

Calculate the Energy Flow of Transformers, Antenna Systems, and Photons by Redefining the Radiated Electromagnetic Field of Plane-sheet Current

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Abstract The author found that many important unsolved problems in today's physics are actually caused by the unreasonable definition of radiated electromagnetic field in classical electromagnetic fields. For example, the wave particle duality problem and many different interpretations of quantum mechanics. The definition of radiated electromagnetic field in classical electromagnetic field theory has deviated from the original definition of quasi static electromagnetic field. This deviation leads to the failure of all energy theorems, including Poynting theorem and mutual energy flow theorem. The solution of Maxwell's equation cannot be fully satisfied. Either the boundary conditions cannot be satisfied, or the formula of electromagnetic field calculated by vector potential cannot be satisfied. The author chooses the electromagnetic field of plane sheet current as an example to study this problem. The author redefines the electromagnetic field in electromagnetic radiation, so that the newly defined radiated electromagnetic field is a more reasonable seamless generalization of quasi-static electromagnetic field. Two plane-sheet currents can form a transformer, so this method can also study the energy flow from the primary coil to the secondary coil of the transformer. When there is a certain distance between the secondary coil and the primary coil, the primary coil becomes a transmitting antenna and the secondary coil becomes a receiving antenna. Therefore, this method is also suitable for the study of antenna systems. As we all know, the energy of photons is emitted from the light source and absorbed by the light sink. The light source is equivalent to the transmitting antenna, and the light sink is equivalent to the receiving antenna. Therefore, the method introduced in this paper is also suitable for the photonic system including one light source and one light sink. According to the new definition of radiated electromagnetic field in this paper, the mutual energy flow is generated at the source of the electromagnetic wave, then propagates to the sink of the electromagnetic wave and is absorbed at the sink. This can explain the fact that photons are generated at the light source and annihilated at the light sink. This paper supports the author's view that the energy flow of photons is mutual energy flow. Mutual energy flow is the common product of retarded wave and advanced wave. The author supports that the advanced wave is a real and objective existence. The examples given in this paper show that the interpretation of mutual energy flow proposed by the author can be regarded as an improved transactional interpretation of quantum mechanics proposed by Cramer.

Keywords: Maxwell equation, reciprocity theorem, conservation of energy, Poynting theorem, energy flow, transformer, primary coil, secondary coil, transmitting antenna, receiving antenna, retarded wave, retarded potential, advanced wave, advanced potential, absorber, radiator, emitter, photons, quantum, electromagnetic wave, electromagnetic field, transactional interpretation

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1. Introduction

1.1. Dilemma of Classical Electric Field Theory

Although Maxwell's electromagnetic theory has won a great victory, it is not impeccable. Maxwell's electric field theory still can't solve the problem of photons. For

photons, we can only stay at the energy of $\omega\hbar$. We don't know the shape of a photon, we only know its probability distribution. In the classical electromagnetic theory, there is Poynting energy flow, but so far we can't use Poynting energy flow to explain the transformer. We can't use Poynting energy flow to explain how energy flows from the primary coil to the secondary coil of the transformer. In 1987, the author proposed the mutual energy theorem [8,25,26], and recently (2017) proposed the concept of mutual energy flow and the mutual energy flow theorem

[9,10,11,12]. Therefore, the author tries to use the mutual energy flow theorem to explain the energy flow between electromagnetic wave source and sink, but there are always mistakes. It is found that the solution of the radiated electromagnetic field in the classical electromagnetic field theory does not meet the Poynting theorem. It is found that the classical radiated electromagnetic field does not meet one of the three conditions: (1) the Maxwell equations, (2) the electromagnetic field obtained from the vector potential, and (3) the boundary conditions. Therefore, these solutions are actually invalid.

1.2. The Author's Electromagnetic Mutual Energy Theory

Mutual energy theorem, the theory of mutual energy flow theorem, [9,10,11,12] are the theories that support the existence of advanced waves in electromagnetic field theory. In quantum physics, the theories supporting the existence of advanced waves include Wheeler Feynman's absorber theory 1945 [1,2], and the absorber theory is based on a-action-at-distance theory [7,21,23]. Stephenson established his advanced wave theory around 1980 [22]. Cramer established the transactional interpretation of quantum mechanical transactions in 1986 based on the absorber theory [3,4].

On the other hand, Welch established the time-domain reciprocity theorem in 1960 [24], Rumsey proposed his new reciprocity theorem in 1963 [20], and de Hoop proposed the correlation reciprocity theorem at the end of 1987 [5]. The author proves that the author's mutual energy theorem [8,25,26] can be obtained by Fourier transform of de Hoop's reciprocal energy theorem. Welch's reciprocity theorem is a special case of de Hoop's reciprocity theorem. Rumsey's reciprocity theorem is consistent with the author's mutual energy theorem in form. Therefore, three reciprocity theorems [5,20,24] and one mutual energy theorem [8,25,26] are same physical formula. There are two electromagnetic fields in this formula, one of which is the advanced wave. In the application of electromagnetic field engineering, it is generally not recognized that the advanced wave is the real and objective existence of physics. Therefore, it is generally considered that this advanced wave is a virtual electromagnetic field. Therefore, this theorem should not be called energy theorem. Therefore, it is not surprising that it is called the reciprocity theorems. The advanced wave as virtual electromagnetic field in the reciprocity theorem is allowed.

The author believes that this theorem is an energy theorem. In 2014, the author returned to the subject of electromagnetic mutual energy. Found papers of Welch, Rumsey, and de Hoop. Because the author and the three of them have different positions on the same theorem, the author thinks it is the energy theorem, and others all think it is the reciprocity theorem. It is therefore necessary to clarify this fact. The author first proves that the mutual energy theorem is a sub-theorem of Poynting's theorem. It is generally recognized that Poynting's theorem is an energy theorem, so the mutual energy theorem is also an energy theorem. Furthermore, the author puts forward the theorem of mutual energy flow, the principle of mutual

energy and the principle of self energy, the laws that the radiation does not overflow the universe. This constitutes a complete electromagnetic theory, which the author calls mutual energy flow theory [9,10,11,12].

Recently, the author began to establish specific examples of mutual energy theory. It tries to solve the problems of how the energy flow of transformer flows from primary coil to secondary coil, and the transmission and reception of antenna radiation. In order to simplify the problem as much as possible, the author selects (1) the radiation and reception of dipole antenna, (2) the radiation and reception of infinite wire conductor, (3) the radiation and reception of infinite plane-sheet current. The plane-sheet is the simplest. In such a simple case, the problems of classical electromagnetic theory are also obviously exposed. In this case, the author must modify the definition of radiated electromagnetic field in order to fill the loophole.

Using the newly defined radiated electromagnetic field, Poynting's theorem and the mutual energy flow theorem are satisfied. In addition, the author's mutual energy theory also supports the author's interpretation of mutual energy flow [9] and Cramer's transactional interpretation of quantum mechanics [3,4].

1.3. Development of Mutual Energy Flow Theory

On the basis of the mutual energy theorem, the author puts forward the electromagnetic theory of mutual energy, including the principle of mutual energy, the law of energy conservation, the law that electromagnetic radiation does not overflow the universe, and the theorem of mutual energy flow. Constitute a complete electromagnetic field theory, referred to as mutual energy flow theory. The author applies the theory of mutual energy flow to quantum mechanics, puts forward the interpretation of mutual energy flow, holds that photons are mutual energy flow, and all particles are mutual energy flow, and tries to solve the problem of wave particle duality [9,10,11,12].

In order to complete the discussion of this paper, the author made several preparations in advance. The first preparation is to study the energy conservation law of the primary coil and secondary coil of the transformer. It is considered that the transmitting antenna and receiving antenna should meet the same law of energy conservation as the transformer system, so it is concluded that the radiation of the receiving antenna is a advanced wave [17,18,19].

Another preparatory work is to discuss the bug of Poynting's theorem. The solution of classical electric field theory does not meet Poynting's theorem. The main work of this paper is to discuss the bugs of Poynting's theorem and establish it in order to make up for this bugs. The author redefines the magnetic field in order to establish Poynting's theorem. This bug is not easy to see. Otherwise, how could it be hidden for more than 100 years [14,15,16].

Another kind of Green function to solve Maxwell's equation involving the mutual energy flow can be found in reference [27]. Mutual energy flow theorem has also been developed as mutual stress flow theorem and has been applied to generalize the Newton third law [28].

This paper attempts to solve the bug of the Poynting theorem by using the infinite plane-sheet current. In fact, this bug also appears in the mutual energy flow theorem. The author believes that Poynting's theorem and mutual energy flow theorem are energy conservation laws, so the solution of Maxwell's equation should first consider satisfying these energy conservation laws, and then satisfy Maxwell's equations. If choosing between Maxwell's equation and the law of conservation of energy (including Poynting's theorem, mutual energy principle and mutual energy flow theorem), the law of conservation of energy should be first satisfied rather than Maxwell's equations.

2. Problems of Radiation Electromagnetic Theory

For radiated electromagnetic field and potential function, we use lowercase letters $\mathbf{e}$, $\mathbf{d}$, $\mathbf{b}$, $\mathbf{h}$, ϕ , $\mathbf{a}$. Capital letters $\mathbf{E}$, $\mathbf{D}$, $\mathbf{B}$, $\mathbf{H}$, Φ , $\mathbf{A}$ are retained to represent quasi-static electromagnetic fields. The author does not think that the radiated electromagnetic field is a direct generalization of the quasi-static electromagnetic field, but regards them as two systems with completely different definitions.

2.1. Maxwell's Equations

So the Gauss law of Maxwell's equations is,

$$\nabla \cdot \mathbf{d} = \rho \quad (1)$$

where $\mathbf{d} = \epsilon_0 \mathbf{e}$. $\mathbf{d}$ is electric displacement. $\mathbf{e}$ is electric field. ρ is charge intensity. ϵ_0 is dielectric constant in the empty space. Definition of magnetic induction intensity is,

$$\mathbf{b} = \nabla \times \mathbf{a} \quad (2)$$

$\mathbf{a}$ is magnetic vector potential. $\mathbf{b} = \mu_0 \mathbf{h}$. $\mathbf{h}$ is magnetic field intensity. μ_0 is permeability in empty space. Definition of electric field,

$$\mathbf{e} = -\nabla \phi - \frac{\partial \mathbf{a}}{\partial t} \quad (3)$$

Faraday's law is,

$$\nabla \times \mathbf{e} = -\frac{\partial}{\partial t} \nabla \times \mathbf{a} = -\frac{\partial \mathbf{b}}{\partial t} \quad (4)$$

Maxwell-Ampere circular law is

$$\nabla \times \mathbf{h} = \mathbf{J} + \frac{\partial \mathbf{d}}{\partial t} \quad (5)$$

The current continuity equation is,

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad (6)$$

Scalar wave equation,

$$\nabla \cdot (-\nabla \phi - \frac{\partial \mathbf{a}}{\partial t}) = \frac{\rho}{\epsilon_0} \quad (7)$$

or

$$(-\nabla \cdot \nabla \phi - \frac{\partial}{\partial t} \nabla \cdot \mathbf{a}) = \frac{\rho}{\epsilon_0} \quad (8)$$

Consider Lorenz gauge condition,

$$\nabla \cdot \mathbf{a} = -\mu_0 \epsilon_0 \frac{\partial \phi}{\partial t} \quad (9)$$

There is,

$$(-\nabla \cdot \nabla \phi + \frac{\partial}{\partial t} (\mu_0 \epsilon_0 \frac{\partial \phi}{\partial t})) = \frac{\rho}{\epsilon_0} \quad (10)$$

or

$$\nabla \cdot \nabla \phi - \mu_0 \epsilon_0 \frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0} \quad (11)$$

The above is the wave equation of the scale potential. The solution of the above wave equation is (1) retarded wave,

$$\begin{aligned} \phi^{(+)} &= \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho(\mathbf{x}', t-r/c)}{r} dV \\ &\rightarrow \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho(\mathbf{x}', t)}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) dV \end{aligned} \quad (12)$$

The symbol $\rightarrow$ is one kind derivation. $\mathbf{k} = k \frac{\mathbf{r}}{r}$, $k = \frac{\omega}{c}$, in the above we have considered $\rho(\mathbf{x}, t) = \rho(\mathbf{x}, t) \exp(j\omega t)$. (2) Advanced wave,

$$\begin{aligned} \phi^{(-)} &= \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho(\mathbf{x}', t+r/c)}{r} dV \\ &\rightarrow \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho(\mathbf{x}', t)}{r} \exp(+j\mathbf{k} \cdot \mathbf{r}) dV \end{aligned} \quad (13)$$

According to Maxwell-Ampere law and the definition of $\mathbf{b}$ there is,

$$\nabla \times \nabla \times \mathbf{a} = \mu_0 \mathbf{J} + \mu_0 \frac{\partial \mathbf{d}}{\partial t} \quad (14)$$

Considering,

$$\nabla \times \nabla \times \mathbf{a} = \nabla \nabla \cdot \mathbf{a} - \nabla^2 \mathbf{a} \quad (15)$$

There is,

$$\nabla \nabla \cdot \mathbf{a} - \nabla^2 \mathbf{a} = \mu_0 \mathbf{J} + \mu_0 \frac{\partial \mathbf{d}}{\partial t} \quad (16)$$

Or

$$-\nabla \nabla \cdot \mathbf{a} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \mathbf{E} + \nabla^2 \mathbf{a} = -\mu_0 \mathbf{J} \quad (17)$$

Considering the Lorenz gauge condition (9),

$$-\nabla(-\mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}) + \mu_0 \epsilon_0 \frac{\partial}{\partial t} (-\nabla \phi - \frac{\partial \mathbf{a}}{\partial t}) + \nabla^2 \mathbf{a} = -\mu_0 \mathbf{J} \quad (18)$$

So there's the wave equation,

$$\nabla^2 \mathbf{a} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{a}}{\partial t^2} = -\mu_0 \mathbf{J} \quad (19)$$

Thus, the retarded wave is obtained,

$$\begin{aligned} \mathbf{a}^{(+)} &= \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\mathbf{x}', t-r/c)}{r} dV \\ &\rightarrow \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\mathbf{x}', \omega)}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) dV \end{aligned} \quad (20)$$

And the solution of the advanced wave.

$$\begin{aligned} \mathbf{a}^{(-)} &= \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\mathbf{x}', t+r/c)}{r} dV \\ &\rightarrow \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\mathbf{x}', \omega)}{r} \exp(+j\mathbf{k} \cdot \mathbf{r}) dV \end{aligned} \quad (21)$$

In the above we have considered that $\mathbf{J}(\mathbf{x}, t) = \mathbf{J}(\mathbf{x}) \exp(j\omega t)$.

2.2. Poynting Theorem

$$\nabla \cdot (\mathbf{e} \times \mathbf{h}) = (\nabla \times \mathbf{e}) \cdot \mathbf{h} - \mathbf{e} \cdot (\nabla \times \mathbf{h}) \quad (22)$$

or

$$\nabla \cdot (\mathbf{e} \times \mathbf{h}) = \left(-\frac{\partial}{\partial t} \mathbf{b}\right) \cdot \mathbf{h} - \mathbf{e} \cdot \left(\mathbf{J} + \frac{\partial}{\partial t} \mathbf{d}\right) \quad (23)$$

or

$$-\nabla \cdot (\mathbf{e} \times \mathbf{h}) = \mathbf{e} \cdot \mathbf{J} + \mathbf{e} \cdot \frac{\partial}{\partial t} \mathbf{d} + \mathbf{h} \cdot \frac{\partial}{\partial t} \mathbf{b} \quad (24)$$

Poynting theorem,

$$-\oint_{\Gamma} (\mathbf{e} \times \mathbf{h}) \cdot \hat{n} d\Gamma = \iiint_V (\mathbf{e} \cdot \mathbf{J} + \mathbf{e} \cdot \frac{\partial}{\partial t} \mathbf{d} + \mathbf{h} \cdot \frac{\partial}{\partial t} \mathbf{b}) dV \quad (25)$$

Similarly, we can get the Poynting theorem of complex numbers,

$$-\oint_{\Gamma} (\mathbf{e} \times \mathbf{h}^*) \cdot \hat{n} d\Gamma = \iiint_V (\mathbf{e} \cdot \mathbf{J}^* + j\omega(-\mathbf{e} \cdot \mathbf{d} + \mathbf{h} \cdot \mathbf{b})) dV \quad (26)$$

In vacuum, for the radiated electromagnetic field, the energy of the magnetic field is equal to that of the electric field, so there is,

$$-\mathbf{e} \cdot \mathbf{d} + \mathbf{h} \cdot \mathbf{b} = 0 \quad (27)$$

For a stable AC system, the energy of space remains stable. The power of the energy part should be zero. Therefore, the above formula is still valid. So, there is complex Poynting theorem in empty space,

$$-\oint_{\Gamma} (\mathbf{e} \times \mathbf{h}^*) \cdot \hat{n} d\Gamma = \iiint_V (\mathbf{e} \cdot \mathbf{J}^*) dV \quad (28)$$

2.3. Failure of Poynting's Theorem

Although we can derive Poynting's theorem from Maxwell's equations, it does not mean that we can find a solution that satisfies Maxwell's equations and Poynting's theorem at the same time. We know that the Poynting energy flow is not zero for any antenna, which can be obtained from the calculation of any antenna for example a dipole antenna, i.e.,

$$\Re(\oint_{\Gamma} (\mathbf{e} \times \mathbf{h}^*) \cdot \hat{n} d\Gamma) > 0 \quad (29)$$

$\Re$ means taking the real part. But,

$$\iiint_V (\mathbf{e} \cdot \mathbf{J}^*) dV \rightarrow \int_C \mathbf{e} \cdot d\mathbf{l}^* = \mathcal{E} I^* \quad (30)$$

$$\mathcal{E} \triangleq \int_C \mathbf{e} \cdot d\mathbf{l} = -M_{2,1} \frac{\partial}{\partial t} I \quad (31)$$

Where $M_{2,1}$ is mutual inductance. The mutual inductance of the primary coil to the secondary coil. Here, we assume that a current element is used to test the electromagnetic field, and this test current element constitutes the secondary coil C_2 .

$$M_{2,1} I = \frac{\mu_0}{4\pi} \int_{C_2} \int_{C_1} \frac{Id\mathbf{l}_1 \cdot d\mathbf{l}_2}{r} \quad (32)$$

Vector potential,

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_{C_1} \frac{Id\mathbf{l}_1}{r} \rightarrow \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}}{r} dV \quad (33)$$

The retarded and advanced vector potentials,

$$\mathbf{a}^{(+)} = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\mathbf{x}', t-r/c)}{r} dV \quad (34)$$

$$\rightarrow \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\mathbf{x}') \exp(-j\mathbf{k} \cdot \mathbf{r})}{r} dV$$

$$\mathbf{a}^{(-)} = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\mathbf{x}', t+r/c)}{r} dV \quad (35)$$

$$\rightarrow \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\mathbf{x}') \exp(+j\mathbf{k} \cdot \mathbf{r})}{r} dV$$

If the time retarded wave is considered, the mutual inductance of the retarded can be written as,

$$M_{2,1}^{(+)} I = \frac{\mu_0}{4\pi} \int_{C_2} \int_{C_1} \exp(-j\mathbf{k} \cdot \mathbf{r}) \frac{Id\mathbf{l}_1 \cdot d\mathbf{l}_2}{r} \quad (36)$$

If the time retarded is considered, the mutual inductance of the retarded can be written as

$$\begin{aligned} \iiint_V (\mathbf{e} \cdot \mathbf{J}^*) dV &= \mathcal{E} I^* = -M_{2,1}^{(+)} \left(\frac{\partial}{\partial t} I\right) I^* \\ &= -\frac{\partial}{\partial t} \frac{\mu_0}{4\pi} \int_{C_2} \int_{C_1} \exp(-j\mathbf{k} \cdot \mathbf{r}) \frac{Id\mathbf{l}_1 \cdot d\mathbf{l}_2}{r} I^* \\ &= -j\omega \frac{\mu_0}{4\pi} \int_{C_2} \int_{C_1} \exp(-j\mathbf{k} \cdot \mathbf{r}) \frac{Id\mathbf{l}_1 \cdot d\mathbf{l}_2}{r} I^* \\ &= -j\omega \frac{\mu_0}{4\pi} \int_{C_2} \int_{C_1} \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{r} I I^* \\ &= -j\omega \frac{\mu_0}{4\pi} M_{2,1} I I^* \end{aligned} \quad (37)$$

In the above $\mathbf{k} \cdot \mathbf{r} \rightarrow 0$ is considered. that means $\frac{2\pi}{\lambda} l \rightarrow 0$, or $l \ll \lambda$. l is the size of C_1 . λ is the wave length. If l very small compare to λ , the retarded factor $\exp(-j\mathbf{k} \cdot \mathbf{r})$ can be omitted. Or

$$\iiint_V (\mathbf{e} \cdot \mathbf{J}^*) dV = -j\omega \frac{\mu_0}{4\pi} M_{2,1} I I^* \quad (38)$$

$\frac{\mu_0}{4\pi} M_{2,1} I I^*$ is a pure real number, so the above formula is an imaginary number. Therefore, there are generally,

$$-\oint_{\Gamma} (\mathbf{e} \times \mathbf{h}^*) \cdot \hat{n} d\Gamma \neq \iiint_V (\mathbf{e} \cdot \mathbf{J}^*) dV \quad (39)$$

In the above, the real part of left is not 0, but the real part of right is 0. Here the author has assume the transmitting antenna has very small in size for example as a dipole antenna. Hence for this kind of antenna retarded factor $\exp(-j\mathbf{k} \cdot \mathbf{r}) \approx 1$. Therefore, Poynting vector theorem fails in general for radiated electromagnetic field. In fact, the radiated electromagnetic field has deviated greatly from the quasi-static electromagnetic field. This is why the author uses lowercase letters to represent the radiated electromagnetic field. Capital letters remain to indicate quasi-static electromagnetic fields.

2.4. Law of Conservation of Energy

We know that the law of conservation of current energy is N elements,

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \iiint_V (\mathbf{e}_i \cdot \mathbf{J}_j^*) dV = 0 \quad (40)$$

Current element $\mathbf{J}_j$ get power from current element $\mathbf{J}_i$ is $\iiint_V (\mathbf{e}_i \cdot \mathbf{J}_j^*) dV$, current element $\mathbf{J}_i$ Will provide the same amount of energy $-\iiint_V (\mathbf{e}_j^* \cdot \mathbf{J}_i) dV$ (providing energy is output energy, and there is a negative sign in front of all formulas. If it is a positive sign, it means energy is consumed), Hence there always is,

$$\iiint_V (\mathbf{e}_i \cdot \mathbf{J}_j^*) dV + \iiint_V (\mathbf{e}_j^* \cdot \mathbf{J}_i) dV = 0 \quad (41)$$

Therefore, the above law of energy conservation (40) is self-evident.

2.5. Mutual Energy Principle

Maxwell's equations can be written as,

$$-\frac{\partial}{\partial t} \mathbf{d} + \nabla \times \mathbf{h} = \mathbf{J} \quad (42)$$

$$-\nabla \times \mathbf{e} - \frac{\partial}{\partial t} \mathbf{b} = 0 \quad (43)$$

or

$$L\xi = \tau \quad (44)$$

where,

$$L = \begin{bmatrix} -\frac{\partial}{\partial t} & \nabla \times \\ -\nabla \times & -\frac{\partial}{\partial t} \end{bmatrix}, \tau = \begin{bmatrix} \mathbf{J} \\ 0 \end{bmatrix}, \xi = \begin{bmatrix} \mathbf{e} \\ \mathbf{h} \end{bmatrix} \quad (45)$$

We can prove the following mathmatic formula,

$$-\oint_{\Gamma} (\mathbf{e}_1 \times \mathbf{h}_2 + \mathbf{e}_2 \times \mathbf{h}_1) \cdot \hat{n} d\Gamma = \iiint_V \begin{pmatrix} \xi_1 \cdot L\xi_2 + L\xi_2 \cdot \xi_1 + \mathbf{e}_1 \cdot \frac{\partial}{\partial t} \mathbf{d}_2 \\ + \mathbf{e}_2 \cdot \frac{\partial}{\partial t} \mathbf{d}_1 + \mathbf{h}_1 \cdot \frac{\partial}{\partial t} \mathbf{b}_2 + \mathbf{h}_2 \cdot \frac{\partial}{\partial t} \mathbf{b}_1 \end{pmatrix} dV \quad (46)$$

Considered the Maxwell's equations (44), there is,

$$-\oint_{\Gamma} (\mathbf{e}_1 \times \mathbf{h}_2 + \mathbf{e}_2 \times \mathbf{h}_1) \cdot \hat{n} d\Gamma = \iiint_V \begin{pmatrix} \mathbf{e}_1 \cdot \mathbf{J}_2 + \mathbf{J}_2 \cdot \mathbf{e}_1 + \mathbf{e}_1 \cdot \frac{\partial}{\partial t} \mathbf{d}_2 \\ + \mathbf{e}_2 \cdot \frac{\partial}{\partial t} \mathbf{d}_1 + \mathbf{h}_1 \cdot \frac{\partial}{\partial t} \mathbf{b}_2 + \mathbf{h}_2 \cdot \frac{\partial}{\partial t} \mathbf{b}_1 \end{pmatrix} dV \quad (47)$$

The above can be written as,

$$-\sum_{i=1}^2 \sum_{j=1, j \neq i}^2 \oint_{\Gamma} (\mathbf{e}_i \times \mathbf{h}_j) \cdot \hat{n} d\Gamma = \sum_{i=1}^2 \sum_{j=1, j \neq i}^2 \iiint_V (\mathbf{e}_i \cdot \mathbf{J}_j + \mathbf{e}_i \cdot \frac{\partial}{\partial t} \mathbf{d}_j + \mathbf{h}_i \cdot \frac{\partial}{\partial t} \mathbf{b}_j) dV \quad (48)$$

The above can be generalized as,

$$-\sum_{i=1}^N \sum_{j=1, j \neq i}^N \oint_{\Gamma} (\mathbf{e}_i \times \mathbf{h}_j) \cdot \hat{n} d\Gamma = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \iiint_V (\mathbf{e}_i \cdot \mathbf{J}_j + \mathbf{e}_i \cdot \frac{\partial}{\partial t} \mathbf{d}_j + \mathbf{h}_i \cdot \frac{\partial}{\partial t} \mathbf{b}_j) dV \quad (49)$$

The above is the mutual energy principle. Here Γ is a closed surface which is the boundary of the volume V.

2.6. The Principle of Mutual Energy Supports the Law of Conservation of Energy

Changed to the frequency domain, there is the mutual energy principle,

$$-\oint_{\Gamma} (\mathbf{e}_1 \times \mathbf{h}_2^* + \mathbf{e}_2^* \times \mathbf{h}_1) \cdot \hat{n} d\Gamma = \iiint_V (\mathbf{e}_1 \cdot \mathbf{J}_2^* + \mathbf{e}_2^* \cdot \mathbf{J}_1) dV \quad (50)$$

If for $\xi_1 = [\mathbf{e}_1, \mathbf{h}_1]^T$ and $\xi_2 = [\mathbf{e}_2, \mathbf{h}_2]^T$, one is the retarded wave and the other is the advanced wave. There is,

$$(\xi_1, \xi_2) = \oint_{\Gamma} (\mathbf{e}_1 \times \mathbf{h}_2^* + \mathbf{e}_2^* \times \mathbf{h}_1) \cdot \hat{n} d\Gamma = 0 \quad (51)$$

This is because the retarded wave and the advanced wave do not reach the big sphere with infinite radius at the same time, hence the inner product (ξ_1, ξ_2) is zero at the sphere surface. Hence there is,

$$\iiint_V (\mathbf{e}_1 \cdot \mathbf{J}_2^* + \mathbf{e}_2^* \cdot \mathbf{J}_1) dV = 0 \quad (52)$$

This is the energy conservation law,

$$-\iiint_V (\mathbf{e}_2^* \cdot \mathbf{J}_1) dV = \iiint_V (\mathbf{e}_1 \cdot \mathbf{J}_2^*) dV \quad (53)$$

or

$$\sum_{i=1}^2 \sum_{j=1, j \neq i}^2 \iiint_V (\mathbf{e}_i \cdot \mathbf{J}_j^*) dV = 0 \quad (54)$$

The above can be generalized to,

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \iiint_V (\mathbf{e}_i \cdot \mathbf{J}_j^*) dV = 0 \quad (55)$$

The above is the energy conservation law Eq.(40). Therefore, the principle of mutual energy supports the law of conservation of energy. The principle of mutual energy supports the law of conservation of energy. Therefore, the principle of mutual energy (49) is also the law of conservation of energy.

2.7. Conflict between Mutual Energy Principle and Poynting's Vector Theorem

Suppose there are N current elements in the space, so there are superposition principle, that means,

$$\mathbf{e} = \sum_{i=1}^N \mathbf{e}_i, \quad \mathbf{h} = \sum_{i=1}^N \mathbf{h}_i, \quad \mathbf{J} = \sum_{i=1}^N \mathbf{J}_i \quad (56)$$

Substitute to the Poynting theorem(25)

$$\begin{aligned} & -\sum_{i=1}^N \sum_{j=1}^N \oint_{\Gamma} (\mathbf{e}_i \times \mathbf{h}_j) \cdot \hat{n} d\Gamma \\ & = \sum_{i=1}^N \sum_{j=1}^N \iiint_V (\mathbf{e}_i \cdot \mathbf{J}_j + \mathbf{e}_i \cdot \frac{\partial}{\partial t} \mathbf{d}_j + \mathbf{h}_i \cdot \frac{\partial}{\partial t} \mathbf{b}_j) dV \end{aligned} \quad (57)$$

Comparing with the mutual energy principle (49). The above formulas and (49) are all the law of conservation of energy of N current elements. Their difference,

$$\begin{aligned} & -\sum_{i=1}^N \oint_{\Gamma} (\mathbf{e}_i \times \mathbf{h}_i) \cdot \hat{n} d\Gamma \\ & = \sum_{i=1}^N \iiint_V (\mathbf{e}_i \cdot \mathbf{J}_i + \mathbf{e}_i \cdot \frac{\partial}{\partial t} \mathbf{d}_i + \mathbf{h}_i \cdot \frac{\partial}{\partial t} \mathbf{b}_i) dV \end{aligned} \quad (58)$$

should not transfer energy flow, or the following Poynting theorem for current $\mathbf{J}_i$ does not transfer the energy. Or the following formula does not transfer energy,

$$\begin{aligned} & -\oint_{\Gamma} (\mathbf{e}_i \times \mathbf{h}_i) \cdot \hat{n} d\Gamma \\ & = \iiint_V (\mathbf{e}_i \cdot \mathbf{J}_i + \mathbf{e}_i \cdot \frac{\partial}{\partial t} \mathbf{d}_i + \mathbf{h}_i \cdot \frac{\partial}{\partial t} \mathbf{b}_i) dV \end{aligned} \quad (59)$$

The above is Poynting theorem for current element $\mathbf{J}_i$, we know that usually there is,

$$\oint_{\Gamma} (\mathbf{e}_i \times \mathbf{h}_i) \cdot \hat{n} d\Gamma \neq 0 \quad (60)$$

Because Poynting energy flow must be to spread energy. This leads to contradictions and conflicts. In the frequency domain, Poynting's theorem can be written as,

$$-\oint_{\Gamma} (\mathbf{e}_i \times \mathbf{h}_i^*) \cdot \hat{n} d\Gamma = \iiint_V (\mathbf{e}_i \cdot \mathbf{J}_i^*) dV \quad (61)$$

In the above we have known that, for a small antenna (for example a dipole antenna)

$$\Re(\iiint_V (\mathbf{e}_i \cdot \mathbf{J}_i^*) dV) = 0 \quad (62)$$

$\Re$ is operator taking the real part. This one is still correct. The mistake is at the Poynting energy flow,

$$\Re(\oint_{\Gamma} (\mathbf{e}_i \times \mathbf{h}_i^*) \cdot \hat{n} d\Gamma) \neq 0$$

This item should be zero, but it is not zero. The author realized early that self energy flow should not transfer energy flow (If it transfer the energy then the Poynting theorem of N charge Eq.(57) will transfer more energy than the mutual energy principle Eq.(49). To overcome this contradiction, time reversal wave was added to electromagnetic field theory to solve the problem [9]. However, it was later found that time reversal wave could not solve all problems. It's like programming has encountered bug, patched bug, and caused other bugs. Therefore, the author hopes to modify the definition of electromagnetic field to solve the problem Thoroughly.

3. Review of the Quasi-static Electromagnetic Field Theory

Let's take a step back from Maxwell's electromagnetic radiation field theory and start from the quasi-static electromagnetic field theory to see whether Poynting's theorem, mutual energy principle, mutual energy flow theorem and other energy theorems remain valid.

3.1. Quasi Static Electromagnetic Field

Quasi static electromagnetic field is the electromagnetic field defined by taking away the displacement current from Maxwell's equation.

Gauss law,

$$\nabla \cdot \mathbf{D} = \rho \quad (63)$$

Where $\mathbf{D} = \epsilon_0 \mathbf{E}$, $\mathbf{E}$ is electric field. ρ is charge intensity. $\mathbf{D}$ is electric displacement.

Definition of magnetic induction intensity,

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (64)$$

$\mathbf{A}$ is magnetic vector potential. $\mathbf{B} = \mu_0 \mathbf{H}$. $\mathbf{H}$ is magnetic field intensity. Faraday's law and definition of electric field is,

$$\mathbf{E} = -\nabla \Phi - \frac{\partial}{\partial t} \mathbf{A} \quad (65)$$

Hence, there is

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \nabla \times \mathbf{A} = -\frac{\partial}{\partial t} \mathbf{B} \quad (66)$$

Ampere circular law

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (67)$$

In the magnetic quasi-static electromagnetic field theory, the displacement current $\frac{\partial}{\partial t} \mathbf{D}$ is take away from the above formula. The current continuity equation is,

$$\nabla \cdot \mathbf{J} = -\frac{\partial}{\partial t} \rho \quad (68)$$

In this situation, we assume $\frac{\partial}{\partial t} \rho = 0$, hence there is,

$$\nabla \cdot \mathbf{J} = 0 \quad (69)$$

3.2. The Scale Potential and Vector Potential

$$\nabla \cdot \mathbf{E} = \nabla \cdot (-\nabla \Phi - \frac{\partial}{\partial t} \mathbf{A}) = (-\nabla \cdot \nabla \Phi - \frac{\partial}{\partial t} \nabla \cdot \mathbf{A}) \quad (70)$$

Use Coulomb gauge condition,

$$\nabla \cdot \mathbf{A} = 0 \quad (71)$$

hence,

$$\nabla \cdot \nabla \Phi = -\frac{\rho}{\epsilon_0} \quad (72)$$

hence,

$$\Phi = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho}{r} dV \quad (73)$$

Ampere circular equation can be written as,

$$\nabla \times \nabla \times \mathbf{A} = \mu_0 \mathbf{J} \quad (74)$$

Considering,

$$\nabla \times \nabla \times \mathbf{A} = \nabla \nabla \cdot \mathbf{A} - \nabla^2 \mathbf{A} = -\nabla^2 \mathbf{A} \quad (75)$$

There is,

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} \quad (76)$$

$$\mathbf{A} = \frac{\mu_0}{4\pi\epsilon_0} \iiint_V \frac{\mathbf{J}}{r} dV \quad (77)$$

3.3. Poynting Theorem in Magnetic Quasi-static Electromagnetic Field Theory

Considering,

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = (\nabla \times \mathbf{E}) \cdot \mathbf{H} - \mathbf{E} \cdot (\nabla \times \mathbf{H}) \quad (78)$$

or

$$-\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{E} \cdot \mathbf{J} + \mathbf{H} \cdot \frac{\partial}{\partial t} \mathbf{B} \quad (79)$$

There is the Poynting theorem for magnetic quasi-static electromagnetic field,

$$\oint_{\Gamma} (\mathbf{E} \times \mathbf{H}) \cdot \hat{n} d\Gamma + \iiint_V (\mathbf{E} \cdot \mathbf{J} + \mathbf{H} \cdot \frac{\partial}{\partial t} \mathbf{B}) dV = 0 \quad (80)$$

3.4. The Mutual Energy Theorem

Consider there are two current elements in the empty space, there is the superposition principle,

$$\mathbf{E} = \sum_{i=1}^2 \mathbf{E}_i, \quad \mathbf{H} = \sum_{i=1}^2 \mathbf{H}_i, \quad \mathbf{J} = \sum_{i=1}^2 \mathbf{J}_i \quad (81)$$

Substitute the above to the Poynting theorem Eq.(80). Poynting's theorem for two current elements,

$$\begin{aligned} & \sum_{i=1}^2 \sum_{j=1}^2 \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma \\ & + \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_j + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_j) dV = 0 \end{aligned} \quad (82)$$

Poynting's theorem for the i -th current element is,

$$\oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma + \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_i + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_i) dV = 0 \quad (83)$$

where $i=1, 2$. The principle of mutual energy is obtained by subtracting the Poynting theorem of two single current elements from the Poynting theorem of two current elements,

$$\begin{aligned} & \sum_{i=1}^2 \sum_{j=1, j \neq i}^2 \oint_{\Gamma} \mathbf{E}_i \times \mathbf{H}_j \cdot \hat{n} d\Gamma \\ & + \iiint_V \mathbf{E}_i \cdot \mathbf{J}_j + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_j dV = 0 \end{aligned} \quad (84)$$

or

$$\begin{aligned} & -\oint_{\Gamma} (\mathbf{E}_1 \times \mathbf{H}_2 + \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{n} d\Gamma \\ & = \iiint_V \left(\mathbf{E}_1 \cdot \mathbf{J}_2 + \mathbf{E}_2 \cdot \mathbf{J}_1 + \mathbf{H}_1 \cdot \frac{\partial}{\partial t} \mathbf{B}_2 + \mathbf{H}_2 \cdot \frac{\partial}{\partial t} \mathbf{B}_1 \right) dV \end{aligned} \quad (85)$$

In the frequency domain, there is,

$$\begin{aligned} & -\oint_{\Gamma} (\mathbf{E}_1 \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_1) \cdot \hat{n} d\Gamma \\ & = \iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2^* + \mathbf{E}_2^* \cdot \mathbf{J}_1) dV \end{aligned} \quad (86)$$

Energy should not overflow the universe. Take Γ as the sphere with infinite radius, there are,

$$\oint_{\Gamma} (\mathbf{E}_1 \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_1) \cdot \hat{n} d\Gamma = 0 \quad (87)$$

$$\oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i^*) \cdot \hat{n} d\Gamma = 0, \quad i=1, 2 \quad (88)$$

We have,

$$\iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2^* + \mathbf{E}_2^* \cdot \mathbf{J}_1) dV = 0 \quad (89)$$

Hence we have the energy conservation law,

$$-\iiint_V (\mathbf{E}_2^* \cdot \mathbf{J}_1) dV = \iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2^*) dV \quad (90)$$

In the time domain there is the energy conservation law,

$$-\int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_2 \cdot \mathbf{J}_1) dV = \int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2) dV \quad (91)$$

We can also prove the mutual energy flow theorem in frequency domain,

$$-\iiint_V (\mathbf{E}_2^* \cdot \mathbf{J}_1) dV = (\xi_1, \xi_2) = \iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2^*) dV \quad (92)$$

where,

$$(\xi_1, \xi_2) = \oint_{\Gamma} (\mathbf{E}_1 \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_1) \cdot \hat{n} d\Gamma \quad (93)$$

In the time domain, the mutual energy flow theorem can be written as,

$$\begin{aligned} & -\int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_2 \cdot \mathbf{J}_1) dV \\ & = (\xi_1, \xi_2) \\ & = \int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2) dV \end{aligned} \quad (94)$$

where the mutual energy flow is defined as,

$$(\xi_1, \xi_2) = \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E}_1 \times \mathbf{H}_2 + \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{n} d\Gamma \quad (95)$$

The proof of the mutual energy flow theorem can be found [9].

3.5. Complex Poynting Theorem in (Magnetic) Quasi-static Electromagnetic Field

$$\oint_{\Gamma} (\mathbf{E} \times \mathbf{H}^*) \cdot \hat{n} d\Gamma + \iiint_V (\mathbf{E} \cdot \mathbf{J}^* + \mathbf{H} \cdot j\omega \mathbf{B}^*) dV = 0 \quad (96)$$

Each of the above items is power. For the energy of space, when the system is stable, the energy of this part $\mathbf{H} \cdot j\omega \mathbf{B}^*$ will be constant, and the power of this part should be zero. So, there is,

$$\iiint_V (\mathbf{H} \cdot \mathbf{B}^*) dV = 0 \quad (97)$$

Hence, there is,

$$-\oint_{\Gamma} (\mathbf{E} \times \mathbf{H}^*) \cdot \hat{n} d\Gamma = \iiint_V (\mathbf{E} \cdot \mathbf{J}^*) dV \quad (98)$$

Like the formula (38),

$$\Re(\iiint_V (\mathbf{E} \cdot \mathbf{J}^*) dV) = 0 \quad (99)$$

For the quasi-static electromagnetic field, both the electric field and magnetic field here decay with $\frac{1}{R^2}$.

$$\lim_{r \rightarrow \infty} \Re \oint_{\Gamma} (\mathbf{E} \times \mathbf{H}^*) \cdot \hat{n} d\Gamma = \lim_{r \rightarrow \infty} \frac{1}{r^2} = 0 \quad (100)$$

Therefore, Poynting's theorem Eq.(98) is at least satisfied. Similarly, for quasi-static electromagnetic field, the mutual energy flow theorem is also still valid as energy conservation law. The proof is left for the reader. Therefore, all energy theorems are still valid for (magnetic) quasi-static electromagnetic fields.

3.6. Mutual Energy Principle in Quasi-static Electromagnetic Field Theory

In the above the radiated electromagnetic field, since the radiation of self energy flow is not zero,

$$\Re \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i^*) \cdot \hat{n} d\Gamma \neq 0 \quad (101)$$

It causes the conflict between the mutual energy principle of N current elements and the Poynting's theorem of N current elements. For quasi-static electromagnetic field, because,

$$\Re \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i^*) \cdot \hat{n} d\Gamma = 0 \quad (102)$$

$$\Re \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_i^*) dV = 0 \quad (103)$$

Therefore, the mutual energy principle of N current elements,

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \left(\oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma + \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_j + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_j) dV \right) = 0 \quad (104)$$

and the Poynting theorem of N current elements,

$$\sum_{i=1}^N \sum_{j=1}^N \left(\oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma + \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_j + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_j) dV \right) = 0 \quad (105)$$

have no any conflict.

4. Reasons why the Radiated Electromagnetic Field does not meet the Energy Law

Let us found the reason why the energy theorem and laws are fail for radiation electromagnetic field $\mathbf{e}$ and $\mathbf{h}$.

4.1. Magnetic Field

The author found that the quasi-static electromagnetic field and the radiated electromagnetic field are not the same thing. In order to distinguish these two cases, capital letters are still used for quasi-static electromagnetic field and lowercase letters are used for radiated electromagnetic field.

$$\mathbf{A}(\mathbf{x}, t) \sim \iiint_V \frac{\mathbf{J}(\mathbf{x}', t)}{r} dV \quad (106)$$

Where $\mathbf{r} = \mathbf{x} - \mathbf{x}'$, $r = \sqrt{(\mathbf{r} \cdot \mathbf{r})}$. $\mathbf{x}'$ is an integral variable. $\mathbf{x}$ is the field point.

$$\mathbf{a}(\mathbf{x}, t) \sim \iiint_V \frac{\mathbf{J}(\mathbf{x}', t - r/c)}{r} dV \quad (107)$$

Transform to frequency domain, considering AC current, $\mathbf{J}(t) = \mathbf{J}_0 \exp(j\omega t)$.

$$\begin{aligned} \mathbf{J}(t - r/c) &= \mathbf{J}_0 \exp(j\omega(t - r/c)) \\ &= \mathbf{J}_0 \exp(j(\omega t - \omega r/c)) \end{aligned} \quad (108)$$

$$\begin{aligned} &= \mathbf{J}_0 \exp(j(\omega t)) \exp(-j\omega r/c) \\ &= \mathbf{J}(t) \exp(-jkr) = \mathbf{J}(t) \exp(-j\mathbf{k} \cdot \mathbf{r}) \end{aligned} \quad (109)$$

where

$$\begin{aligned} \mathbf{J}(t) &= \mathbf{J}_0 \exp(j(\omega t)) \\ k &= \omega/c, \mathbf{k} = k \frac{\mathbf{r}}{r} \end{aligned} \quad (110)$$

Hence, there is,

$$\mathbf{A}(\mathbf{x}, \omega) \sim \iiint_V \frac{\mathbf{J}(\mathbf{x}', \omega)}{r} dV \quad (111)$$

The vector potential considering the retarded potential is,

$$\mathbf{a}(\mathbf{x}, \omega) \sim \iiint_V \frac{\mathbf{J}(\mathbf{x}', \omega)}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) dV \quad (112)$$

Quasi-static magnetic field is,

$$\begin{aligned} \mathbf{B} &= \nabla \times \mathbf{A}(\mathbf{x}, \omega) \\ &\sim \iiint_V \nabla \left(\frac{1}{r} \right) \times \mathbf{J}(\mathbf{x}', \omega) dV \\ &= - \iiint_V \frac{\mathbf{r}}{r^3} \times \mathbf{J}(\mathbf{x}', \omega) dV \end{aligned} \quad (113)$$

The magnetic field for the radiation is,

$$\begin{aligned} \mathbf{b} &= \nabla \times \mathbf{a}(\mathbf{x}, \omega) \\ &\sim \nabla \times \iiint_V \frac{\mathbf{J}(\mathbf{x}', \omega)}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) dV \\ &= \iiint_V \nabla \left(\frac{1}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) \right) \times \mathbf{J}(\mathbf{x}', \omega) dV \\ &= - \iiint_V \left(\frac{1}{r^2} + \frac{j\mathbf{k}}{r} \right) \frac{\mathbf{r}}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) \times \mathbf{J}(\mathbf{x}', \omega) dV \end{aligned} \quad (114)$$

We find that,

$$\begin{aligned} \lim_{\mathbf{k} \cdot \mathbf{r} \rightarrow 0} \mathbf{a} &\sim \lim_{\mathbf{k} \cdot \mathbf{r} \rightarrow 0} \iiint_V \frac{\mathbf{J}(\mathbf{x}', \omega)}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) dV \\ &= \iiint_V \frac{\mathbf{J}(\mathbf{x}', \omega)}{r} dV \end{aligned} \quad (115)$$

$\mathbf{k} \cdot \mathbf{r} \rightarrow 0$ means $\frac{2\pi}{\lambda} r \sim \frac{l}{\lambda} \rightarrow 0$, that is $l \ll \lambda$. l is the size of equipment for example a transformer. λ is the wave length of the electromagnetic wave. Hence, there is,

$$\lim_{\mathbf{k} \cdot \mathbf{r} \rightarrow 0} \mathbf{a} = \mathbf{A} \quad (116)$$

That means $\mathbf{a}$ is a generation of vector potential $\mathbf{A}$. But

$$\begin{aligned} \lim_{\mathbf{k} \cdot \mathbf{r} \rightarrow 0} \mathbf{b} &= - \lim_{\mathbf{k} \cdot \mathbf{r} \rightarrow 0} \iiint_V \left(\frac{1}{r^2} + \frac{j\mathbf{k}}{r} \right) \frac{\mathbf{r}}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) \times \mathbf{J}(\mathbf{x}', \omega) dV \\ &= - \iiint_V \left(\frac{1}{r^2} + \frac{j\mathbf{k}}{r} \right) \frac{\mathbf{r}}{r} \times \mathbf{J}(\mathbf{x}', \omega) dV \\ &= - \iiint_V \left(\frac{1}{r^2} \frac{\mathbf{r}}{r} \times \mathbf{J}(\mathbf{x}', \omega) dV - \iiint_V \frac{j\mathbf{k}}{r} \frac{\mathbf{r}}{r} \times \mathbf{J}(\mathbf{x}', \omega) dV \right) \\ &= \mathbf{B} - \iiint_V \frac{j\mathbf{k}}{r} \frac{\mathbf{r}}{r} \times \mathbf{J}(\mathbf{x}', \omega) dV \end{aligned} \quad (117)$$

Hence, there is

$$\lim_{\mathbf{k} \cdot \mathbf{r} \rightarrow 0} \mathbf{b} \neq \mathbf{B} \quad (118)$$

Therefore, although $\mathbf{a}$ can be regarded as a seamless generalization of $\mathbf{A}$, $\mathbf{b}$ is no longer the seamless generalization of $\mathbf{B}$ of quasi-static electromagnetic field, because they are different even when $\mathbf{k} \cdot \mathbf{r} \rightarrow 0$. Therefore, it is completely a magnetic field under another definition. This is also the reason why the radiated electric field is represented by lowercase letters in this paper.

4.2. Electric Field

$$\mathbf{E} = \mathbf{E}^{(S)} + \mathbf{E}^{(I)} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t} \quad (119)$$

$$\mathbf{e} = \mathbf{e}^{(S)} + \mathbf{e}^{(I)} = -\nabla\phi - \frac{\partial\mathbf{a}}{\partial t} \quad (120)$$

Superscript (S) indicates electrostatic field, and (I) indicates inductive. Therefore $\mathbf{E}^{(S)}$ is a static electric field and $\mathbf{E}^{(I)}$ is an induced electric field,

$$\mathbf{E}^{(S)} = -\nabla\Phi = - \iiint_V \nabla \left(\frac{1}{r} \right) \rho dV = \iiint_V \frac{\mathbf{r}}{r^3} \rho dV \quad (121)$$

$$\mathbf{E}^{(I)} = -\frac{\partial\mathbf{A}}{\partial t} \rightarrow -j\omega\mathbf{A} \quad (122)$$

$$\begin{aligned} \mathbf{e}^{(S)} &= -\nabla\phi = - \iiint_V \nabla \left(\frac{1}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) \right) \rho dV \\ &= \iiint_V \left(\frac{\mathbf{r}}{r^3} + \frac{j\mathbf{k}}{r} \right) \exp(-j\mathbf{k} \cdot \mathbf{r}) \rho dV \end{aligned} \quad (123)$$

$$\mathbf{e}^{(I)} = -\frac{\partial\mathbf{a}}{\partial t} \rightarrow -j\omega\mathbf{a} \quad (124)$$

Hence there is,

$$\lim_{\mathbf{k} \cdot \mathbf{r} \rightarrow 0} \mathbf{e}^{(I)} = \mathbf{E}^{(I)} \quad (125)$$

and

$$\begin{aligned} \lim_{\mathbf{k} \cdot \mathbf{r} \rightarrow 0} \mathbf{e}^{(S)} &= \lim_{\mathbf{k} \cdot \mathbf{r} \rightarrow 0} \iiint_V \left(\frac{\mathbf{r}}{r^3} + \frac{j\mathbf{k}}{r} \right) \exp(-j\mathbf{k} \cdot \mathbf{r}) \rho dV \\ &= \mathbf{E}^{(S)} + \iiint_V \left(\frac{j\mathbf{k}}{r} \right) \frac{\mathbf{r}}{r} \rho dV \end{aligned} \quad (126)$$

Hence, there is

$$\lim_{\mathbf{k} \cdot \mathbf{r} \rightarrow 0} \mathbf{e}^{(S)} \neq \mathbf{E}^{(S)} \quad (127)$$

From this, we can see that only $\mathbf{e}^{(I)}$ can be regarded as the seamless generalization of $\mathbf{E}^{(I)}$. But $\mathbf{b}$ cannot be seen as a seamless generalization of $\mathbf{B}$. $\mathbf{e}^{(S)}$ can not be seen as seamless generalization of $\mathbf{E}^{(S)}$.

4.3. Redefining the Electromagnetic Field in the Case of Radiation

Since the electromagnetic field provided by the existing classical electromagnetic field theory is inconsistent with the quasi-static electromagnetic field at $\mathbf{k} \cdot \mathbf{r} \rightarrow 0$, the Poynting theorem and mutual energy principle cannot be satisfied. The author decided to redefine the electromagnetic field for radiation. The electromagnetic field defined by the author in the case of radiation is represented by capital letters $\mathbf{E}$, $\mathbf{D}$, $\mathbf{H}$, $\mathbf{B}$, $\mathbf{A}$, Φ . Since the author has not found a general method to directly give the above electromagnetic field, the author studies the electromagnetic field from the simplest situation. The electromagnetic field of the plane-sheet

current is the easiest. The electromagnetic field of the infinite plane-sheet current is considered in the following.

5. The Electromagnetic Field of Plane Sheet Current

The last two chapters tell us that for the radiated electromagnetic field $\mathbf{e}$ and $\mathbf{h}$ satisfying Maxwell's equations, all energy theorems are invalid. However, for the quasi-static electromagnetic field, $\mathbf{E}$, $\mathbf{H}$ energy theorem still holds for all energy theorems and laws. In order to simplify the problem, we choose to solve the electromagnetic field of the plane-sheet current. The electromagnetic field of plane-sheet current is very simple and can be used as a touchstone to test our theory.

5.1. Electromagnetic Field of Single Infinite Plane-sheet Current

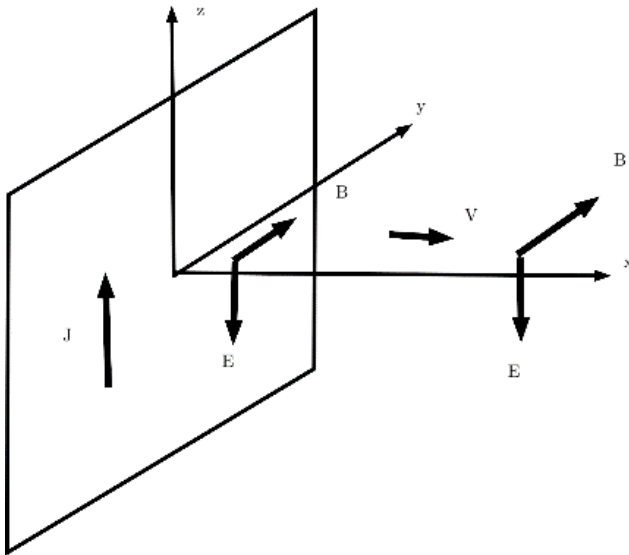


Figure 1. Electromagnetic field of single plane-sheet current.

In the figures 18-3 to 18-6 of the collection of Feynman lecture [6], describe an example of electromagnetism, where there is an infinite plane-sheet current, and then try to find its magnetic field and electric field, see Figure 1. Now let us to find the electromagnetic field in the space $x > 0$ on the right of the current.

5.1.1. Feynman's Method

Feynman believes that the initial phase of the magnetic field is consistent with the plane-sheet current. Then move according to the plane wave,

$$\mathbf{h} = h_0 \exp(j(\omega t - kx))\hat{y} \quad (128)$$

$$h_0 = h(x=0) \sim I \quad (129)$$

$\sim$ is symbol only keep the phase information and does not interesting the information of the values. The electric field is also a plane wave, and the direction is $(-\hat{z})$,

$$\mathbf{e} = e_0 \exp(j(\omega t - kx))(-\hat{z}) \quad (130)$$

Consider Faraday's law

$$\nabla \times \mathbf{e} = -\omega j \mu_0 \mathbf{h} \quad (131)$$

or

$$-jk\hat{x} \times e_0 \exp(-jkx)(-\hat{z}) = -\omega j \mu_0 h_0 \hat{y} \exp(-jkx) \quad (132)$$

or

$$e_0 = \eta_0 h_0 \quad (133)$$

$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$. So $e_0 = e(x=0)$ and $h_0 = h(x=0)$ has the same phase. Hence formula (132) becomes,

$$e(x=0) \sim h(x=0) \sim I \quad (134)$$

This method do not satisfy the electric field calculated according the vector potential,

$$\mathbf{e} = -\frac{\partial}{\partial t} \mathbf{a} \quad (135)$$

5.1.2. Another Consideration

The author considers that if the electric field of wire antenna is calculated, it is usually used

$$\mathbf{e} = -\frac{\partial}{\partial t} \mathbf{a} - \nabla \phi \quad (136)$$

to calculate the electromagnetic field. For planar-sheet current, $\nabla \phi$ should be zero. So there

$$\mathbf{e} = -\frac{\partial}{\partial t} \mathbf{a} \quad (137)$$

This author also see some people think that the electric field should be calculated according to the magnetic vector potential, so

$$\mathbf{e} = -\frac{\partial}{\partial t} \mathbf{a} = -j\omega \mathbf{a} \sim -j\mathbf{a} \sim -jI(\hat{z}) \sim jI(-\hat{z}) \quad (138)$$

$$e(x=0) \sim jI \quad (139)$$

Then it is obtained according to (133),

$$h(x=0) \sim jI \quad (140)$$

In this way, the electromagnetic field $h(x=0)$, $e(x=0)$ have phase j with the current. We all follow the classical electromagnetic theory. This method do not satisfied the boundary condition,

$$\hat{n} \times (\mathbf{h}_2 - \mathbf{h}_1) = \mathbf{J} \quad (141)$$

Why do we get two different results? The reason is that the classical electromagnetic theory is not always consistent.

Both methods try to keep the Poynting vector real, so that at least the energy flow points out of the plane-sheet current.

5.1.3. The Consideration of the Author

We know that plane currents can be composed of many small current elements, as all small current elements have their own magnetic field in phase with the current element, see Figure 2. The electric field near the element current itself is induced by the magnetic vector potential, and its

phase is j . All current elements added together should be consistent with the situation of a small current element.

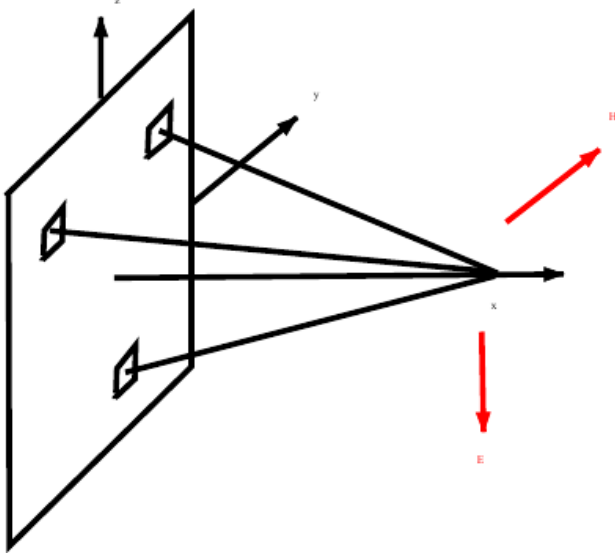


Figure 2. Electromagnetic field of single plane-sheet current is consistent by many smaller current elements

In above section 5.1.1 and 5.1.2, we get two different electromagnetic fields. The author believes that neither is right. But in Feynman's method (1), the calculation of magnetic field is correct. The magnetic field and current have the same phase. Then propagate according to the plane wave. The author believes that the electric field method of others in method (2) is correct. Therefore, there are,

$$\mathbf{E}(x=0) \sim j(-\hat{z}) \quad (142)$$

$$\mathbf{H}(x=0) \sim 1\hat{y} \quad (143)$$

So Poynting vector at the plane $x=0$ is

$$\mathbf{E} \times \mathbf{H}^* \sim j\hat{x} \quad (144)$$

Therefore, the radiated power of the electromagnetic field of this current is reactive power. This is consistent with the viewpoint of mutual energy theory put forward by the author. Poynting energy flow of the current does not transfer energy, because it belong to the self-energy flow.

5.2. Comparison of the 3 Methods

The result of the above 3 methods are listed on the following table.

	method 1	method 2	method 3
boundary condition	Yes	No	Yes
Maxwell's equation	Yes	Yes	No
from vector potential	No	Yes	Yes
Poynting theorem	No	No	Yes
Poynting energy flow	active	active	reactive
MEF theorem	No	No	Yes
MEF	reactive	reactive	active

In the above table, Yes means satisfying. No means unsatisfying. "MEF" means Mutual energy flow. "MEF

theorem" means the mutual energy flow theorem. The items related to MEF and MEF theorem in the table will be discussed in the next chapter, but they are also listed in this table now.

Method 1 is Feynman's method, in which does not consider the vector potential of electromagnetic field. Method 2 is above another consideration, the boundary conditions are not satisfied. The key is that these two methods can not satisfy Poynting theorem and mutual energy flow theorem (39). They offer active Poynting energy flow that is correct according to the classical electromagnetic field theory (in classical electromagnetic field theory they believe the energy flow is the energy flow corresponding to Poynting vector).

The third method is that Maxwell's equation is not satisfied. The author believes that the third method is correct. The electromagnetic field should satisfy the law of conservation of energy. Maxwell's equations does not necessarily need to be satisfied. Although the differential relation of Maxwell equation is satisfied in the first two methods, either the boundary conditions or the relationship of vector potential can not be satisfied. Such a solution is also invalid.

5.3. Step Current Situation

Another case discussed here is step current. This is very good example, we can clearly see the errors in the definition of radiated electromagnetic field in the classical electromagnetic theory. Assume the current is same as before are infinite plane-sheet current,

$$I(t) = I_0 U(t) \quad (145)$$

$$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \quad (146)$$

There are also three cases for the definition of the electromagnetic field.

5.3.1. Method 1

The magnetic field is still zero when $x > ct$, because the signal has not transmitted to here. Here c is light speed.

$$h \sim \begin{cases} 0 & x > ct \\ 1 & x \leq ct \end{cases} \quad (147)$$

The electric field is consistent with the magnetic field, this is according to the Maxwell's equations,

$$\nabla \times \mathbf{e} = -\frac{\partial}{\partial t} \mathbf{b} \quad (148)$$

$$(-j\mathbf{k}) \times \mathbf{e} = -j\omega \mathbf{b} \quad (149)$$

$$\frac{\mathbf{k}}{\omega} \times \mathbf{e} = \mathbf{b} \quad (150)$$

$\mathbf{e}$ and $\mathbf{b}$ has same phase. If $\mathbf{e}$ and $\mathbf{b}$ have same phase for all frequency, they should has same shape in time domain,

$$\mathbf{e} \sim \begin{cases} 0 & x > ct \\ 1 & x \leq ct \end{cases} \quad (151)$$

This scheme is because the magnetic field and electric field are required to maintain the same phase. If all frequencies maintain the same phase, the electric field and magnetic field should be consistent. This also ensures that the Poynting energy flow is real and positive.

This method does not satisfy the calculation of electric field according to vector potential, i.e.,

$$\mathbf{e} \neq -\frac{\partial}{\partial t} \mathbf{a} \quad (152)$$

5.3.2. Method 2

The electric field is selected to satisfy the relationship of vector potential,

$$\mathbf{e}(x=0) = -\frac{\partial}{\partial t} \mathbf{a} \sim -\frac{\partial}{\partial t} J(t)\hat{z} = \delta(t)(-\hat{z}) \quad (153)$$

$$e(x=0) = \delta(t) \quad (154)$$

$$e(x,t) = \delta(t-ct) \quad (155)$$

We can assume that the magnetic field is consistent with the above electric field, which can ensure that the Poynting vector is real and positive.

$$h(x,t) = \delta(t-ct) \quad (156)$$

In this method, the differential expression of Maxwell's equation is satisfied, and the formula of calculating electric field from vector potential is satisfied, but the boundary condition of magnetic field

$$\hat{n} \times (\mathbf{h}_2 - \mathbf{h}_1) = \mathbf{J} \quad (157)$$

can not be satisfied.

5.3.3. Method 3

Electric field according to,

$$\mathbf{E}(x=0) = -\frac{\partial}{\partial t} \mathbf{A} \sim -\frac{\partial}{\partial t} J(t)\hat{z} = \delta(t)(-\hat{z}) \quad (158)$$

$$\mathbf{E}(x,t) = \delta(t-ct)(-\hat{z}) \quad (159)$$

Magnetic field according to,

$$\mathbf{H}(x,t) \sim \hat{y} \begin{cases} 0 & x > ct \\ 1 & x \leq ct \end{cases} = (1-U(t-ct))\hat{y} \quad (160)$$

The author supports method 3. From the feeling, method 3 is also a more correct choice. The tables of these three methods to deal with this situation are consistent with the above tables, so they will not be listed. Which item in the table about mutual energy flow is the content of the next section. Because the mutual energy flow must be investigated in the transformer environment.

For this example, method 3 is correct, indisputable and self-evident. Only the current finally stabilizes and the magnetic field must finally stabilize. Therefore, the magnetic field cannot be a δ function. We can know from the experience of the transformer that the electric field can only be a δ function and cannot be permanently stable. Therefore, the first two schemes are misplaced. Only the

third option is feasible. But the classical electromagnetic field theory made a mistake here. For the planar electromagnetic field, the first method is selected for the classical electromagnetic field theory. Corresponding to dipole or wire antenna, the second method is selected for classical electromagnetic field theory. Both methods can ensure that the Poynting vector is active power. Using two different methods to deal with electromagnetic field itself makes the theory very inconsistent. Therefore, the author strongly advocates the third method.

According to the method 3, the electric field E and the magnetic field H has 90 degree phase difference and, hence, the power of this wave is reactive power. The reactive power do not carry the energy, this satisfy the author's point of view, that is the self-energy flow do not carry the energy! The energy should be transferred only by the mutual energy flow, which will be shown in next section.

5.4. Electromagnetic Field Defined by the Author, Including Advanced Wave

In the previous sub-sections, we only discussed the radiated electromagnetic field propagating to the right (positive x-axis direction) of the plane-sheet current. We only discussed retarded waves in the place $x > 0$. In fact, the advanced wave is also included in the author's electromagnetic field theory. In the article of advanced wave [13], the author proposed that the retarded wave and advanced wave together constitute a wave moving to the left and a wave moving to the right.

When it comes to current, it will produce a wave to the left and a wave to the right at the same time. Among them, the rightward wave at $-\infty < x < 0$ is the advanced wave, and at $0 \leq x < \infty$ is the retarded wave. The leftward wave $0 \leq x < \infty$ is the advanced wave, and the $-\infty < x < 0$ is the retarded wave.

We assume that the current in next section only produces rightward wave. We assume that there is another current absorbing electromagnetic waves on the right side of this plane-sheet current, and there is nothing on the left side. Therefore, the current can only send electromagnetic waves moving to the right.

In fact, current can still send electromagnetic waves moving to the left, but because no other current absorbs the energy of these electromagnetic waves. These electromagnetic waves become ineffective electromagnetic waves. Because the power of these electromagnetic waves is reactive power. In the author's electromagnetic mutual energy theory, the radiated energy flow of this electromagnetic wave belongs to self energy flow, so it is reactive power.

Even for the electromagnetic field of plane-sheet current, the classical electromagnetic theory can not satisfy Maxwell's equations, boundary condition and vector potential formula at the same time. The solution of classical electromagnetic field does not satisfy Poynting's theorem. Therefore, the author redefined the electromagnetic fields for plane-sheet current. According to the new definition, both electric field and magnetic field are plane waves, so there are,

$$\mathbf{J} = \exp(-j\omega t)\hat{z} \quad (161)$$

Rightward wave is,

$$\mathbf{E}^{(+)} = \mathbf{E}_0 \exp(j(\omega t - kx)) \quad -\infty < x < \infty \quad (162)$$

$$\mathbf{H}^{(+)} = \begin{cases} \mathbf{H}_0 \exp(j(\omega t - kx)) & x \geq 0 \\ -\mathbf{H}_0 \exp(j(\omega t - kx)) & x < 0 \end{cases} \quad (163)$$

The above magnetic field is determined by boundary conditions,

$$\hat{n} \times \mathbf{H}_0 = \frac{1}{2} \mathbf{J} \quad (164)$$

Hence,

$$\mathbf{H}_0 = \frac{1}{2} J \hat{y} \quad (165)$$

The author believes, that the electric field does not follow Maxwell's equation,

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B}$$

but determined according to the vector potential,

$$\mathbf{E}_0 = -\frac{\partial}{\partial t} \mathbf{A} = -j\omega \mathbf{A} \sim jJ(-\hat{z}) \quad (166)$$

So E_0 and H_0 maintains a 90 degree phase difference. This difference is maintained throughout the wave propagation. So Poynting energy flow, or self energy flow,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}^* = \mathbf{E}_0 \times \mathbf{H}_0^* = E_0 H_0^* \hat{x} = (jJ)(\frac{1}{2}J)^* \hat{x} \sim j\hat{x} \quad (167)$$

is a pure imaginary number,

$$\Re(\mathbf{S}) = 0 \quad (168)$$

$\Re$ is the operator of tanking the real part. This is just consistent with the conclusion in the author's mutual energy theory, that is, the self energy flow corresponding to Poynting energy flow is zero. This is completely different from the electromagnetic field calculated by the classical electric field theory. In the classical electromagnetic field, the radiation of the real part of the Poynting's vector is not zero.

Now we give the wave to the left,

$$\mathbf{E}^{(-)} = \mathbf{E}_0 \exp(j(\omega t + kx)) \quad -\infty < x < \infty \quad (169)$$

$$\mathbf{H}^{(-)} = \begin{cases} \mathbf{H}_0 \exp(j(\omega t + kx)) & x \geq 0 \\ -\mathbf{H}_0 \exp(j(\omega t + kx)) & x < 0 \end{cases} \quad (170)$$

where,

$$\mathbf{E}_0 = -\frac{\partial}{\partial t} \mathbf{A} = -j\omega \mathbf{A} \sim jJ(-\hat{z}) \quad (171)$$

$$\mathbf{H}_0 = \frac{1}{2} J \hat{y} \quad (172)$$

In the transformer mode, if the current $\mathbf{J}$ is on the primary coil and the secondary coil is on the right of the primary coil, the current generates a wave to the right and does not generate a wave to the left. At this time, the wave to the left is an invalid wave. If this wave has sent out, it can be ignored because it has reactive power. If there are secondary coils on both sides of the coil, both left and

right waves must be considered. At this time, since the energy is divided on both sides, the energy of the wave should be halved, and the electric field and magnetic field should be divided by $\sqrt{2}$. The jump of magnetic field at plane current is very important, which just determines the generation and absorption of electromagnetic wave mutual energy flow in current. This will be seen more clearly in next section.

6. Electromagnetic Field of Two Infinite Plane-sheet Transformers

6.1. Mutual Energy Flow from Primary Coil to Secondary Coil

We give 3 definitions for the electromagnetic field. For transformer environment, there are too many errors in the first two methods, so it is not necessary to study them. The author only considers the method 3 in section 5.1.3.

The following Figure 3 is the author's example. Here we need two plane-sheet to illustrate the problem. One plate is the primary coil of the transformer and the other is the secondary coil.

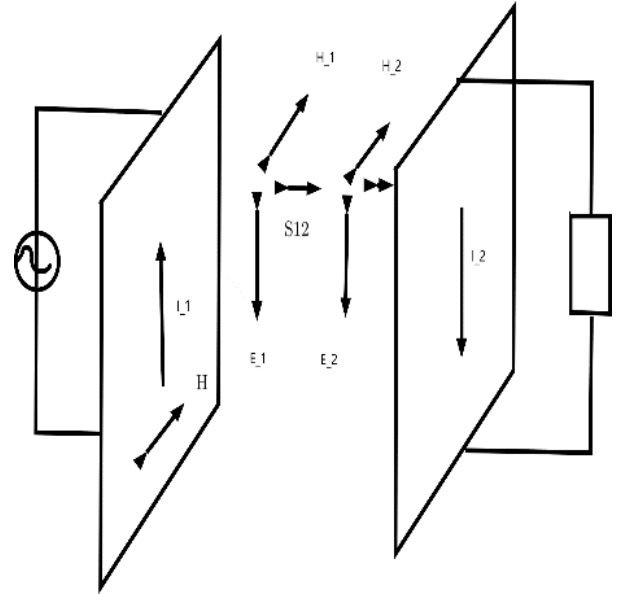


Figure 3. Double plate transformer.

We assume that the primary coil is an infinite conductor plate with current flowing to the z -axis. The secondary coil is a infinite large plate close to the first one and then loaded a resistant, as shown in Figure 3. H_1 should be the same as the plane current J_1 in phase, assuming

$$J_1 \sim 1 \quad (173)$$

and

$$H_1 \sim J_1 \quad (174)$$

The electric field,

$$E_1 \sim jJ_1 \quad (175)$$

(Note that the author and Feynman have different views).

Current J_2 and E_1 have the same phase. Here we have assume the resistance R is very large, $R \gg \omega L_2$. Here L_2 is the self inductance of the secondary coil. The phase of the current of the secondary coil is same to the electric field E_1 ,

$$J_2 \sim E_1 \sim jJ_1 \quad (176)$$

The electric field of the secondary coil is,

$$E_2 \sim (-j)J_2 = (-j)(jJ_1) \quad (177)$$

The magnetic field of secondary coil has the same phase with the current J_2 ,

$$H_2 \sim J_2 \sim jJ_1, \quad (178)$$

so there are still the mutual energy flow,

$$\begin{aligned} S_m &= \frac{1}{2}(\mathbf{E}_1 \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_1) \\ &= \frac{1}{2}(E_1 H_2^* + E_2^* H_1) \hat{x} \sim \frac{1}{2}((j)(j)^* + (-jj)^* 1) \hat{x} = \hat{x} \end{aligned} \quad (179)$$

Therefore, we find that the mutual energy flow is indeed the energy flow from the primary coil to the secondary coil.

We can imagine moving the secondary coil some distance away from the primary coil. Since the electromagnetic field generated by the infinite plate current must be a plane wave, the propagation of this plane wave will ensure the magnetic field H_1 and electric field E_1 Always maintain a 90 degree phase difference. E_2 and H_2 also always maintain a phase difference of 90 degrees. In this case, it is obvious that what we need is to calculate the distant magnetic field according to the electromagnetic field retardation,

$$H_1(x) = H_1(x=0) \exp(-jkx) \sim J_1 \exp(-jkx) \quad (180)$$

$$E_1 \sim jJ_1 \exp(-jkx) \quad (181)$$

$$E_2 \sim -jJ_2 \exp(+jk(l-x)) \quad (182)$$

$$H_2 \sim J_2 \exp(+jk(l-x))$$

Considering

$$\begin{aligned} J_2 &\sim E_1(x=l) = jJ_1 \exp(-jkl) \\ E_2 &\sim -jJ_2 \exp(+jk(l-x)) \\ &= -j(jJ_1 \exp(-jkl)) \exp(+jk(l-x)) \\ &\sim -jjJ_1 \exp(-jkx) \end{aligned} \quad (183)$$

$$\begin{aligned} H_2 &\sim J_2 \exp(+jk(l-x)) \\ &\sim jJ_1 \exp(-jkl) \exp(+jk(l-x)) \\ &\sim jJ_1 \exp(-jkx) \end{aligned} \quad (184)$$

Mutual energy flow is calculated,

$$\begin{aligned} S_m(x) &= \frac{1}{2}(\mathbf{E}_1 \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_1) \\ &= \frac{1}{2}(E_1 H_2^* + E_2^* H_1) \hat{x} \\ &\sim \frac{1}{2}((j \exp(-jkx))(j \exp(-jkx))^* \\ &\quad + (-jj \exp(-jkx))(1 \exp(-jkx))^*) \hat{x} \\ &= \hat{x} \end{aligned} \quad (185)$$

In the above formula, the mutual energy flow flows from the primary coil to the secondary coil.

If we following classical electromagnetic field theory, instead of calculating the magnetic vector potential according to the retarded magnetic vector potential

$$\mathbf{a}_1^{(+)} \sim \exp(-jkx) \hat{z} \quad (186)$$

Then the magnetic field is calculated from the vector potential, we get,

$$\mathbf{h}_1 = \nabla \times \mathbf{a}_1^{(+)} = -jk \hat{x} \times \exp(-jkx) \hat{z} = jk \exp(-jkx) \hat{y} \quad (187)$$

$$h_1 \sim j \exp(-jkx) \quad (188)$$

In this way, the magnetic field has a phase difference j from the current. Then the mutual energy flow will be reactive. That is clear wrong.

6.2. The Mutual Energy Flow at the Outside of Place from Primary Coil to Secondary Coil

Assume that the front of the primary coil is facing the secondary coil and the front of the secondary coil is facing the primary coil. In this subsection, we calculate the mutual energy flow on the outside of the primary coil and the secondary coil. We assume that the primary coil produces a wave moving to the right, which is directed towards the secondary coil. The secondary coil also generates a wave moving to the right. Between the two coils, the wave of the primary coil is the retarded wave and the secondary coil is the advanced wave. On the right side of the secondary coil, the wave of the primary coil is still a retarded wave, but the wave of the secondary coil is change to a retarded wave. On the left side of the primary coil, the wave of the secondary coil is still the advanced wave, but the wave of the primary coil changes to the advanced wave too.

The primary coil current is on the plane of $x=0$. The secondary coil is on the plane of $x=L$. On the right side of the secondary coil, i.e. $x>L$, due to the magnetic field H_2 direction reversal, see (163,170), other quantities E_1 , H_1 , E_2 . It still keeps no changing, so mutual energy flow $S_m(x>L)=0$.

On the left side of the primary coil, similarly, due to the magnetic field H_1 reversal occurs, see (163,170), but E_2 , H_2 , E_1 . Keep no changing, so there is also $S_m(x<0)=0$. So, there is,

$$S_m \sim \begin{cases} 0 & x < 0 \\ \hat{x} & 0 \leq x \leq L \\ 0 & x > L \end{cases} \quad (189)$$

This shows that the mutual energy flow is generated on the primary coil, then it propagates to the secondary coil, and is completely absorbed on the secondary coil. This also shows that the electromagnetic fields $\mathbf{E}$ and $\mathbf{H}$ newly defined by the author can explain the energy flow of the transformer. If the two coils are separated by a considerable distance, the primary coil can also be regarded as a transmitting antenna and the secondary coil can be regarded as a receiving antenna. Therefore, this mutual energy flow is also the energy flow from the transmitting antenna to the receiving antenna. In fact, the primary coil can also be regarded as a light source, and the secondary coil can also be regarded as a sink of light. The mutual energy flow can be understood as photons. Therefore, photons are generated on the primary coil (or transmitting antenna), move to the secondary coil, and then annihilate on the secondary coil (or receiving antenna).

6.3. Poynting Energy Flow and the Mutual Energy Flow

The author finds that the Poynting energy flow of the classical electromagnetic theory is just consistent with the author's mutual energy flow. Consider the second method according to the formula (139, 140)

$$e(x=0) \sim jI \quad (190)$$

$$h(x=0) \sim jI \quad (191)$$

The Poynting vector can be even

$$\mathbf{S} = \mathbf{e} \times \mathbf{h}^* = (jI)(jI)^* \hat{x} \sim \hat{x}$$

According to the method defined by the author, corresponding to the transformer, the situation. According to (175, 178)

$$E_1(x=0) \sim jI$$

$$H_2(x=0) \sim jI$$

The first part of mutual energy flow,

$$S_{12} = \mathbf{E}_1 \times \mathbf{H}_2^* = (jI)(jI)^* \hat{x} \sim \hat{x}$$

In this way, the Poynting energy flow calculated by the classical electromagnetic theory is consistent with the first part of the mutual energy flow calculated according to the author's method. This paper only discusses the electromagnetic field of plane-sheet current. If for the general case, at least the far field of Poynting energy flow and mutual energy flow is consistent. Since we have no general method to calculate the radiated electromagnetic field $\mathbf{E}$ and $\mathbf{H}$, we only know how to calculate the electromagnetic field $\mathbf{E}$ and $\mathbf{H}$ for the case of flat plane-sheet current. But we have a general method to calculate $\mathbf{e}$ and $\mathbf{h}$. Therefore, it is not necessary for us to give up the classical electromagnetic theory. The classical

electromagnetic theory also provides the calculation method of mutual energy flow.

When we calculate the mutual energy flow, we always need to provide a secondary coil or an absorber. For classical electromagnetic fields, only one light source or transmitting antenna is often provided. In fact, classical electromagnetic theory needs an implicit environment to absorb all electromagnetic waves. For classical electromagnetic theory, it is often necessary to have a Silver-Muller far-field radiation condition. This condition actually means that there are enough absorbers in the far field to absorb all electromagnetic waves emitted by the light source. We can still apply Poynting's theorem to calculate the radiant energy flow of electromagnetic field. However, we should keep a clear mind. This Poynting energy flow is actually the mutual energy flow (S_{12}) between the light source and the sink.

6.4. The Transactional Interpretation of Quantum Mechanics

Above, we studied the energy flow of double plane-sheet current transformer according to our newly defined electromagnetic field. The nature of mutual energy flow is very close to the transactional interpretation of quantum mechanics. In the transactional interpretation [3,4], it is the superposition of retarded wave and advanced wave. In the above electromagnetic theory, there are two energy flows S_{12} and S_{21} are superimposed on each other. In the transactional interpretation, the advanced wave emitted by the emitter and the advanced wave of the absorber have a phase difference of 180 degrees, so it is offset, and the retarded wave emitted by the absorber and the retarded wave of the emitter have a phase difference of 180 degrees, so it is offset. These 180 phase differences are difficult to explain. In the author's above theory, the magnetic field just turns on both sides of the plane-sheet current, which explains the phase difference of 180. Therefore, the author's electromagnetic field theory provides support for the transactional interpretation of quantum mechanical. The transactional interpretation of quantum mechanics is a qualitative theory. The electromagnetic theory proposed by the author is a quantitative theory, which can be regarded as the concrete realization of transactional interpretation. Considering the author's interpretation of quantum mechanics with mutual energy flow, there are still some detailed differences with transactional interpretation of quantum mechanics. Therefore, the author's interpretation of quantum mechanics can be called mutual energy flow interpretation [9].

7. Conclusion

It is found that the solution of radiated electromagnetic field in classical electromagnetic field theory does not meet one of three conditions: (1) Maxwell's equations, (2) electromagnetic field obtained by vector potential method, and (3) boundary conditions of magnetic field. Although these solutions and Poynting vector can be used to calculate the radiant energy flow of distant

electromagnetic field, it can not satisfy Poynting theorem. The classical electromagnetic field theory can not explain the energy flow from the primary coil to the secondary coil of the transformer. The author establishes the mutual energy flow theorem of electromagnetic field, and the classical electromagnetic field can not meet the mutual energy flow theorem. The author studies this phenomenon and finds that the radiated electromagnetic field of classical electromagnetic field is not a seamless generalization of the concept of quasi-static electromagnetic field. It should be seen as a completely different system. Therefore, the electromagnetic field of classical electromagnetic theory is represented by lowercase letters to show that there is a great difference between this electromagnetic field and quasi-static electromagnetic field. It is found that corresponding to the quasi static electromagnetic field, the electromagnetic field can satisfy three conditions at the same time: (1) Maxwell's equations, (2) the electromagnetic field obtained by the vector potential method, and (3) the boundary conditions of the magnetic field. It can also satisfy Poynting theorem and mutual energy flow theorem. The author believes that the self-energy flow does not transfer energy, which is also satisfied with the quasi-static electromagnetic field.

The electromagnetic field of infinite plane-sheet current is the simplest case. Corresponding to this case, everyone agrees that the electromagnetic field in this case should be plane wave. Therefore, as long as the electromagnetic field near the current is determined, the electromagnetic field at any time and at any position can be calculated according to the plane wave. The author defines a new method to determine the electromagnetic field. The magnetic field is determined according to the Ampere circular law, so the magnetic field near the current is in phase with the current. The electromagnetic field is not solved according to Maxwell's equations, but according to magnetic vector potential, so the electric field and current have phase angle $-j$. In this way, the electromagnetic and magnetic fields have phase $-j$, so the real part of the Poynting energy flow is exactly zero. This is in line with the author's view that self-energy flow does not transfer energy.

The author further considers the electromagnetic field of two infinite plane-sheet currents. Two flat plane-sheet currents can be regarded as a transformer, one of which is the primary coil and the other is the secondary coil. According to the electromagnetic field defined by the author, and then using the mutual energy flow theorem, the energy flow from the primary coil to the secondary coil can be calculated correctly. This energy flow is generated on the primary coil and absorbed on the secondary coil. Therefore, the author can explain the generation and absorption of energy flow. This is completely different from Poynting's vector. Poynting's vector can only explain the direction of energy, but it can not explain the generation and absorption of energy flow.

The author found that the mutual energy flow defined by the author is just consistent with the Poynting energy of the classical electromagnetic field theory. In other words, there is no need to give up classical electromagnetic theory. Because the classical

electromagnetic field has a definite method. At present, the electromagnetic field proposed by the author can only be obtained in special cases, such as the electromagnetic field of plane-sheet current.

However, the author's newly defined mutual energy flow of electromagnetic field $S_m = E_1 \times H_2^* + E_2^* \times H_1$, it can be mainly applied to the interpretation of quantum mechanics, including the collapse of wave, the energy flow of photon and the concept of advanced wave. It can also be applied to the interpretation of the energy flow from the primary coil to the secondary coil of the transformer, the energy flow from the transmitting antenna to the receiving antenna, and the energy flow of photons.

References

- [1] Wheeler. J. A. and Feynman. R. P. *Rev. Mod. Phys.*, 17:157, 1945.
- [2] Wheeler. J. A. and Feynman. R. P. *Rev. Mod. Phys.*, 21: 425, 1949.
- [3] John Cramer. The transactional interpretation of quantum mechanics. *Reviews of Modern Physics*, 58: 647-688, 1986.
- [4] John Cramer. An overview of the transactional interpretation. *International Journal of Theoretical Physics*, 27: 227, 1988.
- [5] Adrianus T. de Hoop. Time-domain reciprocity theorems for electromagnetic fields in dispersive media. *Radio Science*, 22(7): 1171-1178, December 1987.
- [6] Feynman. The feynman lectures on physics, volume ii mainly electromagnetism and matter.
- [7] A. D. Fokker. *Zeitschrift fuer Physik*, 58:386, 1929.
- [8] Shuang ren Zhao. The application of mutual energy theorem in expansion of radiation fields in spherical waves. *ACTA Electronica Sinica, P.R. of China*, 15(3): 88-93, 1987.
- [9] Shuang ren Zhao. A new interpretation of quantum physics: Mutual energy flow interpretation. *American Journal of Modern Physics and Application*, 4(3):12-23, 2017.
- [10] Shuang ren Zhao. Photon can be described as the normalized mutual energy flow. *Journal of Modern Physics*, 11(5): 668-682, 2020.
- [11] Shuang ren Zhao. A solution for wave-particle duality using the mutual energy principle corresponding to schroedinger equation. *Physics Tomorrow Letters*, 2020.
- [12] Shuang ren Zhao. Huygens principle based on mutual energy flow theorem and the comparison to the path integral. *Physics Tomorrow Letters*, pages 09-06, JANUARY 2021.
- [13] Shuang ren Zhao. antenna system, and photon energy flow (2) advanced wave. not finished, 2022.
- [14] Shuang ren Zhao. The contradictions in poynting theorem and classical electromagnetic field theory. *International Journal of Physics*, 10(5): 242-251, 2022.
- [15] Shuang ren Zhao. The paradox that induced electric field has energy in maxwell theory of classical electromagnetic field is shown and solved. *International Journal of Physics*, 10(4): 204-217, 2022.
- [16] Shuang ren Zhao. The theory of mutual energy flow proves that macroscopic electromagnetic waves are composed of photons. *International Journal of Physics*, 10(5): 218-235, 2022.
- [17] Shuang ren Zhao. Energy conservation law and energy flow theorem for transformer, antenna and photon. *Theoretical Physics Letters*, 11(2): 55-66, 2023.
- [18] Shuang ren Zhao. Energy flow and photons from primary coil to secondary coil of transformer. 11(1): 24-39, 2023.
- [19] Shuang ren Zhao. Experiment to prove the existence of the advanced wave and experiment to prove the wrong definition of magnetic field in maxwell's theory. *Theoretical Physics Letters*, 11(2): 73-80, 2023.
- [20] V.H. Rumsey. A short way of solving advanced problems in electromagnetic fields and other linear systems. *IEEE Transactions on antennas and Propagation*, 11(1): 73-86, January 1963.

- [21] K. Schwarzschild. *Nachr. ges. Wiss. Gottingen*, pages 128,132, 1903.
- [22] Lawrence M. Stephenson. The relevance of advanced potential solutions of maxwell's equations for special and general relativity. *Physics Essays*, 13(1), 2000.
- [23] H. Tetrode. *Zeitschrift fuer Physik*, 10:137, 1922.
- [24] W. J. Welch. Reciprocity theorems for electromagnetic fields whose time dependence is arbitrary. *IRE trans. On Antennas and Propagation*, 8(1): 68-73, January 1960.
- [25] Shuangren Zhao. The application of mutual energy formula in expansion of plane waves. *Journal of Electronics, P. R. China*, 11(2): 204-208, March 1989.
- [26] Shuangren Zhao. The simplification of formulas of electromagnetic fields by using mutual energy formula. *Journal of Electronics, P.R. of China*, 11(1): 73-77, January 1989.
- [27] Shuang-ren, Solve the Maxwell's equations and Schrodinger's equation but avoiding the Sommerfeld radiation condition, *Theoretical Physics Letters*, 10(5), 2022
- [28] Shuang-ren Zhao, Mutual stress flow theorem of electromagnetic field and extension of Newton's third law, *Theoretical Physics Letters*, 10(7), 2022.



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