim_{R \rightarrow \infty} \left(-\eta_0 \frac{J}{2} \left(\exp(-jkx) - \exp\left(-jk\sqrt{x^2 + R^2}\right) \right) \right) \hat{z} \end{aligned} \quad (64)$$

The latter term in the above equation diverges, but the average value of the latter term is 0. Therefore, it can also be considered as 0 this way

$$\mathbf{E}^\infty = \eta_0 \frac{J}{2} \exp(-jkx) (-\hat{z}) \quad (65)$$

From this, the magnetic field can be calculated,

$$\begin{aligned} \mathbf{H} &= \frac{1}{\mu_0} \nabla \times \mathbf{A}(t, x) \\ \mathbf{H} &= \frac{1}{\mu_0} \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} = \frac{1}{\mu_0} \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & A_z \end{bmatrix} \\ &= -\frac{1}{\mu_0} \frac{\partial}{\partial x} A_z \hat{y} \\ &= -\frac{1}{\mu_0} \frac{\partial}{\partial x} \left(\frac{1}{jk} \frac{\mu_0 J}{2} \left(\exp(-jkx) - \exp\left(-jk\sqrt{x^2 + R^2}\right) \right) \right) \hat{y} \\ &= -\frac{1}{\mu_0} \left(\frac{1}{jk} \frac{\mu_0 J}{2} J(-jk \exp(-jkx)) \right. \\ &\quad \left. - (-jk) \frac{R}{\sqrt{x^2 + R^2}} \exp\left(-jk\sqrt{x^2 + R^2}\right) \right) \hat{y} \\ &= \left(\frac{J}{2} \left(\exp(-jkx) - \frac{R}{\sqrt{x^2 + R^2}} \exp\left(-jk\sqrt{x^2 + R^2}\right) \right) \right) \hat{y} \\ \mathbf{H}^\infty &= \lim_{R \rightarrow \infty} \mathbf{H} = \frac{J}{2} (\exp(-jkx)) \hat{y} \end{aligned} \quad (67) \quad (68)$$

We can also calculate the electric field from the magnetic field,

$$\begin{aligned} \frac{\partial}{\partial t} \epsilon_0 \mathbf{E}^\infty &= \nabla \times \mathbf{H}^\infty \\ &= \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x^\infty & H_y^\infty & H_z^\infty \end{bmatrix} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & H_y^\infty & 0 \end{bmatrix} \\ &= \hat{z} \frac{\partial}{\partial x} H_y^\infty \\ &= \hat{z} \frac{\partial}{\partial x} \frac{J}{2} \exp(-jkx) = -jk \hat{z} \frac{J}{2} \exp(-jkx) \end{aligned} \quad (69)$$

Or

$$j\omega\epsilon_0 \mathbf{E}^\infty = -jk \hat{z} \frac{J}{2} \exp(-jkx) \quad (70)$$

$$\mathbf{E}^\infty = -\frac{k}{\omega\epsilon_0} \frac{J}{2} \exp(-jkx) \hat{z} \quad (71)$$

$$\frac{k}{\omega\epsilon_0} = \frac{\omega\sqrt{\mu_0\epsilon_0}}{\omega\epsilon_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0 \quad (72)$$

$$\mathbf{E}^\infty = -\eta_0 \frac{J}{2} \exp(-jkx) \hat{z} \quad (73)$$

Considering that we are calculating the retarded potential,

$$\mathbf{E}^{r\infty} = \eta_0 \frac{J}{2} \exp(-jkx) (-\hat{z}) \quad (74)$$

$$\mathbf{H}^{r\infty} = \lim_{R \rightarrow \infty} \mathbf{H} = \frac{J}{2} (\exp(-jkx)) \hat{y} \quad (75)$$

The superscript r represents retardation, while the superscript ∞ represents the electromagnetic field of an infinite plate current. The above calculation results (74,75) are consistent with the previous formulas (47,48). Just consider,

$$E_0 = \eta_0 \frac{J}{2} \quad (76)$$

It is worth mentioning that in the above calculation, if the magnetic field is calculated first, and then the electric field is calculated by the magnetic field, the divergence problem encountered in calculating the electric field can be avoided. However, the results are consistent with our direct neglect of divergent items with an average of 0.

2.6. Advanced Potential

Considering the advanced potential,

$$\mathbf{A}^a = \frac{\mu_0}{4\pi} \int_{\sigma} \frac{J}{r} \exp(jkr) d\sigma \hat{z} \quad (77)$$

Similar to the previous method, the advanced potential can be obtained,

$$A^a(t, x) = -\frac{1}{jk} \frac{\mu_0 J}{2} \left(\exp(+jkx) - \exp\left(+jk\sqrt{x^2 + R^2}\right) \right) \quad (78)$$

$$\mathbf{E}^a = -j\omega \mathbf{A}^a(t, x) \quad (79)$$

$$\mathbf{E}^{a\infty} = \lim_{R \rightarrow 0} \mathbf{E}^{a\infty} \quad (80)$$

The second term diverges, but it averages 0. Ignore the second term with an average of zero,

$$\mathbf{E}^{a\infty} = +\eta_0 \frac{J}{2} \exp(+jkx) \hat{z} \quad (81)$$

(29) Strictly speaking, divergent terms cannot be ignored. But it is also possible to handle it this way in engineering. Consider Faraday's law (29)

$$\begin{aligned}
\mathbf{H} &= \frac{1}{-j\omega\mu_0} \nabla \times \mathbf{E} = \frac{1}{-j\omega\mu_0} \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix} \\
&= \frac{1}{-j\omega\mu_0} \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & E_z \end{bmatrix} = \frac{1}{-j\omega\mu_0} \hat{y} \left(-\frac{\partial}{\partial x} E_z \right) \\
&= \frac{1}{j\omega\mu_0} \hat{y} \frac{\partial}{\partial x} E_z
\end{aligned}$$

So,

$$\begin{aligned}
\mathbf{H}^{a\infty} &= \lim_{R \rightarrow \infty} \mathbf{H} = \frac{1}{j\omega\mu_0} \hat{y} \frac{\partial}{\partial x} E_z^{a\infty} \\
&= \frac{1}{j\omega\mu_0} \hat{y} (jk) E_z^{a\infty} \\
&= \frac{k}{\omega\mu_0} \hat{y} E_z^{a\infty} = \frac{1}{\eta_0} \hat{y} E_z^{a\infty} \\
&= \frac{J}{2} (\exp(+jkx)) \hat{y}
\end{aligned} \tag{82}$$

2.7. Superposition of Retarded and Advanced Waves

According to the viewpoint of Wheeler Feynman absorber theory [2,3], current radiates half the retarded wave and half the advanced wave. The author agrees with this viewpoint, that is,

$$\mathbf{E}^{\infty} = \frac{1}{2} (\mathbf{E}^{r\infty} + \mathbf{E}^{a\infty}) \tag{83}$$

$$\mathbf{H}^{\infty} = \frac{1}{2} (\mathbf{H}^{r\infty} + \mathbf{H}^{a\infty}) \tag{84}$$

When the wavelength is very large, i.e

$$kr = \frac{2\pi}{\lambda} r \rightarrow 0 \tag{85}$$

there is,

$$\begin{aligned}
\lim_{kr \rightarrow 0} \mathbf{E}^{\infty} &= \\
\lim_{kr \rightarrow 0} \frac{1}{2} \left(-\eta_0 \frac{J}{2} \exp(-jkx) \hat{z} + \eta_0 \frac{J}{2} \exp(+jkx) \hat{z} \right) &= \\
= \frac{1}{2} \left(-\eta_0 \frac{J}{2} + \eta_0 \frac{J}{2} \right) \hat{z} &= 0
\end{aligned} \tag{86}$$

$$\begin{aligned}
\lim_{kr \rightarrow 0} \mathbf{H}^{\infty} &= \\
= \lim_{kr \rightarrow 0} \frac{1}{2} \left(\frac{J}{2} (\exp(-jkx)) \hat{y} + \frac{J}{2} (\exp(+jkx)) \hat{y} \right) &= \\
= \frac{1}{2} \left(\frac{J}{2} + \frac{J}{2} \right) \hat{y} = \frac{J}{2} \hat{y}
\end{aligned} \tag{87}$$

From above, we can see that the electric field is completely cancelled and the magnetic field is superimposed. This seems to be a bit problematic. This makes the half retarded and half advanced scheme proposed by Wheeler Feynman cannot be realized when

$$r \ll \lambda \tag{88}$$

The condition we proposed item 1.2 cannot be met.

2.8. Correction of Magnetic Field

The previous calculations $\mathbf{E}^{r\infty}$, $\mathbf{H}^{r\infty}$ are the retarded electric and magnetic fields of infinite plate currents calculated according to Maxwell's electromagnetic field theory. This electric field and magnetic field are in phase. Such calculations can be found in Feynman Lectures on Physics [25]. Feynman specifically brought up this issue for discussion, and it seems that it is very entangled with this issue. According to Maxwell's theory,

$$\mathbf{E}^{r\infty} = -\eta_0 \frac{J}{2} \exp(-jkx) \hat{z} \tag{89}$$

$$\mathbf{H}^{r\infty} = \lim_{R \rightarrow \infty} \mathbf{H} = \frac{J}{2} (\exp(-jkx)) \hat{y} \tag{90}$$

Due to the fact that the plane wave generated by the flat plate current is itself a far-field, radiating electromagnetic field. According to the author's theory, a correction should be made to the magnetic field,

$$\mathbf{E}^{cr\infty} = \mathbf{E}^{r\infty} = -\eta_0 \frac{J}{2} \exp(-jkx) \hat{z} \tag{91}$$

$$\mathbf{H}^{cr\infty} = (-j) \mathbf{H}^{r\infty} = (-j) \frac{J}{2} (\exp(-jkx)) \hat{y} \tag{92}$$

The superscript *c* represents corrected, the superscript *r* represents retarded, the superscript *a* represents advanced, and the superscript ∞ represents the electric and magnetic fields generated by infinite plate currents. We calculate the electric and magnetic fields in the x-direction of the flat plate current. See Eq.(81)

$$\mathbf{E}^{ca\infty} = \mathbf{E}^{a\infty} = +\eta_0 \frac{J}{2} \exp(+jkx) \hat{z} \tag{93}$$

$$\mathbf{H}^{a\infty} = \frac{J}{2} (\exp(+jkx)) \hat{y} \tag{94}$$

For the advanced wave correction factor, it is not $(-j)$ but (j) , therefore there is,

$$\mathbf{H}^{ca\infty} = (j) \mathbf{H}^{a\infty} = j \frac{J}{2} (\exp(+jkx)) \hat{y} \tag{95}$$

Consider adding a phase factor to both the electric and magnetic fields, which will not change the phase difference between them. Only this phase difference is important. The electric and magnetic fields after increasing the phase factor are electromagnetic fields with "...". That is,

$$\mathbf{E}^{cr\infty} = j \mathbf{E}^{r\infty} = -j\eta_0 \frac{J}{2} \exp(-jkx) \hat{z} \tag{96}$$

$$\mathbf{H}^{cr\infty} = j\mathbf{H}^{cr\infty} = j(-j)\frac{J}{2}(\exp(-jkx))\hat{y} \quad (97)$$

$$\mathbf{E}^{ca\infty} = -j\mathbf{E}^{ca\infty} = -j\eta_0\frac{J}{2}\exp(+jkx)\hat{z} \quad (98)$$

$$\mathbf{H}^{ca\infty} = -j\mathbf{H}^{ca\infty} = -jj\frac{J}{2}(\exp(+jkx))\hat{y} \quad (99)$$

$\mathbf{E}'$, $\mathbf{H}'$ are electric and magnetic fields, considering,

$$\begin{aligned} \mathbf{E}^{cr\infty} &= j\mathbf{E}^{cr\infty} \\ &= -\eta_0\frac{J}{2}\exp\left(j\frac{\pi}{2}\right)\exp(-jkx)\hat{z} \\ &= -\eta_0\frac{J}{2}\exp\left(-j\left(kx - \frac{\pi}{2}\right)\right)\hat{z} \\ &= -\eta_0\frac{J}{2}\exp\left(-j\left(k\left(x - \frac{2\pi}{4k}\right)\right)\right)\hat{z} \\ &= -\eta_0\frac{J}{2}\exp\left(-j\left(k\left(x - \frac{\lambda}{4}\right)\right)\right)\hat{z} \end{aligned}$$

So the ratio of $\mathbf{E}'^{cr\infty}$ to $\mathbf{E}^{cr\infty}$ moves the current plane in the $-x$ direction

$$\mathbf{E}^{cr\infty} = \mathbf{E}^{cr\infty}\left(x - \frac{\lambda}{4}\right)$$

Similarly, we have

$$\mathbf{E}^{ca\infty} = \mathbf{E}^{ca\infty}\left(x - \frac{\lambda}{4}\right)$$

This way,

$$\mathbf{E}^{c\infty} = \frac{1}{2}(\mathbf{E}^{cr\infty} + \mathbf{E}^{ca\infty}) \quad (100)$$

$$\mathbf{H}^{c\infty} = \frac{1}{2}(\mathbf{H}^{cr\infty} + \mathbf{H}^{ca\infty}) \quad (101)$$

Hence,

$$\mathbf{E}^{cr\infty} = -j\eta_0\frac{J}{2}\exp(-jkx)\hat{z} \quad (102)$$

$$\mathbf{H}^{cr\infty} = \frac{J}{2}(\exp(-jkx))\hat{y} \quad (103)$$

$$\mathbf{E}^{ca\infty} = -j\eta_0\frac{J}{2}\exp(+jkx)\hat{z} \quad (104)$$

$$\mathbf{H}^{ca\infty} = \frac{J}{2}(\exp(+jkx))\hat{y} \quad (105)$$

Hence,

$$\mathbf{E}^{c\infty} = -j\frac{1}{2}\left(\eta_0\frac{J}{2}\exp(-jkx)\hat{z} + \eta_0\frac{J}{2}\exp(+jkx)\hat{z}\right) \quad (106)$$

$$\mathbf{H}^{c\infty} = \frac{1}{2}\left(\frac{J}{2}\exp(-jkx)\hat{y} + \frac{J}{2}\exp(+jkx)\hat{y}\right) \quad (107)$$

$$\lim_{kr \rightarrow 0} \mathbf{E}^{c\infty} = j\frac{\eta_0}{2}J\left(-\hat{z}\right) \quad (108)$$

$$\lim_{kr \rightarrow 0} \mathbf{H}^{c\infty} = \frac{J}{2}\hat{y} \quad (109)$$

In this way, electromagnetic waves appear to emanate from the plane of $(x = -\frac{\lambda}{4})$. For this type of wave, the waves are superimposed instead of offsetting.

We can see that according to the author's revised plane wave, the retarded electric field and the advanced electric field remain superimposed at $kr \rightarrow 0$, and the retarded magnetic field and the advanced magnetic field also superimpose, rather than cancel. In this way, we have implemented the theory of half retarded and half advanced proposed by Wheeler Feynman [2,3].

In addition, according to the author's electromagnetic theory, the electric and magnetic fields of plane waves maintain a 90 degree phase difference. This is consistent with the quasi-static situation. Under quasi-static conditions, electromagnetism also lags behind the magnetic field by a phase of $j = \exp\left(j\frac{\pi}{2}\right)$. In this way,

half retarded waves and half advanced waves can degenerate into quasi-static electromagnetic fields when the condition $r \ll \lambda$ is met.

3. Simple Circuit

The following figure 2 is a simple circuit, for which we can consider the problem under magnetic quasi-static conditions.

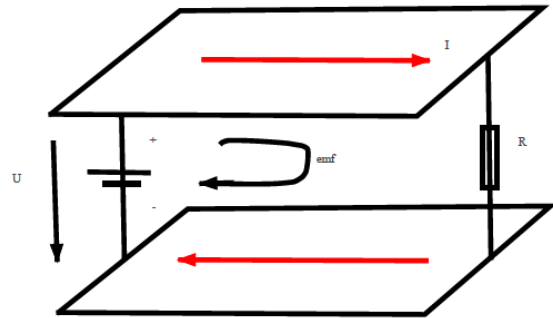


Figure 2. Simple circuit. It is composed of a power supply and a resistor, connected by a wide wire as shown in the figure. Line width L_y , line height L_z , with current I_0 . The voltage drop on the line is U

Consider a simple circuit composed of a power supply and a resistor, for this circuit. Obviously, we are considering that the voltage between the two wires is U . So the electrostatic field is

$$\mathbf{E}_s = \frac{U}{L_z}\left(-\hat{z}\right) = -\hat{z}\frac{U}{L_z} \quad (110)$$

That is to say, the electric field should be calculated based on voltage. Alternatively, the electric field is calculated by the electric potential ϕ . $\mathbf{E}_s$ is static electric field that is related to scalar potential ϕ . We are applying

$$\mathbf{E}_s = -\nabla\phi \quad (111)$$

$$U = \phi_{up} - \phi_{down} \quad (112)$$

The above is for the electric field. The magnetic field is calculated by the ampere circuital theorem. For upper current conductors,

$$\mathbf{H}_{up} \approx \frac{J}{2} \hat{y} \quad (113)$$

$$\mathbf{H}_{down} \approx \frac{J}{2} \hat{y} \quad (114)$$

The symbol “ $\approx$ ” in the above equation is because the width of the wire is finite, so the approximate value obtained above is.

$$\mathbf{H} = \mathbf{H}_{up} + \mathbf{H}_{down} \quad (115)$$

$$\mathbf{H} \approx 2 \frac{J}{2} \hat{y} = J \hat{y} = \frac{I}{L_y} \hat{y} \quad (116)$$

So the wire power

$$\begin{aligned} W &= UI^* \\ W &= \frac{UU^*}{R} \end{aligned} \quad (117)$$

On the other hand, Poynting vector,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}^* = -\hat{z} \frac{U}{L_z} \times \frac{I^*}{L_y} \hat{y} = \hat{x} \frac{UI^*}{L_z L_y} \quad (118)$$

We can calculate the power based on the Poynting vector, and the output power is,

$$\begin{aligned} W &= \iint_{\sigma} \mathbf{S} \cdot \hat{x} d\sigma \\ &= \iint_{\sigma} \frac{UI^*}{L_z L_y} d\sigma = \frac{UI^*}{L_z L_y} L_z L_y = UI^* \end{aligned} \quad (119)$$

The two methods for calculating power are consistent. From the above simple circuit method, we can see that the magnetic field of the circuit should be calculated according to the current through the ampere circuital law. The electrostatic field is calculated based on the potential $-\nabla\phi$. Our question is, does this circuit have an induced electric field? Of course, if we assume that the current is alternating current. So the magnetic field will induce an induced electric field. We know,

$$\oint_C \mathbf{E}_i \cdot d\mathbf{l} = -j\omega\mu_0 \oint_{\Gamma} \mathbf{H} \cdot \hat{n} d\Gamma \sim -jI \quad (120)$$

It can be seen that the induced electric field near the power source is

$$\mathbf{E}_i \sim -jI \hat{z} = jI \begin{pmatrix} -\hat{z} \end{pmatrix}$$

The Poynting vector can be calculated from the induced electric field, which is

$$\mathbf{E}_i \times \mathbf{H}^* \sim j \quad (121)$$

The Poynting vector of an induced electric field is a pure imaginary number and does not transfer energy. We

will calculate the transmission line later. Transmission lines are very similar to simple circuit. The transmission line should consider the transmission of waves. But there are some properties that should be preserved. For example, a magnetic field can be calculated from current. The Poynting vector composed of induced electric and magnetic fields is a pure imaginary number. Energy should be transmitted through the Poynting vector composed of a static electric field $\mathbf{E}_s$ and a magnetic field $\mathbf{H}$. That is to say, the transmission line should meet the conditions in the wave field (88) and can degenerate to the simple circuit mentioned above.

4. Review of Using Circuit Methods to Study Transmission Line Problems

Few people use Maxwell's equations to solve transmission line problems. In most cases, the circuit method is used. This chapter reviews this method. We are considering a small section of the transmission line, Δx . Assuming the impedance per unit length of this circuit is

$$Z = R + j\omega L \quad (122)$$

The conductivity per unit length is,

$$Y = g + j\omega C \quad (123)$$

Further consider the input voltage and current as,

$$U_{in} = U(x) \quad (124)$$

$$I_{in} = I(x) \quad (125)$$

The output voltage is

$$U_{out} = U(x + \Delta x) \quad (126)$$

$$I_{out} = I(x + \Delta x) \quad (127)$$

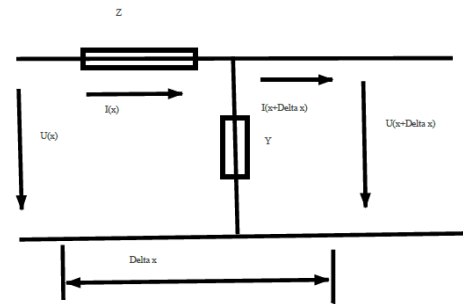


Figure 3. The voltage drop on the transmission line is $U(x)$, and the current is $I(x)$

Using the Kirchhoff voltage theorem,

$$U(x) = IZ\Delta x + U(x + \Delta x) \quad (128)$$

or

$$-IZ\Delta x = U(x + \Delta x) - U(x) \quad (129)$$

Or

$$-IZ = \frac{U(x+\Delta x) - U(x)}{\Delta x} \quad (130)$$

or

$$-IZ = \frac{\partial}{\partial x} U(x) \quad (131)$$

According to Kirchhoff's current theorem,

$$I(x) - I(x+\Delta x) = U(x)Y\Delta x \quad (132)$$

or

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = -U(x)Y \quad (133)$$

or

$$\frac{\partial}{\partial x} I = -U(x)Y \quad (134)$$

Consider (130) Consider

$$I = -\frac{1}{Z} \frac{\partial}{\partial x} U(x) \quad (135)$$

Substitution (134)

$$\frac{\partial}{\partial x} \left(-\frac{1}{Z} \frac{\partial}{\partial x} U(x) \right) = -U(x)Y \quad (136)$$

or

$$\frac{\partial^2}{\partial x^2} U(x) = ZYU(x) \quad (137)$$

If R and g are both small quantities, they can be ignored,

$$ZY = (R + j\omega L)(g + j\omega C) \approx j\omega Lj\omega C = -\omega^2 LC \quad (138)$$

Hence,

$$\frac{\partial^2}{\partial x^2} U(x) = -\omega^2 LCU(x) \quad (139)$$

or

$$k = \omega^2 LC \quad (140)$$

$$\frac{\partial^2}{\partial x^2} U(x) + k^2 U(x) = 0 \quad (141)$$

The solution is

$$U(x) = U_0 \exp(-jkx) \quad (142)$$

Similarly, it can be concluded that,

$$I(x) = I_0 \exp(-jkx) \quad (143)$$

Consider,

$$\begin{aligned} I(x) &= -\frac{1}{Z} \frac{\partial}{\partial x} U(x) = -\frac{1}{Z} (-jk) U(x) = -\frac{1}{j\omega L} (-jk) U(x) \\ &= \frac{1}{\omega L} k U(x) = \frac{1}{\omega L} \omega \sqrt{LC} U(x) = \sqrt{\frac{C}{L}} U(x) \end{aligned} \quad (144)$$

or

$$I(x) = \sqrt{\frac{C}{L}} U(x) \quad (145)$$

$$U(x) = \sqrt{\frac{L}{C}} I(x) \quad (146)$$

It can be seen that $U(x)$ and $I(x)$ are in phase.

$$Z_0 = \sqrt{\frac{L}{C}} \quad (147)$$

The characteristic impedance of a transmission line. Where C is the capacitance per unit length. L is the inductance per unit length. Consider a special case where the radius of the wire is assumed to be a . The distance between two wires is d , so the capacitance per unit length is

$$C = \frac{2\pi\epsilon_0}{\ln \frac{d}{a}} \quad (148)$$

The inductance per unit length is,

$$L = \frac{\mu_0}{2\pi} \ln \frac{d}{a} \quad (149)$$

The characteristic impedance is

$$Z_0 = \sqrt{\frac{\frac{\mu_0}{2\pi} \ln \frac{d}{a}}{\frac{2\pi\epsilon_0}{\ln \frac{d}{a}}}} = \frac{\ln \frac{d}{a}}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \quad (150)$$

5. Using the Electromagnetic Field Theory in Today's Textbook to Calculate Parallel Transmission Lines

The transmission line consists of an upper conductor and a lower conductor, forming a waveguide device. This device can confine waves within the transmission line. At the transmission line $x=0$, there is a current J in the direction of the current along the z -axis. This current generates a magnetic vector potential A .

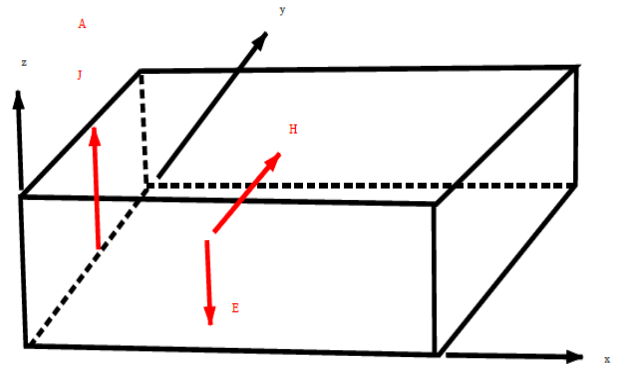


Figure 4. Parallel transmission line, line width L_y , line height L_z , with current I_0 . The voltage drop on the line is U

5.1. Parallel Transmission Lines in Electromagnetic Theory Textbooks

Our textbooks generally approach transmission line problems by considering TEM waves, where the electric field follows the z -axis direction and the magnetic field follows the y -axis direction,

$$\mathbf{E} = E_z(x, t) \hat{z} \quad (151)$$

$$\mathbf{H} = H_y(x, t) \hat{y} \quad (152)$$

Satisfy Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mu_0 \mathbf{H} \quad (153)$$

$$\nabla \times \mathbf{H} = +\frac{\partial}{\partial t} \epsilon_0 \mathbf{E} \quad (154)$$

The above equation assumes that $\mathbf{J}$ and ρ are both 0. Therefore, ϕ is 0.

$$\nabla \times \mathbf{E} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & E_z \end{bmatrix} \quad (155)$$

$$= -\hat{y} \left(\frac{\partial}{\partial x} E_z \right)$$

$$\nabla \times \mathbf{H} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{bmatrix} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & H_y & 0 \end{bmatrix} \quad (156)$$

$$= \hat{z} \frac{\partial}{\partial x} H_y$$

The Maxwell equation is transformed into (153,154),

$$\frac{\partial}{\partial x} E_z = \frac{\partial}{\partial t} \mu_0 H_y \quad (157)$$

$$\frac{\partial}{\partial x} H_y = \frac{\partial}{\partial t} \epsilon_0 E_z \quad (158)$$

Or

$$\frac{\partial^2}{\partial x^2} E_z = \frac{\partial}{\partial t} \mu_0 \frac{\partial}{\partial x} H_y = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E_z \quad (159)$$

Or

$$\frac{\partial^2}{\partial x^2} E_z = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E_z \quad (160)$$

Or

$$\frac{\partial^2}{\partial x^2} E_z = \mu_0 \epsilon_0 (j\omega)^2 E_z \quad (161)$$

Or

$$\frac{\partial^2}{\partial x^2} E_z + k^2 E_z = 0 \quad (162)$$

wherein

$$k^2 = \mu_0 \epsilon_0 \omega^2$$

The above equation is the Helmholtz equation, and its solution is,

$$E_z = E_0 \exp(-jkx) \quad (163)$$

Similarly, for magnetic fields,

$$H_y = H_0 \exp(-jkx) \quad (164)$$

Consider (157),

$$\frac{\partial}{\partial x} E_z = \frac{\partial}{\partial t} \mu_0 H_y \quad (165)$$

Or

$$-jkE_z = j\omega\mu_0 H_y \quad (166)$$

Or

$$-kE_z = \omega\mu_0 H_y \quad (167)$$

Or

$$-E_z = \frac{\omega\mu_0}{k} H_y = \frac{\omega\mu_0}{\omega\sqrt{\epsilon_0\mu_0}} H_y = \sqrt{\frac{\mu_0}{\epsilon_0}} H_y = \eta_0 H_y \quad (168)$$

$$E_z = -\eta_0 H_y \quad (169)$$

This indicates that the electric field and magnetic field are in phase. We assume that,

$$H_y = H_0 \exp(-jkx) \quad (170)$$

Obtain,

$$E_z = -\eta_0 H_0 \exp(-jkx) \quad (171)$$

The reference literature for solving transmission line problems using the field method can be seen from [26], and Liang Changhong particularly compared the circuit method with the field method,

$$\left[\begin{array}{l} \frac{\partial}{\partial x} (-U(x)) = j\omega LI \\ \frac{\partial}{\partial x} I = j\omega C (-U(x)) \end{array} \right] \text{ vs } \left[\begin{array}{l} \frac{\partial}{\partial x} E_z = j\omega\mu_0 H_y \\ \frac{\partial}{\partial x} H_y = j\omega\epsilon_0 E_z \end{array} \right] \quad (172)$$

If we take a double wire transmission line as an example, assuming the radius of the wire is a and the distance between the two wires is d , the capacitance of the wire is

$$C = \frac{2\pi\epsilon_0}{\ln \frac{d}{a}} \quad (173)$$

Inductance is

$$L = \frac{\mu_0}{2\pi} \ln \frac{d}{a} \quad (174)$$

Hence,

$$\begin{aligned} \left[\begin{aligned} \frac{\partial}{\partial x}(-U(x)) &= j\omega LI \\ \frac{\partial}{\partial x} I &= j\omega C(-U(x)) \end{aligned} \right] \\ = \left[\begin{aligned} \frac{\partial}{\partial x}(-U(x)) &= j\omega \frac{\mu_0}{2\pi} \ln \frac{d}{a} I \\ \frac{\partial}{\partial x} I &= j\omega \frac{2\pi\epsilon_0}{\ln \frac{d}{a}} (-U(x)) \end{aligned} \right] \end{aligned} \quad (175)$$

or

$$\begin{aligned} \frac{\partial^2}{\partial x^2}(-U(x)) &= j\omega \frac{\mu_0}{2\pi} \ln \frac{d}{a} \frac{\partial}{\partial x} I \\ &= j\omega \frac{\mu_0}{2\pi} \ln \frac{d}{a} \left(j\omega \frac{2\pi\epsilon_0}{\ln \frac{d}{a}} (-U(x)) \right) \\ &= -\omega^2 \mu_0 \epsilon_0 (-U(x)) \end{aligned} \quad (176)$$

or

$$\frac{\partial^2}{\partial x^2} U(x) + \omega^2 \mu_0 \epsilon_0 U(x) = 0 \quad (177)$$

lets,

$$k^2 = \omega^2 \mu_0 \epsilon_0$$

we obtain,

$$U(x) = U_0 \exp(-jkx) \quad (178)$$

This indicates that the field method can obtain the same results as the circuit method. If considering the following changes

$$I, -U(x), L, C \leftrightarrow H_y, E_z, \mu_0, \epsilon_0 \quad (179)$$

The left and right sides of the formula (172) are equivalent.

5.2. Problems with Methods in Parallel Transmission Line in Textbooks

This method seems to have no problem calculating the electric and magnetic fields according to Maxwell's equations. Actually, it is an incorrect methode (the author believes it is incorrect). For transmission lines, the electric field consists of two parts: the static electric field and the induced electric field. This is already evident in the previous chapter 3. The transmission line should be able to degenerate into a simple circuit. Because there is voltage between the upper and lower wires. This voltage will generate an static electric field, which is generated by electric potential and is also in the same phase as the current, that is,

$$\mathbf{E}_s = -\nabla \phi \quad (180)$$

Additionally, at wire $x = 0$, there is a power source, or equivalent power source. This power supply has current. This current should contribute to the magnetic vector potential

$$\mathbf{J} = J_0 \hat{z} \quad (181)$$

We should calculate the induced electric field based on the magnetic vector potential $\mathbf{A}$. This method is exactly the same as the one we use to solve antenna problems. Otherwise, one method is used for solving the antenna, and another method is used for calculating the transmission line. That is in correct.

The author believes that electrostatic fields

$$E_{sz} = -\frac{(\phi_{up} - \phi_{down})}{L_z} = -\frac{U}{L_z} \quad (182)$$

$\mathbf{E}_s$ It's an electrostatic field. ϕ_{up} is the potential of the upper wire. ϕ_{down} is the potential of the lower wire.

$$\begin{aligned} U &\equiv \phi_{up} - \phi_{down} \\ U &= U_0 \exp(-jkx) \end{aligned} \quad (183)$$

U_0 is the initial voltage, which is a constant.

$$E_{sz} = -\frac{1}{L_z} U_0 \exp(-jkx) \quad (184)$$

$$\begin{aligned} \mathbf{E}_s &= \frac{1}{L_z} U_0 \exp \exp(-jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \\ &= E_{s0} \exp(-jkx) \begin{pmatrix} -\hat{z} \end{pmatrix} \end{aligned} \quad (185)$$

wherein,

$$E_{s0} = \frac{1}{L_z} U_0 \quad (186)$$

At this point, we will find that the curl of this electrostatic field is not zero

$$\begin{aligned} \nabla \times \mathbf{E}_s &= -\hat{y} \left(\frac{\partial}{\partial x} E_z \right) = -\hat{y} \frac{\partial}{\partial x} (E_{s0} \exp(-jkx)(-1)) \\ &= \hat{y} \frac{\partial}{\partial x} (E_{s0} \exp(-jkx)) \\ &= -jk E_{s0} \exp(-jkx) \hat{y} \end{aligned} \quad (187)$$

The author believes that this part of the electrostatic field cannot follow the Faraday formula

$$\nabla \times \mathbf{E}_s = -j\omega \frac{\partial}{\partial t} \mathbf{B} \quad (188)$$

Contribute to the magnetic field. Therefore, the above formula cannot be used to calculate the magnetic field. This is because Maxwell's original equation was

$$\mathbf{E} = -\nabla \phi - \frac{\partial}{\partial t} \mathbf{A} \quad (189)$$

In this equation, we can only use $-\nabla \phi$ to find the electrostatic field $\mathbf{E}_s$.

$$\mathbf{E}_s = -\nabla \phi \quad (190)$$

We can only use vector potential $\mathbf{A}$ to calculate the induced electric field

$$\mathbf{E}_i = -\frac{\partial}{\partial t} \mathbf{A} \quad (191)$$

This formula is equivalent to Faraday's law

$$\nabla \times \mathbf{E}_i = -\frac{\partial}{\partial t} \mathbf{B} = -j\omega \mathbf{B} \quad (192)$$

Therefore, we can only using the curl of induced electric field $\mathbf{E}_i$ to calculate the magnetic field $\mathbf{B}$. Alternatively, use the curl of vector potential $\mathbf{A}$ to find the magnetic field $\mathbf{B}$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (193)$$

The curl of the electrostatic field in formula (187) is not 0, which is a very special case. In this case, write again

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B} \quad (194)$$

It's not correct because the above equation means,

$$\nabla \times \mathbf{E}_i + \nabla \times \mathbf{E}_s = -\frac{\partial}{\partial t} \mathbf{B} \quad (195)$$

And at this point

$$\nabla \times \mathbf{E}_s \neq 0 \quad (196)$$

The equation (189) is the Faraday's law in Maxwell's own Maxwell equation, while the equation (194) is a modified Maxwell's equation by Maxwell's descendants. At this point, it can be seen that Maxwell's own Maxwell's equation is more advanced than that of Maxwell's descendants.

5.3. Revise the Above Theory

The equation (151,152) actually means,

$$\mathbf{E}_i = E_{zi}(x,t)\hat{z} \quad (197)$$

$$\mathbf{H} = H_y(x,t)\hat{y} \quad (198)$$

The subscript (i) means the induced electric field $\mathbf{E}_i$ instead of the electric field $\mathbf{E}$. The induced electric field and magnetic field satisfy the following relationship,

$$\nabla \times \mathbf{E}_i = -\frac{\partial}{\partial t} \mu_0 \mathbf{H} \quad (199)$$

$$\nabla \times \mathbf{H} = +\frac{\partial}{\partial t} \epsilon_0 \mathbf{E}_i \quad (200)$$

Comparing the above equation with (153,154) shows the difference between the two. The solution of the above equation

$$H_y = H_0 \exp(-jkx) \quad (201)$$

$$E_{iz} = -\eta_0 H_0 \exp(-jkx) \quad (202)$$

The above two formulas are the solution obtained based on the retarded wave that satisfies the Maxwell's equations. If written completely, it should be

$$\mathbf{H}_{yMaxwell}^{(r)} = H_0 \exp(-jkx) \quad (203)$$

$$\mathbf{E}_{izMaxwell}^{(r)} = -\eta_0 H_0 \exp(-jkx) \quad (204)$$

The superscript (r) represents retarded, and the subscript i represents induced. The subscript $Maxwell$ represents the value calculated according to Maxwell's electromagnetic theory. The magnetic field in the above equation is already correct, so we need to modify the induced electric field according to (21,22)

$$\mathbf{E}_i^{(r)} = j\mathbf{E}_{iMaxwell}^{(r)} = j\eta_0 H_0 \exp(-jkx) \left(-\hat{z} \right) \quad (205)$$

$$\mathbf{H}^{(r)} = \mathbf{H}_{Maxwell}^{(r)} = H_0 \exp(-jkx) \hat{y} \quad (206)$$

In this way, we obtain a magnetic field and an induced electric field. We noticed that after the correction, the phase of the induced electric field is consistent with that of a simple circuit, as described in chapter 3.

The Poynting vector is calculated from electrostatic and magnetic fields,

$$\begin{aligned} \mathbf{S} &= \mathbf{E}_s \times \mathbf{H}^* = E_{s0} \exp(-jkx) (-\hat{z}) \times (H_0 \exp(-jkx) \hat{y})^* \\ &= E_{s0} H_0^* \hat{x} \sim \hat{x} \end{aligned} \quad (207)$$

In the above equation,

$$\mathbf{E}_s \sim \mathbf{U} \sim \mathbf{I} \sim \mathbf{H} \quad (208)$$

That is to say, electrostatic field $\mathbf{E}_s$, the phase of is consistent with the phase of the voltage and the phase of the current, therefore it is also consistent with the phase of the magnetic field $\mathbf{H}$. The above equation (207) is a real number, so this Poynting vector transmits active power. If we calculate based on the induced electric field,

$$\begin{aligned} \mathbf{E}_i \times \mathbf{H}^* &= j\eta_0 H_0 \exp(-jkx) (-\hat{z}) \times H_0 \exp(-jkx) \hat{y} \\ &= j\eta_0 H_0 H_0^* \hat{x} \sim j\hat{x} \end{aligned} \quad (209)$$

The above equation is a pure imaginary number, indicating that reactive power does not transfer energy. Therefore, the power corresponding to the induced electric field is reactive power. That is to say, the author believes that there is still an induced electric field within the transmission line. Comparing to the electrostatic field $\mathbf{E}_s$, the phase of the induced electric field $\mathbf{E}_i$ lags by

$$j = \exp\left(j\frac{\pi}{2}\right).$$

6. Calculate the Electric and Magnetic Fields According to the Circuit Method

We hope to solve this problem in multiple ways. Firstly, we consider that the voltage and current have been solved

according to the circuit problem in Chapter 4. Assuming we have already calculated,

$$I = I_0 \exp(-jkx) \quad (210)$$

$$U = U_0 \exp(-jkx) \quad (211)$$

Now we need to calculate the electric and magnetic fields. among

$$\frac{U_0}{I_0} = Z = \sqrt{\frac{L}{C}} \quad (212)$$

is the impedance of the transmission line. Here we assume that there is no reflection.

6.1. Calculation of Electrostatic Field

$$\mathbf{E}_s = -\frac{U}{L_z} \hat{z} \quad (213)$$

So the electrostatic field is,

$$\mathbf{E}_s = \frac{U}{L_z} (-\hat{z}) = \frac{U_0 \exp(-jkx)}{L_z} (-\hat{z}) \quad (214)$$

L_z It is the height of the transmission line.

6.2. Calculation of Magnetic Field

Considering the contribution of the above current to the magnetic field, due to the infinite length of the wire, the vector potential is infinite. Consider parallel transmission lines being relatively wide. L_y compared to L_z is much larger, so we can approximately obtain

$$J_x = \frac{I}{L_y} \quad (215)$$

$$\mathbf{H} = \frac{1}{2} J_x \hat{y} = \frac{1}{2} \frac{I}{L_y} \hat{y} \quad (216)$$

We know that the contribution of the lower wire to the magnetic field is the same as that of the upper wire, so the total magnetic field is,

$$\mathbf{H} = \frac{I}{L_y} \hat{y} = \frac{1}{L_y} I_0 \exp(-jkx) \hat{y} \quad (217)$$

6.3. Calculating Power

The average power is

$$\begin{aligned} P &= UI^* = U_0 \exp(-jkx) (I_0 \exp(-jkx))^* \\ &= U_0 I_0^* = U_0^2 Z \end{aligned} \quad (218)$$

We will calculate the power based on the electric and magnetic fields

$$\mathbf{E}_s \times \mathbf{H}^* = \frac{U_0 \exp(-jkx)}{L_z} (-\hat{z}) \times \left(\frac{1}{L_y} I_0 \exp(-jkx) \hat{y} \right)^* \quad (219)$$

So

$$\mathbf{S} = \mathbf{E}_s \times \mathbf{H}^* = \frac{U_0 I_0^*}{L_z L_y} \hat{x} \quad (220)$$

$$P = \oint_{\Gamma} \mathbf{S} \cdot \hat{n} d\Gamma = L_z L_y \frac{U_0 I_0^*}{L_z L_y} \hat{x} \cdot \hat{x} = U_0 I_0^* \quad (221)$$

Therefore, the power calculated using Poynting vector is consistent with the power directly calculated using voltage and current.

6.4. Calculation of Induced Electric Field

Now we know that magnetic fields and induced electric fields can be calculated according to Faraday's law in Maxwell's equation,

$$\nabla \times \mathbf{E}_i = -j\omega\mu_0 \mathbf{H} \quad (222)$$

$$\begin{aligned} \nabla \times \mathbf{E}_i &= \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{ix} & E_{iy} & E_{iz} \end{bmatrix} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & E_{iz} \end{bmatrix} \\ &= -\hat{y} \left(\frac{\partial}{\partial x} E_{iz} \right) \end{aligned} \quad (223)$$

$$-\left(\frac{\partial}{\partial x} E_{iz} \right) = -j\omega\mu_0 H_y \quad (224)$$

$$-(-jkE_{iz}) = -j\omega\mu_0 H_y \quad (225)$$

$$-(kE_{iz}) = \omega\mu_0 H_y \quad (226)$$

$$E_{iz} = -\frac{\omega\mu_0}{k} H_y = -\eta_0 H_y \quad (227)$$

This means that

$$E_{izMaxwell}^{(r)} = -\eta_0 H_{yMaxwell}^{(r)} \quad (228)$$

wherein,

$$H_{yMaxwell}^{(r)} = \frac{1}{L_y} I_0 \exp(-jkx) \quad (229)$$

$$E_{izMaxwell}^{(r)} = -\eta_0 \frac{1}{L_y} I_0 \exp(-jkx) \quad (230)$$

We know that the above magnetic field calculation is correct, so we need to make corrections to the induced electric field (21,22)

$$E_{iz}^{(r)} = jE_{izMaxwell}^{(r)} = -j\eta_0 \frac{1}{L_y} I_0 \exp(-jkx) \quad (231)$$

$$H_y^{(r)} = H_{yMaxwell}^{(r)} = \frac{1}{L_y} I_0 \exp(-jkx) \quad (232)$$

So we see that if we calculate the Poynting vector,

$$\begin{aligned}
\mathbf{S} &= E_{iz}^{(r)} \hat{z} \times H_y^{(r)} \hat{y} \\
&= \left(-j\eta_0 \frac{1}{L_y} I_0 \exp(-jkx) \right) \left(\frac{1}{L_y} I_0 \exp(-jkx) \right)^* (-\hat{x}) \quad (233) \\
&= j\eta_0 \frac{1}{L_y} \frac{1}{L_y} I_0 I_0^* \hat{x}
\end{aligned}$$

The Poynting vector composed of induced electric and magnetic fields is a pure imaginary number. It is reactive power and does not transfer energy. These results are consistent with the 5.3 section of the previous chapter. Here, we calculate the magnetic field from the current and the induced electric field from the magnetic field, and the calculated induced electric field must be corrected. At this point, the induced electric field is corrected.

7. Calculate the Transmission Line Based on the Curl of the Magnetic Vector Potential

In this chapter, we do not consider the contribution of the upper and lower wires to the magnetic vector potential. Two wires act as waveguides to confine waves in one-dimensional space. Electromagnetic waves propagate along the x-axis. It can be considered that the magnetic vector potential propagates along x.

7.1. Calculating Vector Potential

The transmission line problem is a one-dimensional problem, where electromagnetic waves can only propagate in the x direction. The vector potential generated by this problem should be consistent with the vector potential generated by an infinite plate. The vector potential of an infinite plate current is,

$$\mathbf{A} \sim \int_V \frac{\mathbf{J}}{r} \exp(-jkx) dV \quad (234)$$

$$\mathbf{A} \sim J_0 \hat{z} \exp(-jkx) \quad (235)$$

Now we consider the magnetic vector potential of the current extending along the z-axis at the source $x = 0$, where the current is I_0 , the surface current density is,

$$J_0 = \frac{I_0}{L_y} \quad (236)$$

Hence,

$$\mathbf{J} = J_0 \hat{z} \quad (237)$$

Consider that the magnetic vector potential should propagate along the x-axis without decay. Therefore, the magnetic vector is,

$$\mathbf{A} \sim A_0 \exp(-jkx) \hat{z} = J_0 \hat{z} \exp(-jkx) \quad (238)$$

7.2. Calculation of Induced Electric Field

The induced electric field is,

$$\begin{aligned}
\mathbf{E}_i &= -\frac{\partial}{\partial t} \mathbf{A} \sim -j\omega A_0 \hat{z} \exp \exp(-jkx) \quad (239) \\
&= j\omega A_0 \exp(-jkx) (-\hat{z})
\end{aligned}$$

The induced electric field calculated from the magnetic vector potential does not need to be corrected.

7.3. Calculation of Magnetic Field from Vector Potential

The magnetic field can have the curl calculation of vector potential,

$$\mathbf{H} \sim \frac{1}{\mu_0} \nabla \times \mathbf{A} = \frac{1}{\mu_0} \nabla \times (A_0 \hat{z} \exp(-jkx)) \quad (240)$$

$$= \frac{1}{\mu_0} \nabla \exp(-jkx) \times A_0 \hat{z} \quad (241)$$

$$= \frac{1}{\mu_0} \exp(-jkx) (-jk\hat{x}) \times A_0 \hat{z} \quad (242)$$

$$= jk \frac{1}{\mu_0} A_0 \exp(-jkx) \hat{y} \quad (243)$$

assume,

$$k \frac{1}{\mu_0} A_0 = H_0$$

So, according to Maxwell's electromagnetic theory

$$\mathbf{H} \sim jH_0 \exp(-jkx) \hat{y} \quad (244)$$

We know that the induced electric field can be obtained from the Maxwell equation $\mathbf{E}_i$ in the $(-\hat{z})$ direction, the magnitude includes a spatial impedance η_0 . phase that is completely consistent with $\mathbf{H}$, that is,

$$\mathbf{E}_i = j\eta_0 H_0 \exp(-jkx) (-\hat{z}) \quad (245)$$

The above equation is based on Maxwell's electromagnetic theory to obtain the induced electric field, which is in phase with the magnetic field. Numerically, there is an additional impedance factor η_0 . Our research is only interested in phase, and the above equation is consistent with the formula (239).

7.4. Calculation of Static Electric Field

There is a current on the transmission line and a voltage U between two wires. This voltage can of course form an electrostatic field, which is consistent with the previous calculation.

$$U = U_0 \exp(-jkx) \quad (246)$$

$$\mathbf{E}_s = \frac{-U}{L_z} \hat{z} = \frac{U_0}{L_z} \exp(-jkx) (-\hat{z}) \quad (247)$$

7.5. Correction of Magnetic Field

Since we calculate the induced electric field based on the magnetic vector $\mathbf{A}$, the phase of the induced electric

field calculated is correct, and the phase of the magnetic field calculated from the induced electric field must be corrected,

$$\mathbf{H}_{real} = (-j) \mathbf{H}_{Maxwell} \quad (248)$$

$$\mathbf{H}_{Maxwell} = jJ_0 \exp(-jkx) \hat{y} \quad (249)$$

$$\mathbf{H}_{real} = (-j) jJ_0 \exp(-jkx) \hat{y} = J_0 \exp(-jkx) \hat{y} \quad (250)$$

$\mathbf{H}_{real}$ is the true magnetic field, which is also the magnetic field obtained after correction according to the author's electromagnetic field theory. It can be seen that the corrected magnetic field and current are in phase.

7.6. Discussion

According to classic electromagnetic field theory textbooks such as literature [26,27], the electromagnetic field satisfies the Maxwell equation,

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mu_0 \mathbf{H} \quad (251)$$

$$\nabla \times \mathbf{H} = +\frac{\partial}{\partial t} \epsilon_0 \mathbf{E} \quad (252)$$

The Maxwell equation above ignores the current J and charge density ρ . Due to ignoring ρ . Therefore, the electrostatic field generated by charges has also been ignored.

$$\mathbf{E}_s = -\nabla \phi \quad (253)$$

In fact, the transmission line is composed of two conductors, and there is an electrostatic field between them, which is generated by $-\nabla \phi$. This part of the field is in the same phase as the voltage and current on the transmission line. such as

$$\phi = z \frac{U}{L_z} \exp(-jkx) \quad (254)$$

This indicates that the electric potential varies linearly with the variable z .

$$E_{zs} = -\frac{\partial}{\partial z} \phi = -\frac{\partial}{\partial z} \left(z \frac{U}{L_z} \exp(-jkx) \right) \quad (255)$$

$$= -\frac{U}{L_z} \exp(-jkx)$$

from (245) we can obtain,

$$\mathbf{E}_i = -\frac{\partial}{\partial t} \mathbf{A} \quad (256)$$

$$\sim j\eta_0 J_0 \exp(-jkx) (-\hat{z})$$

Therefore, according to the author's theory, there are actually two types of electric fields within the transmission line, the electrostatic field $\mathbf{E}_s$ and induced electric field $\mathbf{E}_i$. According to general textbooks, there is only one type of electric field within the transmission line, whether it is an induced electric field or an electrostatic field. The induced electric field calculated by the author has a phase factor j comparing to current. The induced

electric field in general textbooks [26,27] does not exist. The electric field in general textbooks [26,27] does not have this phase factor comparing to current. Therefore, both methods can be verified through experiments.

Since the induced electric field can be regarded as a circuital field, we can measure it using a coil. When measuring with a coil, it is impossible for the electrostatic field to enter the measurement system.

8. Experiments and Measurements

Assuming we use an electromagnetic signal with a wavelength of 1 meter, the frequency of this meter wave is

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{1} = 300 \text{ MHz} \quad (257)$$

A transmission line of 5 meters is sufficient. It is already 5 times the wavelength, which can be considered long enough. For the convenience of measurement, double parallel wire transmission lines are selected. Assuming the radius of two wires a is one millimeter,

$$a = 0.001 \text{ Metre} \quad (258)$$

Assuming the distance d between the wires is one centimeter,

$$d = 0.01 \text{ Metre} \quad (259)$$

Calculate capacitance per unit length,

$$\begin{aligned} C &= \frac{2\pi\epsilon_0}{\ln \frac{d}{a}} = \frac{2\pi}{\ln 10} 8.854187817 \times 10^{-12} \\ &= 2.416 \times 10^{-11} \\ &= 24.16 \times 10^{-12} = 24.16 \text{ pF} \end{aligned} \quad (260)$$

Calculate the inductance per unit length,

$$\begin{aligned} L &= \frac{\mu_0}{2\pi} \ln \frac{d}{a} = \frac{4\pi \times 10^{-7}}{2\pi} \ln 10 = 2 \times 10^{-7} * \ln 10 \\ &= 4.60517 \times 10^{-7} \\ &= 0.460517 \times 10^{-6} = 0.460517 \mu\text{H} \end{aligned} \quad (261)$$

The characteristic impedance of the transmission line is,

$$Z = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.460517 \times 10^{-6}}{2.416 \times 10^{-11}}} = 136.06 \Omega \quad (262)$$

Therefore, a 136Ω resistor should be connected at the end of the transmission line.

8.1. Measurement of Magnetic Field

Magnetic field is the easiest to measure, using traditional current measurement equipment that actually uses Hall elements to measure the magnetic field around wires. Therefore, this device precisely provides the phase of the magnetic field. This phase is the phase of the current on the wire and also the phase of the magnetic field.

8.2. Measurement of Electrostatic Field

The phase of the electrostatic field can be obtained by measuring the voltage between two wires. This phase should be the same as the previously measured magnetic field phase.

8.3. Measurement of Induced Electric Field

The induced electric field can be measured using coil type magnetic field measurement equipment. Because this method of measuring magnetic field actually calculates the magnetic field by measuring the induced electric field (induced electromotive force). Therefore, we can use it to directly measure the phase of the induced electric field. Compare the phase of the induced electric field with the phase of the current.

According to classical electromagnetic field theory, this part of the induced electric field is generally ignored in textbooks. Or do not mention this part of the induced electric field at all. Alternatively, it can be considered that induced electric fields and static electric fields are the same thing, which is an electric field. The author believes that there is a phase lag between the induced electric field and the magnetic field, which is the factor of (j) . Therefore, induced electric fields and electrostatic fields are completely different. This is reflected in a simple circuit. The author believes that the transmission line problem must be able to degenerate into a simple circuit. Therefore, the induced electric field should lag behind the current by $j = \exp\left(j\frac{\pi}{2}\right)$.

8.4. Direct Measurement of Induced Electric Field Using an Electric Field Near Field Measuring Pen

The electric field near-field measurement pen can be used to measure the electric field. This measurement should include both electrostatic and induced electric fields, which are the sum of electrostatic and induced electric fields. According to classical electromagnetic theory, there is only one type of electric field on a transmission line, which is in phase with the current. According to the author's theory, there are electrostatic fields and induced electric fields on the transmission line, and the phase of the induced electric field is 90 degrees lagging than that of the magnetic field. Since the values of the two electric fields are basically the same, the phase factor will become 45 degrees. Namely

$$\mathbf{E} = \mathbf{E}_s + \mathbf{E}_i \quad (263)$$

$$\begin{aligned} E &= \eta H_0 \hat{y} + j\eta H_0 \hat{y} \\ &= \eta H_0 (1 + j) = \eta H_0 \exp\left(j\frac{\pi}{4}\right) \end{aligned} \quad (264)$$

Therefore, according to the author's method, it is calculated that the total electric field measured by the near-field measurement pen should have a phase lag of $\exp\left(j\frac{\pi}{4}\right)$. According to classical electromagnetic theory, this phase is 0.

9. Conclusion

The author has made revisions to the classical electromagnetic field theory, which includes incorporating advanced waves into the theory. In order to ensure that the electromagnetic theory including advanced and retarded waves remains self consistent. The author has made corrections to the solution of the Maxwell equation, mainly by correcting the phase of the far-field magnetic field. The Poynting vector of the antenna in the far field after correction is reactive power. At the same time, the mutual energy flow becomes active power. This revised theory is difficult to be verified through measurement. For example, for transmitting and receiving antennas, it is necessary to measure the far-field of the transmitting antenna. However, the far-field magnetic field of the transmitting antenna is very weak and difficult to measure. Both electric and magnetic fields are easy to measure on transmission lines. Therefore, this article chooses to further calculate the transmission line using the author's electromagnetic theory. The author also reviewed the calculation of transmission lines based on classical electromagnetic theory. Comparing the calculation results of the two, according to the author's electromagnetic theory, there is not only an electrostatic field but also an induced electric field on the transmission line. If we discover an induced electric field through measurement and the induced electric field lags behind the current by a

factor $j = \exp\left(j\frac{\pi}{2}\right)$, it indicates that the author's electromagnetic theory is correct. The author is preparing to complete this experiment in the next step. This experiment is relatively easy, and the author encourages readers to do it themselves too.

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