

Definition, Measurement and Calibration of Magnetic Field of Electromagnetic Wave – Correct the Defects of Maxwell's Classical Electromagnetic Field Theory

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Abstract The research shows that the theory of quasi-static electromagnetic field and the theory of radiated electromagnetic field satisfying Maxwell's equation are two different systems. In these two systems, the definition formula of electric field and magnetic field is identical. The author found that the physical meaning of the definition of the electric field is the same. But the meaning of the definition of magnetic field is different. The author puts forward two new axioms of electromagnetic field theory, (1) N current elements should satisfy the law of conservation of energy. (2) The energy of electromagnetic radiation should not overflow the universe. The theory of quasi-static electromagnetic field can well satisfy these two laws. However, the theory of radiated electromagnetic field satisfying Maxwell's equation does not satisfy these two electromagnetic field laws. These two electromagnetic field laws are self-evident. It is also related. As long as one is true, the other is also true. The conflict between the theory of satisfying Maxwell's equation and these two laws shows that Maxwell's electromagnetic theory has problems. The author first thought that maybe the electromagnetic wave is a wave of reactive power, that is, the electric field and magnetic field of the electromagnetic wave maintain a 90-degree phase difference. The average value of energy transmitted by this wave is 0. However, Maxwell's equation does not support this kind of wave, so the author adds time reversal wave to Maxwell's electromagnetic theory. The time reversal wave causes the reverse collapse of the electromagnetic wave. In this way, electromagnetic waves will not overflow the universe. The propagation of electromagnetic wave energy can be completed by mutual energy flow. The author believes that the mutual energy flow is photon. Recently, the author studied the energy flow transfer from the primary coil to the secondary coil of the transformer. Studying the energy flow transfer from the transmitting antenna to the receiving antenna, it is found that the electromagnetic wave should be reactive power, otherwise it is difficult to establish the physical model of the receiving electromagnetic wave. Therefore, the author began to replace the concept of reverse collapse of electromagnetic wave with reactive power electromagnetic wave. Recently, the author studied the measurement of magnetic field and found that the quasi-static magnetic field can be measured by ring current or straight wire. The values obtained by the two methods are identical. However, for the magnetic field of electromagnetic wave, the measurement with ring antenna and straight-wire antenna has different phases. If the correct method for measuring magnetic field should be straight-wire antenna rather than circular current antenna, then the definition of magnetic field of electromagnetic wave must be revised. The author revised the definition of magnetic field of electromagnetic wave. After the revision, the two laws of electromagnetic field proposed by the author were satisfied. The author believes that the definition of magnetic field of electromagnetic wave in Maxwell's electromagnetic theory is wrong. However, the definition of electric field in Maxwell's electromagnetic wave theory is still valid. This ensures that Maxwell's electromagnetic theory is still correct for most engineering problems. Because the reception of electromagnetic waves mainly involves the induced electromotive force, which is only related to the electric field and has nothing to do with the definition of magnetic field. The problems caused by this defect of Maxwell's theory are mainly problems related to wave-particle duality. The correction of this defect leads us to correctly explain the collapse of waves, the shape of photons, quantum entanglement and other phenomena. If the electromagnetic wave is a reactive power wave, in fact, the wave of other particles, such as the wave of electrons satisfies Schrodinger equation and Dirac equation, may also be reactive power. In short, due to the modification of the definition of magnetic field of electromagnetic wave, the definition of many physical quantities will be affected.

Keywords: magnetic field, electromagnetic wave, photon, energy flow, Poynting, transformer, antenna, reciprocity theorem, mutual energy theorem, energy conservation law, universe

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1. Introduction

1.1. Existing Theories of Electromagnetic Wave and Photon

According to quantum mechanics, electromagnetic wave is a probability wave, which will collapse to an absorber charge to form a photon. That is to say, photons are the products of wave collapse. The wave collapses onto an absorber charge of the receiving antenna, and this charge gets the energy of the electromagnetic wave.

However, according to classical electromagnetic theory, electromagnetic waves are energy waves that carry this electromagnetic energy. According to the classical electromagnetic theory, the changing current on the transmitting antenna causes the generation of spatial electromagnetic waves, which can propagate in space without the transmitting antenna. Electromagnetic waves propagate in the ether according to Maxwell’s idea. Now we have abandoned the concept of ether, but ether still emerges as a new identity of the electromagnetic field. Therefore, electromagnetic waves propagate in space or in an electromagnetic field. Electromagnetic waves travel through space irrespective of the source from which they originate. Just as sound waves move away from the source of sound. When the electromagnetic wave propagates to the receiving antenna, part of the energy of the electromagnetic wave is transferred to the receiving antenna. The receiving antenna receives the electromagnetic signal. It can be seen that the views of classical electromagnetic theory and quantum theory are different.

Are electromagnetic waves energy or probability? In addition to the above two points of view, there is also a point of view that there is an advanced wave. Because the solution of Maxwell equation has retarded wave and advanced wave. Wheeler Ferman proposed in their absorber theory that currents produce half retarded and half advanced waves [1,2]. Of course, both the retarded wave and the advanced wave are moving in all directions in space, that is, both the source and sink produce spherical waves. Wheeler Ferman’s absorption theory is based on the theory of action-at-a-distance [8,25,27]. Cramer has improved the models of Wheeler and Feynman, and it also believes that current can produce retarded and advanced waves [5,6]. However, in his particle model, a particle consists of a retarded wave from the source and a handshake from the sink. Instead of firing in all directions, these retarded waves can be directed. That is, the waves emitted by the source and sink are almost planar. The retarded wave of the source points to the sink, and the wave of the sink points to the source. In this view, both the source and sink emit electromagnetic waves. Receiving electromagnetic waves by sinks is also an active process. It is done by transmitting an advanced wave.

Wheeler Feynman believes that classical electromagnetic theory is wrong. They believe that electromagnetic waves

solved according to Maxwell’s equation should not be treated as a physical quantity. They believe that interaction is a real physical quantity. The resulting electromagnetic wave should be considered a record of interaction [1,2]. Cramer’s transactional interpretation of quantum mechanics is also a qualitative theory. Neither of the two theories specifically corrects Maxwell’s electromagnetic theory.

In 1987, the author proposed the electromagnetic mutual energy theorem [14,30,31]. Thirty years later and before 2015, the author read Welch’s time domain reciprocity theorem [28] and de Hoop’s related reciprocity theorem [7], and found that the two theorems are the inverse Fourier transformation of the mutual energy theorem proposed by the author. So these three theorems can be seen as one. These three theorems involve the advanced wave. Advanced waves violate causal relationships and are not generally considered to be true physical quantities. So it seems correct to call these theorems reciprocity. As a reciprocity theorem, it is not necessary that both quantities in the theorem are physical quantities. One can be virtual. So is it a mistake for the author to call this theorem as an energy theorem? With this in mind, the author has started to re-study the subject of mutual energy [12,16,18-24]. The author first noticed Wheeler’s and Feynman’s absorber theory and Cramer’s transactional interpretation, and was greatly encouraged to learn that there are other important scientists who believe that the advanced wave is an objective existence of physics. The author read Stephenson’s introduction to the advance wave [26] and was also greatly affected. Around 2016, the author first proved that the mutual energy theorem is an energy theorem. Then the mutual energy flow theorem [15] is found. The authors found that the shape of the mutual energy flow is similar to that of the photons. A further quantum mechanical interpretation based on electromagnetic mutual energy flow is proposed. In this interpretation, the collapse of the wave is equal to the reverse collapse of the wave plus the mutual energy flow. In addition, the author found that the mutual energy theorem is not only an energy theorem, but also a law of conservation of energy. It is not only the law of conservation of energy, but also the localized law of conservation of energy. This self-explanatory law of conservation of energy cannot be demonstrated from the Maxwell’s equations. The authors have modified Maxwell’s electromagnetic theory and proposed the concepts of time reversal wave and electromagnetic wave reversal collapse. The authors found that the reverse collapse of waves together with the mutual energy flow can play the same role as the wave collapse. On this basis, the author has established a whole new set of electromagnetic theory [16-24].

The new electromagnetic field theory proposed by the author supports Whiler and Feynman’s absorber theory and Cramer’s view that the advanced wave is a physical and objective existence. The author’s particle model of mutual energy flow replaces the particle model in

Cramer's quantum mechanical transactional interpretation. One of the most important things in quantum mechanical interpretation is to explain the collapse of waves. In the author's model, the collapse of a wave consists of the reverse collapse of the wave and the mutual energy flow. The reverse collapse of the wave is accomplished by the time reversal wave. The author further discovered that the reverse collapse of electromagnetic waves and waves can be replaced by a reactive power wave around 2022. Reactive power waves are not supported by Maxwell's electromagnetic theory. Therefore, in the author's electromagnetic theory, the theory of Maxwell equation is also modified.

Recently, the authors found that their theory of quasi-static and magnetic quasi-static electromagnetic fields is completely different from that of Maxwell's theory of radiated electromagnetic fields after adding displacement current. The measurements of magnetic fields should not be the same for these two completely different systems. For quasi-static fields, The curl of the electric field does give the value of the magnetic field

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \iint_{\Gamma} \mathbf{B} \cdot \hat{n} d\Gamma \quad (1)$$

Where C is the boundary of the surface Γ . Γ is also a coil of a measuring circuit. Assume there is only one circle.

$$\oint \mathbf{E} \cdot d\mathbf{l} = -j\omega B_n S$$

$B_n = \mathbf{B} \cdot \hat{n}$, S is the area of the surface. $B_n = \mathbf{B} \cdot \hat{n}$,

$$B_n = \frac{\oint_C \mathbf{E} \cdot d\mathbf{l}}{-j\omega S}$$

The above magnetic field measurement is used for quasi-static magnetic field. The author believes that this definition is not suitable for radiated electromagnetic fields with retardation factors.

1.2. The Author's Electromagnetic Theory

Few people have studied how an antenna receives electromagnetic waves, and we know that when studying a transmitting antenna, it can be said that the effect of the receiving antenna on the transmitting antenna can be ignored. This seems correct (and indeed incorrect) because the receiving antenna has an impact on the transmitting antenna, and this effect is important. However, the research on the receiving antenna is different. We must consider both the electromagnetic field of the transmitting antenna and the electromagnetic field of the receiving antenna. The problem becomes very complex. If only the inductive electromotive force on the receiving antenna is considered, the problem does not seem complex. The inductive electromotive force on the receiving antenna can be calculated based on the far field of the transmitting antenna. However, the author is concerned with how the energy flow from the electromagnetic field flows into the receiving antenna. The issue involving energy flow involves both electric and magnetic fields.

At the receiving antenna, the electromagnetic field of the transmitting antenna can be seen as a plane wave, so it

can be considered that the incoming field of the receiving antenna is a plane wave. Consider the simplest case, where the receiving antenna is a cylinder of good conductor or a small square piece of metal. The electromagnetic field of the receiving antenna near the receiving antenna can be seen as a quasi-static field. The purpose of this paper is to calculate the energy flow of the electromagnetic field and see how this energy flow flows into the receiving antenna.

Most people may think of the Poynting vector and the Poynting theorem. After all, the Poynting Vector and the Poynting Theorem are often very successful in describing the energy flow of an electromagnetic field. Especially for a simple current circuit with concentrated parameters including power supply, resistance, inductance and capacitor. However, for an antenna system, which includes two separate circuits, one transmitting antenna and one receiving antenna, almost no one has studied it. The author finds that at this point the Poynting vector actually needs to be subdivided. It is divided into self-energy flow and mutual energy flow. Self-energy flow is the energy flow composed of the electromagnetic fields of the transmitting antenna or receiving antenna itself. Mutual energy flow is a hybrid energy flow composed of the interaction between the electromagnetic field of the transmitting antenna and the electromagnetic field of the receiving antenna. So there is the Poynting vector,

$$\begin{aligned} \mathbf{S} &= \mathbf{E} \times \mathbf{H} \\ &= (\mathbf{E}_1 + \mathbf{E}_2) \times (\mathbf{H}_1 + \mathbf{H}_2) \\ &= \mathbf{E}_1 \times \mathbf{H}_1 + \mathbf{E}_2 \times \mathbf{H}_2 + \mathbf{E}_1 \times \mathbf{H}_2 + \mathbf{E}_2 \times \mathbf{H}_1 \end{aligned} \quad (1)$$

Subscript 1 represents the transmitting antenna, and subscript 2 represents the receiving antenna. The author calls

$$Q_{11} = \oint_{\Gamma_1} (\mathbf{E}_1 \times \mathbf{H}_1) \cdot \hat{n} d\Gamma \quad (2)$$

$$Q_{22} = \oint_{\Gamma_2} (\mathbf{E}_2 \times \mathbf{H}_2) \cdot \hat{n} d\Gamma \quad (3)$$

as the self-energy flow. They are the self-energy flow of the transmitting antenna and the receiving antenna. And call

$$Q_m = \oint_{\Gamma} (\mathbf{E}_1 \times \mathbf{H}_2 + \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{n} d\Gamma \quad (4)$$

as mutual energy flow. The division above does not seem magical. However, the author inherits Wheeler Feynman's absorber theory [1,2] (1945), John Cramer's Quantum Mechanical Transactional Interpretation [5,6] (1986), and supports that any change in current of current element produce not only retarded waves, but also advanced waves. For a system consisting of a transmitting antenna and a receiving antenna, the transmitting antenna generates a retarded wave and the receiving antenna generates an advanced wave. Of course, the transmitting antenna also produces advanced waves and the receiving antenna produces retarded waves, but these waves are invalid. If a retarded wave does not find an advanced wave to synchronize with, the retarded wave is invalid. The same is true for advanced waves. The advanced waves in most textbooks about electromagnetic fields are not recognized as physical objects because time in the advance wave takes a negative value and violates a

causal relationship. The advanced wave is considered a virtual quantity. Considered all these, the above formula (4) is not so simple.

In 1987, the author proposed the mutual energy theorem [14,30,31]. In 2017, the author further discovered that this theorem is not only the energy theorem, but also the law of conservation of energy. Not only the law of conservation of energy, but also the localized law of conservation of energy [15]. Maxwell's equation does not prove this Law of conservation of energy, it can only prove that it is an energy theorem (not a law). The author thought there was a problem with Maxwell's electromagnetic theory and began to revise it. The key to this law of conservation of energy is that the energy flow (2,3) does not transfer energy. All energy is transferred by the formula (4). Therefore, the author considers that perhaps the self-energy flow is reactive power, that is, the electric field and the magnetic field keep 90 degree phase difference. However, the solution of Maxwell's equation does not support this kind of electromagnetic field. According to Maxwell's electromagnetic theory, the far field of an antenna is that the electric field and the magnetic field keep the same phase. To keep the self-energy flow from transferring energy, the author adds a time-reversal wave to the electromagnetic field system. Time reversal waves can form the reverse collapse of waves. Reverse collapse combined with mutual energy flow can well explain the concept of wave collapse in quantum mechanics [15]. Recently, the author found that the far field radiation of the antenna should be reactive power wave [16-24]. The electric field and magnetic field of the electromagnetic wave in the solution of Maxwell's equation keep the same phase, so they are active power waves. This is actually wrong.

Therefore, the author assumes that there is a plane electromagnetic wave shining on the receiving antenna. According to the author's electromagnetic theory, the plane wave is a reactive power wave, that is, the electric field and magnetic field maintain a 90-degree phase difference. It is assumed that the electromagnetic field generated on the receiving antenna is near field, so it can be calculated according to magnetic quasi-static electromagnetic field.

The author believes that the system composed of transmitting antenna and receiving antenna should be similar to the system composed the primary coil and secondary coil of a transformer, because if the transformer secondary coil is moved from the transformer primary coil for a distance, the primary coil will become transmitting antenna, and the secondary coil will become receiving antenna. For the transmission and reception of electromagnetic waves, it refers to the transmission of energy flow from the transmitting antenna to the receiving antenna, and also refers to the transmission of energy flow from the primary coil of the transformer to the secondary coil.

The second chapter gives the background knowledge. The third chapter gives the existing electromagnetic theory. The fourth chapter introduces the energy law of electromagnetic field proposed by the author. Chapter 5 is energy formula of radiated electromagnetic field. Chapter 6 is a calculation example of the emission and reception of electromagnetic waves. Chapter 7 redefines magnetic field. Finally, there is a summary.

2. Background Knowledge

2.1. Faraday Electromagnetic Induction

In 1845, Neumann expressed Faraday's electromagnetic induction theorem in formula [10]

$$\mathcal{E}_{2,1} = -\frac{d}{dt} \oint_{C_2} \frac{\mu_0}{4\pi} \oint_{C_1} \frac{1}{r} d\mathbf{l}_1 \cdot d\mathbf{l}_2 \quad (5)$$

$\mathcal{E}_{2,1}$ is the inductive EMF from primary coil C_1 to the secondary coil C_2 . Where, C_1 and C_2 are two closed wire coils. For simplicity, assume that the wire coils have only one loop.

$$r = \sqrt{\|\mathbf{x}_1 - \mathbf{x}_2\|}$$

$\mathbf{x}_1$ is a point on curve C_1 . $\mathbf{x}_2$ is a point on curve C_2 . In 1846 Weber gave another Faraday's

$$\mathcal{E}_{2,1} = -\frac{d}{dt} \oint_{C_2} \frac{\mu_0}{4\pi} \oint_{C_1} \frac{1}{r} (d\mathbf{l}_1 \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} \cdot d\mathbf{l}_2 \quad (6)$$

Two different magnetic vector potentials can be defined.

$$\mathbf{A}_{1N} = \frac{\mu_0}{4\pi} \oint_{C_1} \frac{1}{r} I_1 d\mathbf{l}_1 \rightarrow \frac{\mu_0}{4\pi} \iiint_{V_1} \frac{1}{r} \mathbf{J}_1 dV \quad (7)$$

$$\mathbf{A}_{1W} = \frac{\mu_0}{4\pi} \oint_{C_1} \frac{1}{r} I_1 (d\mathbf{l}_1 \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} \rightarrow \frac{\mu_0}{4\pi} \iiint_{V_1} \frac{1}{r} (\mathbf{J}_1 \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} dV$$

If loop C_1 is a current loop, V_1 is a region. “ $\rightarrow$ ” stands for a promotion. Here is the line current $\oint \dots I_1 d\mathbf{l}$ is generalized to body current $\iiint \dots \mathbf{J}_1 dV$. If C_1 is a closed loop, and Helmholtz proves that the induced electromotive force of the two formulas above is equal. It is generally accepted that Neumann's vector potential is correct when the loop is not closed. Therefore, the vector potential of Weber is not mentioned below. However, we cannot forget Weber's important contribution to the theory of electromagnetic field.

2.2. Electromagnetic Field

For an electromagnetic field, in quasi-static state, suppose there are two coils, C_1 and C_2 . There is an inductive EMF of,

$$\mathcal{E}_{2,1} = -\frac{d}{dt} \oint_{C_2} \frac{\mu_0}{4\pi} \oint_{C_1} \frac{1}{r} I_1 d\mathbf{l}_1 \cdot d\mathbf{l}_2 \quad (8)$$

$\mathcal{E}_{2,1}$ is the electromotive force generated by current I_1 on coil 2. Or,

$$\oint_{C_2} \mathbf{E}_1 \cdot d\mathbf{l} = -\frac{d}{dt} \oint_{C_2} \mathbf{A}_1 \cdot d\mathbf{l}_2 \quad (9)$$

Or

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \oint_{C_1} \frac{1}{r} I_1 d\mathbf{l}_1$$

Or

$$\oint_{C_2} \mathbf{E}_1 \cdot d\mathbf{l} = -\oint_{C_2} \frac{\partial}{\partial t} \mathbf{A}_1 \cdot d\mathbf{l}_2$$

Or

$$\oint_{C_2} (\mathbf{E}_1 + \frac{\partial}{\partial t} \mathbf{A}_1) \cdot d\mathbf{l} = 0$$

Or

$$\nabla \times (\mathbf{E}_1 + \frac{\partial}{\partial t} \mathbf{A}_1) = 0$$

Considering that the curl of a something is zero, it should be a gradient function, so there is,

$$\mathbf{E}_1 + \frac{\partial}{\partial t} \mathbf{A}_1 = -\nabla \phi_1$$

Or

$$\mathbf{E}_1 = -\nabla \phi_1 - \frac{\partial}{\partial t} \mathbf{A}_1$$

Consider,

$$\begin{aligned} \nabla \times \mathbf{A}_1 &= \nabla \times \left(\frac{\mu_0}{4\pi} \iiint_{V_1} \frac{1}{r} \mathbf{J}_1 dV \right) \\ &= \frac{\mu_0}{4\pi} \iiint_{V_1} \nabla \times \left(\frac{1}{r} \mathbf{J}_1 \right) dV \\ &= \frac{\mu_0}{4\pi} \iiint_{V_1} \nabla \left(\frac{1}{r} \right) \times \mathbf{J}_1 dV \\ &= \frac{\mu_0}{4\pi} \iiint_{V_1} \left(-\frac{\mathbf{r}}{r^3} \right) \times \mathbf{J}_1 dV \\ &= \frac{\mu_0}{4\pi} \iiint_{V_1} \mathbf{J}_1 \times \frac{\mathbf{r}}{r^3} dV \end{aligned} \quad (11)$$

According to Biot-Savart's law, we know that the magnetic field is,

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} \iiint_{V_1} \mathbf{J}_1 \times \frac{\mathbf{r}}{r^3} dV \quad (12)$$

Compare (11) and (12), so there is a magnetic

$$\mathbf{B}_1 = \nabla \times \mathbf{A}_1 \quad (13)$$

The above formula can be regarded as the definition of magnetic field. Note that this formula is obtained under quasi-static conditions! Therefore,

$$\mathbf{B} \triangleq \nabla \times \mathbf{A} \quad (14)$$

“ $\triangleq$ ” means definition. Among them,

$$\mathbf{A} = \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \mathbf{J} dV \quad (15)$$

In the following discussion, the subscript “1” is omitted, but it should be clear that at least two objects, such as a primary coil and a secondary coil, are often needed to establish these formulas, so there are always two subscripts 1,2. Now that we omit the subscripts do not mean that the subscripts do not exist. The formula (15) gives the impression that the magnetic vector $\mathbf{A}$ is caused by the current $\mathbf{J}$, which is not completely correct. The author means that electromagnetic problems are problems of interaction, and interaction requires two objects to

interact. For example, the primary coil and secondary coil of transformer, transmitting antenna and receiving antenna, etc. So even if there is no subscripts in the formula (15), in fact, we should reserve a position for the subscripts we should give it.

In the above derivation we known that the reason why we can define a magnetic field using formula (14) is because the curl of $\mathbf{A}$ happens to be equal to the magnetic field defined in Biot-Savart law.

2.3. Retarded Potential

Maxwell's contemporaries, such as Kirchhoff [11] and Lorenz [9], did not establish the concept of electric field and magnetic field, so for them, for example, Ohm's law was used as following,

$$\mathbf{J} = \sigma \mathbf{E}$$

They just write directly as

$$\mathbf{J} = \sigma \left(-\nabla \phi - \frac{\partial}{\partial t} \mathbf{A} \right) \quad (16)$$

σ is the conductivity. The above formula does not introduce the concept of electric field, which was introduced only by Maxwell. However, we can also think that Kirchhoff and Lorenz's concept of electric field is,

$$\mathbf{E} = -\nabla \phi - \frac{\partial}{\partial t} \mathbf{A} \quad (17)$$

Lorenz obtains the retarded potential from the potential function $\mathbf{A}$, ϕ under quasi-static conditions,

$$\mathbf{A}^{(r)} = \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \mathbf{J} dV \rightarrow \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \mathbf{J}(t - r/c) dV$$

$$\phi^{(r)} = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{1}{r} \rho dV \rightarrow \frac{1}{4\pi\epsilon_0} \iiint_V \frac{1}{r} \rho(t - r/c) dV$$

“ $\rightarrow$ ” means some kind of extension. If there is a time factor $\exp(j\omega t)$, for both the current and the charge intensity, then the above form becomes,

$$\mathbf{A}^{(r)} = \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \mathbf{J} dV \rightarrow \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \mathbf{J} \exp(-j\mathbf{k} \cdot \mathbf{r}) dV$$

$$\phi^{(r)} = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{1}{r} \rho dV \rightarrow \frac{1}{4\pi\epsilon_0} \iiint_V \frac{1}{r} \rho \exp(-j\mathbf{k} \cdot \mathbf{r}) dV$$

In the above formula, the time factor $\exp(j\omega t)$ is omitted to simplify the problem. For electric and magnetic fields $\mathbf{E}$, $\mathbf{B}$, the two masters, Kirchhoff and Lorenz, are not defined at all. Because logically, we cannot obtain the following electromagnetic field expression,

$$\mathbf{E}^{(r)} = -\nabla \phi^{(r)} - \frac{\partial}{\partial t} \mathbf{A}^{(r)} \quad (18)$$

$$\mathbf{B}^{(r)} = \nabla \times \mathbf{A}^{(r)} \quad (19)$$

Maxwell actually defines a retarded electrostatic field.

$$\mathbf{E}_s^{(r)} \triangleq -\nabla \phi$$

define the retarded inductive field,

$$\mathbf{E}_i^{(r)} \triangleq -\frac{\partial}{\partial t} \mathbf{A}^{(r)} = -j\omega \mathbf{A}^{(r)}$$

So there are,

$$\mathbf{E}_i^{(r)} = -j\omega \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) dV$$

Hence, we have,

$$\lim_{\lambda \rightarrow \infty} \mathbf{E}_i^{(r)} = -j\omega \left(\frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \right) dV = \mathbf{E}_i$$

$\mathbf{k} = k\hat{r}$, $k = \frac{2\pi}{\lambda}$ is considered here, so $\lambda \rightarrow \infty$, $k \rightarrow 0$ and hence the following is applied,

$$\lim_{\lambda \rightarrow \infty} \exp(-j\mathbf{k} \cdot \mathbf{r}) = 1$$

For a magnetic field defined by Maxwell, consider

$$\begin{aligned} \lim_{\lambda \rightarrow \infty} \mathbf{B}^{(r)} &= \lim_{\lambda \rightarrow \infty} \nabla \times \mathbf{A}^{(r)} \\ &= \lim_{\lambda \rightarrow \infty} \nabla \times \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \mathbf{J} \exp(-j\mathbf{k} \cdot \mathbf{r}) dV \\ &= \lim_{\lambda \rightarrow \infty} \frac{\mu_0}{4\pi} \iiint_V \nabla \times \left(\frac{1}{r} \mathbf{J} \exp(-j\mathbf{k} \cdot \mathbf{r}) \right) dV \\ &= \lim_{\lambda \rightarrow \infty} \frac{\mu_0}{4\pi} \iiint_V \nabla \left(\frac{1}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) \right) \times \mathbf{J} dV \\ &= \lim_{\lambda \rightarrow \infty} \frac{\mu_0}{4\pi} \iiint_V \left[\left(-\frac{\mathbf{r}}{r^3} \exp(-j\mathbf{k} \cdot \mathbf{r}) \right) \right. \\ &\quad \left. + \left[\frac{-j\mathbf{k}}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) \right] \times \mathbf{J} \right] dV \end{aligned}$$

Hence,

$$\begin{aligned} \lim_{\lambda \rightarrow \infty} \mathbf{B}^{(r)} &= \frac{\mu_0}{4\pi} \iiint_V \left(\mathbf{J} \times \frac{\mathbf{r}}{r^3} + \mathbf{J} \times \frac{j\mathbf{k}}{r} \right) dV \\ &= \mathbf{B} + \frac{\mu_0}{4\pi} \iiint_V \left(\mathbf{J} \times \frac{j\mathbf{k}}{r} \right) dV \end{aligned}$$

So there are,

$$\lim_{\lambda \rightarrow \infty} \mathbf{E}_i^{(r)} = \mathbf{E}_i \quad (20)$$

$$\lim_{\lambda \rightarrow \infty} \mathbf{B}^{(r)} \neq \mathbf{B} \quad (21)$$

The retarded inductive electric field becomes a quasi-static inductive electric field at a long wavelength. However, a retarded magnetic field with a long wavelength is not consistent with a quasi-static magnetic field. In the case of magnetic quasi-static state, because the curl of vector potential $\mathbf{A}$ happens to be equal to the magnetic field defined in Biot-Savart law, we define the magnetic field as the curl of $\mathbf{A}$. Now, in the case of a retarded radiated electromagnetic field, the curl of the vector potential $\mathbf{A}^{(r)}$ is not consistent with the magnetic field even at large wavelengths. If we still define the curl of a vector potential $\mathbf{A}^{(r)}$ as a magnetic field, it is obviously unreasonable. In addition,

$$\begin{aligned} \lim_{\lambda \rightarrow \infty} \mathbf{E}_s^{(r)} &= \lim_{\lambda \rightarrow \infty} (-\nabla \phi) \\ &= \lim_{\lambda \rightarrow \infty} \left(-\nabla \frac{1}{4\pi\epsilon_0} \iiint_V \left(\frac{1}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) \right) \rho dV \right) \\ &= -\lim_{\lambda \rightarrow \infty} \frac{1}{4\pi\epsilon_0} \iiint_V \nabla \left(\frac{1}{r} \exp(-j\mathbf{k} \cdot \mathbf{r}) \right) \rho dV \\ &= -\lim_{\lambda \rightarrow \infty} \frac{1}{4\pi\epsilon_0} \iiint_V \left(-\frac{\mathbf{r}}{r^3} \exp(-j\mathbf{k} \cdot \mathbf{r}) \right. \\ &\quad \left. + \frac{1}{r} (-j\mathbf{k}) \exp(-j\mathbf{k} \cdot \mathbf{r}) \right) \rho dV \\ &= \frac{1}{4\pi\epsilon_0} \iiint_V \left(\frac{\mathbf{r}}{r^3} + \frac{j\mathbf{k}}{r} \right) \rho_1 dV \\ \lim_{\lambda \rightarrow \infty} \mathbf{E}_s^{(r)} &= \mathbf{E}_s + \frac{1}{4\pi\epsilon_0} \iiint_V \left(\frac{j\mathbf{k}}{r} \right) \rho dV \end{aligned}$$

$$\lim_{\lambda \rightarrow \infty} \mathbf{E}_s^{(r)} \neq \mathbf{E}_s \quad (22)$$

Therefore, the author considers that the retarded induced electric field $\mathbf{E}_i^{(r)}$ calculated by Maxwell equation is more reliable. The retarded magnetic field $\mathbf{B}^{(r)}$ calculated according to Maxwell's equation is the least reliable. Electrostatic Field $\mathbf{E}_s^{(r)}$ is also not reliable. By saying that they are not reliable, they are not generalizations of quasi-static electromagnetic fields, but another entirely different physical quantity.

2.4. Redefinition of Electromagnetic Wave or Retarded Electric and Magnetic Field by the Author

The author believes that the so-called quasi-static electromagnetic field condition refers to the electromagnetic field at a large wavelength. Therefore, the author wants to consider that the retarded electromagnetic field should be converted into a quasi-static electromagnetic field when the wavelength is very large. That is,

$$\lim_{\lambda \rightarrow \infty} \mathbf{E}_i^{(r)} = \mathbf{E}_i \quad (23)$$

Due to retarded magnetic field $\mathbf{E}_i^{(r)}$. In $\lambda \rightarrow \infty$, the induced magnetic field can be degraded to a magnetic quasi-static, so the relationship between the quasi-static magnetic field and the retarded potential defined below is reliable,

$$\mathbf{E}_i^{(r)} \triangleq -\frac{\partial}{\partial t} \mathbf{A}^{(r)} \quad (24)$$

The symbol " $\triangleq$ " denotes a definition, but the author does not consider the following formula for a magnetic field in the case of a retarded radiated electromagnetic field.

$$\mathbf{B}^{(r)} \triangleq \nabla \times \mathbf{A}^{(r)} \quad (25)$$

since,

$$\lim_{\lambda \rightarrow \infty} \mathbf{B}^{(r)} \neq \mathbf{B} \quad (26)$$

Consider,

$$\lim_{\lambda \rightarrow \infty} \mathbf{E}_s^{(r)} \neq \mathbf{E}_s \quad (27)$$

Nor do we think the following formula is correct,

$$\mathbf{E}_s^{(r)} \triangleq -\nabla \phi^{(r)} \quad (28)$$

However, since there is no problem with the definition of the above form (28) other than the formula (27), it may be advisable to assume that the above form (28) is still valid (in fact $\nabla \phi^{(r)}$ is often not important in the discussion, and sometimes does not appear, for example, planar electromagnetic waves are only related to the retarded inductive field). Therefore, it may be advisable to continue to believe that the following definitions are correct.

$$\mathbf{E}^{(r)} \triangleq -\nabla \phi^{(r)} - \frac{\partial}{\partial t} \mathbf{A}^{(r)} \quad (29)$$

What is more guaranteed is that only inductive electric fields can be extended to cases with retarded fields. That is,

$$\mathbf{E}_i^{(r)} \triangleq -\frac{\partial}{\partial t} \mathbf{A}^{(r)} \quad (30)$$

This paper studies the reception of electromagnetic fields. There is no need to do much research on how the electromagnetic field is radiated. The far field of an antenna can always be seen as approximate plane waves, so assume that there are plane electromagnetic waves, that the direction of the electric field is $(-\hat{z})$, and that the direction of the electromagnetic wave propagation is $(\hat{x})$, so there are,

$$\phi^{(r)} \equiv 0$$

$$\mathbf{A}^{(r)} = A_0 \exp(j(\omega t - kx))\hat{z}$$

$$\mathbf{E} = -\frac{\partial}{\partial t} \mathbf{A}^{(r)} = -j\omega \mathbf{A}^{(r)} = -j\omega A_0 \exp(j(\omega t - kx))\hat{z}$$

the electric field can be written as,

$$\mathbf{E}^{(r)} = jE_0 \exp(-jkx)(-\hat{z})$$

$$E_0 = \omega A_0$$

The j in front of the electric field is for convenience, however, j is an initial phase, which can be freely picked up. The retarded magnetic field according to the Maxwell equation is,

$$\begin{aligned} \mathbf{H}_{\text{maxwell}}^{(r)} &= \frac{1}{\mu_0} \nabla \times \mathbf{A}^{(r)} = -jk \frac{1}{\mu_0} \hat{x} \times A_0 \exp(j(\omega t - kx))\hat{z} \\ &= \frac{1}{\mu_0} jk A_0 \exp(j(\omega t - kx))\hat{y} \\ &= \frac{1}{\mu_0} j\sqrt{\mu_0 \epsilon_0} \omega A_0 \exp(j(\omega t - kx))\hat{y} \\ &= j \frac{1}{\eta_0} \omega A_0 \exp(j(\omega t - kx))\hat{y} \\ &= j \frac{1}{\eta_0} E_0 \exp(j(\omega t - kx))\hat{y} \end{aligned}$$

However, for the reasons of the preceding two sections, the author considers that the retarded magnetic field defined above is unreliable. In this paper, a correction scheme for the above results is proposed. According to this correction scheme, the correct retarded magnetic field should be:

$$\mathbf{H}_{\text{new}}^{(r)} = \frac{1}{\eta_0} E_0 \exp(j(\omega t - kx))\hat{y}$$

This keeps the phase difference between the magnetic field and the electric field at 90 degrees. So the author defined,

$$\mathbf{H}_{\text{new}}^{(r)} \triangleq -j\mathbf{H}_{\text{maxwell}}^{(r)} \quad (31)$$

Hence there is,

$$\mathbf{H}_{\text{new}}^{(r)} = -j(j \frac{1}{\eta_0} E_0 \exp(-jkx))\hat{y} = \frac{1}{\eta_0} E_0 \exp(-jkx)\hat{y}$$

The subscript “new” denotes a retarded magnetic field newly defined by the author. So the Poynting vector,

$$\begin{aligned} \mathbf{S}_{\text{new}} &= \mathbf{E}^{(r)} \times (\mathbf{H}_{\text{new}}^{(r)})^* \\ &= (jE_0 \exp(j(\omega t - kx))(-\hat{z})) \times (\frac{1}{\eta_0} E_0 \exp(j(\omega t - kx))) \\ &= (j \frac{1}{\eta_0} E_0^2) \hat{x} \end{aligned} \quad (32)$$

The above formula is a pure imaginary number. Description of Poynting Vector $\mathbf{S}_{\text{new}}$ is a pure imaginary number, and this wave is reactive power. So no energy is transferred in average. The authors believe that electromagnetic waves should be reactive power and therefore do not transmit energy. This requirement is met by using this newly defined retarded magnetic field. The author's definition of a magnetic field looks pointless and certainly not easy to be acceptable to the reader. The reason for this definition is slowly given below by the author. Hopefully the following reasons will change the reader's view.

3. Existing Electromagnetic Theory

The author briefly reviews some of the existing electromagnetic field theories to see what is not so right.

3.1. Quasi-Static Equation

In the following derivation, the author omits the subscripts, 1,2. The author just omitted the subscripts, but thought they always exist. For an electromagnetic phenomenon, they are an interaction, so there are at least two subscripts, one for the source of emission and one for the sink. The source and sink interact. For quasi-static cases,

$$\mathbf{A} = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}}{r} dV \quad (33)$$

hence,

$$\begin{aligned} \nabla^2 \mathbf{A} &= -\mu_0 \iiint_V (\nabla^2 (-\frac{1}{4\pi r})) \mathbf{J} dV \\ &= -\mu_0 \iiint_V (\delta(\mathbf{x} - \mathbf{x}')) \mathbf{J}(\mathbf{x}') dV \\ &= -\mu_0 \mathbf{J}(\mathbf{x}) \end{aligned} \quad (34)$$

For scalar potentials,

$$\phi = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho}{r} dV \quad (35)$$

Hence,

$$\begin{aligned} \nabla^2 \phi &= -\frac{1}{\epsilon_0} \iiint_V \nabla^2 \left(-\frac{1}{4\pi r}\right) \rho dV \\ &= -\frac{1}{\epsilon_0} \iiint_V \delta(\mathbf{x} - \mathbf{x}') \rho(\mathbf{x}') dV \\ &= -\frac{1}{\epsilon_0} \rho(\mathbf{x}) \end{aligned} \quad (36)$$

So we get the Poisson equation,

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}(\mathbf{x}) \quad (37)$$

$$\nabla^2 \phi = -\frac{1}{\epsilon_0} \rho(\mathbf{x}) \quad (38)$$

And we already know that,

$$\mathbf{E} = -\nabla \phi - \frac{\partial}{\partial t} \mathbf{A} \quad (39)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (40)$$

Consider

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

Or

$$\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A}$$

Hence there is

$$\nabla(\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A} = -\mu_0 \mathbf{J}(\mathbf{x})$$

Consider (40), yes,

$$\nabla(\nabla \cdot \mathbf{A}) - \nabla \times \mathbf{B} = -\mu_0 \mathbf{J}(\mathbf{x}) \quad (41)$$

Considering the Lorenz gauge condition,

$$\nabla \cdot \mathbf{A} = -\mu_0 \epsilon_0 \frac{\partial}{\partial t} \phi \quad (42)$$

The Lorenz gauge condition can be directly demonstrated by the definitions of $\mathbf{A}$ and ϕ formula (33, 35), which require the use of the current continuity equation.

$$\nabla \cdot \mathbf{J} = -\frac{\partial}{\partial t} \rho \quad (43)$$

Further obtained by (41,42),

$$\nabla(-\mu_0 \epsilon_0 \frac{\partial}{\partial t} \phi) - \nabla \times \mathbf{B} = -\mu_0 \mathbf{J}(\mathbf{x})$$

Or,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}(\mathbf{x}) + \nabla(-\mu_0 \epsilon_0 \frac{\partial}{\partial t} \phi)$$

Or,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}(\mathbf{x}) + \mu_0 \epsilon_0 \frac{\partial}{\partial t} (-\nabla \phi) \quad (44)$$

The formula above is Ampere's circuital Law under quasi-static conditions. There is a scalar Poisson equation (36),

$$\nabla \cdot \nabla \phi = -\frac{1}{\epsilon_0} \rho(\mathbf{x})$$

$$\nabla \cdot (-\nabla \phi) = \frac{1}{\epsilon_0} \rho(\mathbf{x})$$

Define,

$$\mathbf{E}_s = -\nabla \phi \quad (45)$$

We get the Gauss law,

$$\nabla \cdot \mathbf{E}_s = \frac{1}{\epsilon_0} \rho(\mathbf{x}) \quad (46)$$

Consider,

$$\nabla \times \mathbf{E} = \nabla \times (-\nabla \phi - \frac{\partial}{\partial t} \mathbf{A}) = -\frac{\partial}{\partial t} \nabla \times \mathbf{A}$$

Consider

$$\nabla \times \mathbf{A} = \mathbf{B}$$

Obtain,

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B} \quad (47)$$

The formula above is Faraday's Law. In addition,

$$\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A}) = 0 \quad (48)$$

The formula above is the law of magnetic gaussian, which together yields a quasi-static equation,

$$\nabla \cdot \mathbf{E}_s = \frac{1}{\epsilon_0} \rho(\mathbf{x}) \quad (49)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (50)$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B} \quad (51)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}(\mathbf{x}) + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \mathbf{E}_s \quad (52)$$

Among them,

$$\mathbf{E}_s = -\nabla \phi \quad (53)$$

$$\mathbf{E}_i = -\frac{\partial}{\partial t} \mathbf{A} \quad (54)$$

$$\mathbf{E} = \mathbf{E}_s + \mathbf{E}_i \quad (55)$$

An example of a quasi-static electromagnetic field is an AC current source in series connected by a capacitor, an inductance, and a load resistance. Note that in this quasi-static equation, the formulas (49) and (52) are both electrostatic fields, $\mathbf{E}_s$. The formula (49-55) represents Maxwell's previous electromagnetic theory. The displacement current already exists in formula (52), but it only contains the static field $\mathbf{E}_s$ does not contain induction field $\mathbf{E}_i$. This part of the displacement current

$\epsilon_0 \frac{\partial}{\partial t} E_s$ is the result of introducing the current continuity equation (43). The current continuity equation was introduced by Kirchhoff. In a sense, this part of displacement current is attributed to Kirchhoff's contribution. In Maxwell's original Maxwell's equations, the current continuity equation is one of them, which takes into account Kirchhoff's contribution to Maxwell's equation. Unfortunately, Maxwell's followers take away the current continuity equation from Maxwell's equations. Since then, Kirchhoff's contribution to electromagnetic theory has been ignored.

3.2. Maxwell's Electromagnetic Theory

Maxwell's equation is actually derived from Maxwell's quasi-static equation by following transformation. Maxwell's equation is not derived from logic. Maxwell's electromagnetic theory can be seen as derived from the generalized quasi-static electromagnetic field equation,

$$E_s \rightarrow E \quad (56)$$

This formula means that Maxwell believes that the following,

$$-\nabla \phi$$

can be replaced by

$$-\nabla \phi - \frac{\partial}{\partial t} A$$

The above transformation has no reason to follow. Some authors believe that this is Maxwell's very clever mistake. That is, the static electric field appearing in Gauss's law (49) and Ampere's circuital law (52) is changed to include both the electrostatic field and the induced electric field, so there is the following Maxwell equation,

$$\nabla \cdot E = \frac{1}{\epsilon_0} \rho(x) \quad (57)$$

$$\nabla \cdot B = 0 \quad (58)$$

$$\nabla \times E = -\frac{\partial}{\partial t} B \quad (59)$$

$$\nabla \times B = \mu_0 J(x) + \mu_0 \epsilon_0 \frac{\partial}{\partial t} E \quad (60)$$

In this way, we get an equation of the radiated electromagnetic field which is completely different from the quasi-static equation. Because of the transformation $E_s \rightarrow E$, the new equation (57-60) is completely different from the original quasi-static equation (49-52). They are two completely different theories of electromagnetic fields. Classical electromagnetic field theory holds that Maxwell equation (57-60) is a more accurate electromagnetic field equation, and quasi-static electromagnetic field equation is an approximation of Maxwell equation, which is also wrong.

3.3. Magnetic Quasi-static Electromagnetic Field

Magnetic quasi-static electromagnetic field is assumed to be quasi-static field if E_s can be ignored, so there are,

$$E_s = 0 \quad (61)$$

This is the case,

$$\nabla \cdot B = 0 \quad (62)$$

$$\nabla \times E = -\frac{\partial}{\partial t} B \quad (63)$$

$$\nabla \times B = \mu_0 J(x) \quad (64)$$

An example of a magnetic quasi-static electromagnetic field can be an AC power source connected to a load that includes inductance and resistance. Another example is a transformer, with the primary coil connect by an AC current source and the secondary connect by a load resistor.

4. Energy Law of Quasi-Static Electromagnetic Field

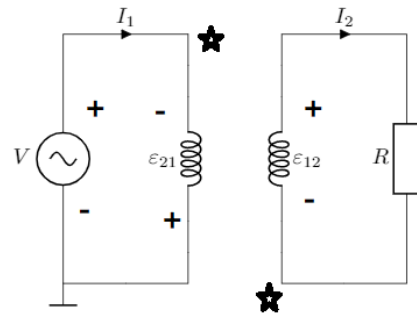


Figure 1. The picture shows a transformer with five stars representing the dotted terminal. The primary coil is connected to an AC current source and the secondary to a load resistance.

4.1. Transformer

Figure 1 shows a transformer. The power provided by the primary coil circuit of the transformer to the primary coil of the transformer is,

$$P_1 = V_1^* I_1$$

Consider Kirchhoff's voltage law for the circuit of primary coil,

$$V_1 + \mathcal{E}_{1,2} = 0$$

$\mathcal{E}_{1,2}$ is the induced electromotive force of the secondary coil current on the primary coil, so there is,

$$V_1 = -\mathcal{E}_{1,2}$$

$$P_1 = -\mathcal{E}_{1,2}^* I_1$$

The output power generated by the induced electromotive force of the secondary coil is,

$$P_2 = \mathcal{E}_{2,1} I_2^*$$

For ideal transforme

$$P_1 = P_2 \quad (65)$$

Or

$$-\mathcal{E}_{1,2}^* I_1 = \mathcal{E}_{2,1} I_2^* \quad (66)$$

The above formula is obviously a law of conservation of energy for transformers. Considering the definition of electromotive force,

$$\mathcal{E}_{1,2} \triangleq \oint_{C_1} \mathbf{E}_2 \cdot d\mathbf{l} = -M_{1,2} \frac{d}{dt} I_2 = -M_{1,2} j\omega I_2$$

$$\mathcal{E}_{2,1} \triangleq \oint_{C_2} \mathbf{E}_1 \cdot d\mathbf{l} = -M_{2,1} \frac{d}{dt} I_1 = -M_{2,1} j\omega I_1$$

Substitute the above two formulas into the formula (66),

$$-(-M_{1,2} j\omega I_2)^* I_1 = (-M_{2,1} j\omega I_1) I_2^*$$

or,

$$M_{1,2}^* = M_{2,1} \quad (67)$$

This is another expression of the energy conservation law formula (66). We know that,

$$M_{2,1} \triangleq \frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{1}{r} d\mathbf{l}_1 \cdot d\mathbf{l}_2$$

$$M_{1,2} \triangleq \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{1}{r} d\mathbf{l}_2 \cdot d\mathbf{l}_1$$

According to the above formula, mutual inductance is a real number, so the formula (67) seems to be obvious, so it is redundant. In fact, it is not. We have further verified the law of conservation of energy (67) under the magnetic quasi-static condition or the environmental conditions of the transformer. This further shows that the formula (66) is indeed the law of conservation of energy.

$\mathbf{E}_1$ is the field of the primary coil and $\mathbf{E}_2$ is the field of the secondary coil. The law of conservation of energy (66) can be rewritten to

$$-\oint_{C_1} \mathbf{E}_2^* \cdot d\mathbf{l}_1 = \oint_{C_2} \mathbf{E}_1 \cdot d\mathbf{l}_2^* \quad (68)$$

Consider the transformation of line current to body current.

$$\oint_C \cdots d\mathbf{l} \rightarrow \iiint_V \cdots \mathbf{J} dV$$

The law of conservation of energy (68) can be written as,

$$\begin{aligned} & -\iiint_V \mathbf{E}_2(\omega)^* \cdot \mathbf{J}_1(\omega) dV \\ & = \iiint_V \mathbf{E}_1(\omega) \cdot \mathbf{J}_2(\omega)^* dV \end{aligned} \quad (69)$$

By doing the inverse Fourier transformation of the above formula,

$$\begin{aligned} & -\int_{t=-\infty}^{\infty} dt \iiint_V \mathbf{E}_2(t) \cdot \mathbf{J}_1(t+\tau) dV \\ & = \int_{t=-\infty}^{\infty} dt \iiint_V \mathbf{E}_1(t+\tau) \cdot \mathbf{J}_2(t) dV \end{aligned} \quad (70)$$

Assume $\tau = 0$, there is,

$$\begin{aligned} & -\int_{t=-\infty}^{\infty} dt \iiint_V \mathbf{E}_2(t) \cdot \mathbf{J}_1(t) dV \\ & = \int_{t=-\infty}^{\infty} dt \iiint_V \mathbf{E}_1(t) \cdot \mathbf{J}_2(t) dV \end{aligned} \quad (71)$$

The formula (71) can also be rewritten to

$$-\sum_{i=1}^2 \sum_{j=1, j \neq i}^2 \int_{t=-\infty}^{\infty} dt \iiint_{V_i} \mathbf{E}_j(t) \cdot \mathbf{J}_i(t) dV \quad (72)$$

Consider $2 \rightarrow N$, the above formula can be further expanded to,

$$-\sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \iiint_{V_i} \mathbf{E}_j(t) \cdot \mathbf{J}_i(t) dV \quad (73)$$

When converting to the Fourier frequency domain,

$$-\sum_{i=1}^N \sum_{j=1, j \neq i}^N \iiint_{V_i} \mathbf{E}_j(\omega) \cdot \mathbf{J}_i(\omega)^* dV \quad (74)$$

The formulas (65-74) all are the laws of energy conservation in the transformer environment. Transformers work under quasi-static conditions, so the above formulas are also the law of energy conservation under quasi-static conditions. Later in this paper, the author will further prove these laws of energy conservation from the quasi-static electromagnetic field equation.

Where (71) is Welch's time domain reciprocity theorem (1960) [28]. The formula (70) is the relevant reciprocity theorem of de Hoop [7]. The formula (69) is the mutual energy theorem (1987) of author (Zhao Shuangren) [14,30,31]. It is also Rumsey's new reciprocity theorem (1963) [32], and the missing second reciprocity theorem (2009) [33]. The formula (69) can be transformed by conjugate transformation through Lorenz reciprocity theorem. Here is the Lorenz reciprocity theorem (1900) [3,4],

$$\iiint_V \mathbf{E}_2(\omega) \cdot \mathbf{J}_1(\omega) dV = \iiint_V \mathbf{E}_1(\omega) \cdot \mathbf{J}_2(\omega) dV$$

As a reciprocity theorem, essentially we think it has two quantities, corresponding to subscript 1 and subscript 2, respectively. One is a physical quantity and the other is a virtual quantity. However, as the energy theorem requires both quantities to be physical quantities, the energy theorem is stronger than the reciprocity theorem. If it goes a step further as the law of conservation of energy, it shows that the formula already contains all the energy of the system, so the energy of the whole system does not change with time. In 2017, the authors found that (73 and 74) are laws of conservation of energy [15]. In the last year or two, the author has deduced this law of conservation of energy flow in transformer environments again [21,22,23,24]. In transformer environments, it is easy to recognize that this formula is indeed the law of conservation of energy.

We know that transformers operate under quasi-static conditions, so we are actually explaining the above law of conservation of energy (65-74) under quasi-static conditions. It is interesting to recognize the evolution of this series of theorems. The earliest is the Lorenz

reciprocity theorem (1900), which is a pure mathematical theorem. Welch's Law of Reciprocity (1960), which is actually the law of the conservation of energy, is only regarded as the law of reciprocity, a law of time-domain reciprocity. Following this is the mutual energy theorem [14], proposed by the author in 1987, which goes a step further, from the reciprocity theorem to the energy theorem. The understanding of a theorem, from its reciprocity to its energy, is a big step forward. By 2017, the author realized that it was the law of conservation of energy [15], which is another big step forward.

These formulas (65-74) are actually laws of conservation of energy, but most people do not even admit that this formula is an energy theorem. The main reason is that this formula involves advanced waves. The electromagnetic fields in this formula are retarded and advanced, which violates our usual understanding of causality. In 1987, the author considered this formula to be very significant as an energy theorem. In 2017, the authors further recognized that these theorems are actually laws of conservation of energy. Since this law of conservation of energy cannot be demonstrated from the Maxwell equation which includes the displacement current, the author has started to modify the Maxwell electromagnetic field theory. The concepts of time reversal wave and electromagnetic wave reversal collapse were initially proposed [15]. But recently (starting in 2021) the author replaced time reversal waves with the concept of reactive power waves [21,22,23,24].

4.2. Poynting Theorem under Magnetic Quasi-static Condition

We know the mathematical formula,

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \nabla \times \mathbf{E} \cdot \mathbf{H} - \mathbf{E} \cdot \nabla \times \mathbf{H}$$

In the above formula, we consider the equation of magnetostatic state (61-64),

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\frac{\partial}{\partial t} \mathbf{B} \cdot \mathbf{H} - \mathbf{E} \cdot \mathbf{J}$$

The corresponding Poynting theorem is obtained,

$$-\oint (\mathbf{E} \times \mathbf{H}) \cdot \hat{n} d\Gamma = \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B} \cdot \mathbf{H} + \mathbf{E} \cdot \mathbf{J} \right) dV \quad (75)$$

4.3. Consider Superposition Principle under Magnetic Quasi-static Condition

It is generally believed that the superposition principle is the basic principle of electromagnetic theory,

$$\mathbf{E} = \sum_{i=1}^N \mathbf{E}_i, \quad \mathbf{H} = \sum_{i=1}^N \mathbf{H}_i, \quad \mathbf{J} = \sum_{i=1}^N \mathbf{J}_i \quad (76)$$

Substituting Poynting theorem under magnetic quasi-static condition,

$$\begin{aligned} & -\sum_{i=1}^N \sum_{j=1}^N \oint (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma \\ & = \sum_{i=1}^N \sum_{j=1}^N \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_j + \mathbf{E}_i \cdot \mathbf{J}_j \right) dV \end{aligned} \quad (77)$$

The above formula is Poynting theorem with N current elements, or Poynting theorem with N coils. We know that the Poynting theorem corresponding to the i -th coil has the following form:

$$-\oint (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma = \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i + \mathbf{E}_i \cdot \mathbf{J}_i \right) dV \quad (78)$$

We can add up N such Poynting's theorem to get,

$$\begin{aligned} & -\sum_{i=1}^N \oint (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma \\ & = \sum_{i=1}^N \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i + \mathbf{E}_i \cdot \mathbf{J}_i \right) dV \end{aligned} \quad (79)$$

The time integral of the above two formulas (77, 79) is,

$$-\sum_{i=1}^N \sum_{j=1}^N \int_{t=-\infty}^{\infty} dt \oint (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma \quad (80)$$

$$= \sum_{i=1}^N \sum_{j=1}^N \int_{t=-\infty}^{\infty} dt \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_j + \mathbf{E}_i \cdot \mathbf{J}_j \right) dV$$

$$-\sum_{i=1}^N \int_{t=-\infty}^{\infty} dt \oint (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma \quad (81)$$

$$= \sum_{i=1}^N \int_{t=-\infty}^{\infty} dt \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i + \mathbf{E}_i \cdot \mathbf{J}_i \right) dV$$

4.4. Proof of (65-74) is an Energy Theorem under Magnetic Quasi Static Conditions

The formula (80) and the formula (81) are both time integrals of Poynting's theorem. The difference between the two formulas should also be equal, that is,

$$\begin{aligned} & -\sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \oint (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma \\ & = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_j + \mathbf{E}_i \cdot \mathbf{J}_j \right) dV \end{aligned} \quad (82)$$

The above surface Γ can be taken as a sphere with infinite radius. Because it is a magnetic quasi-static electromagnetic field, consider that Γ is a sphere with infinite radius, and all the current $\mathbf{J}_i$ are near the center of the ball. For magnetic quasi-static conditions, the electric and magnetic fields on the sphere are

$$\mathbf{E}_i \sim \frac{1}{R^2}$$

$$\mathbf{H}_i \sim \frac{1}{R^2}$$

$$\mathbf{E}_i \times \mathbf{H}_i \sim \frac{1}{R^4}$$

$$\oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma \sim \frac{1}{R^4} 4\pi R^2 \sim \frac{1}{R^2} \rightarrow 0$$

hence, there is,

$$\lim_{R \rightarrow \infty} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma = 0 \quad (83)$$

consider,

$$\begin{aligned} & \int_{t=-\infty}^{\infty} dt \iiint_V \sum_{i=1}^N \sum_{j=1, j \neq i}^N \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_j \right) dV \\ & - \int_{t=-\infty}^{\infty} dt \iiint_V \sum_{i=1}^N \sum_{j=1, j < i}^N \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_j + \mathbf{B}_i \cdot \frac{\partial}{\partial t} \mathbf{H}_j \right) dV \\ & = \int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} \iiint_V \sum_{i=1}^N \sum_{j=1, j < i}^N (\mu_0 \mathbf{H}_i \mathbf{H}_j) dV \\ & = \int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} U = U(\infty) - U(-\infty) = 0 \end{aligned} \quad (84)$$

Among them,

$$U = \iiint_V \sum_{i=1}^N \sum_{j=1, j < i}^N \mu_0 (\mathbf{H}_i \mathbf{H}_j) dV$$

For the same reason as above, both $U(\infty)$ and $U(-\infty)$ are 0. So we get that $U(\infty)$ is the end of the process we are considering, so there is $U(\infty) = 0$. $U(-\infty)$ is the energy of the process before it has started. This energy is $U(-\infty) = 0$.

$$\int_{t=-\infty}^{\infty} dt \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i \right) dV = 0 \quad (85)$$

substituting (83,84) into (82),

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_j) dV = 0 \quad (86)$$

The authors have thus demonstrated that the above formula (86) is an energy theorem under quasi-static conditions. Note that we are only proving that this formula is an energy theorem, because it is proved by two Poynting's theorems, which are also energy theorems. The author has not proven that it is the law of conservation of energy.

4.5. Proof of Law of Conservation of Energy in the Condition of the Magnetostatic Electromagnetic Field

To prove that the above formula is not only an energy theorem but also a law of conservation of energy, the author also needs to prove that each term of the formula (81) is zero, that is,

$$\int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma = 0 \quad (87)$$

$$\int_{t=-\infty}^{\infty} dt \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i \right) dV = 0 \quad (88)$$

$$\int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_i) dV = 0 \quad (89)$$

Because if all three formulas are zero, the subtraction of them from (80) then obtain (81) will remain the law of conservation of energy. Because (80) is the law of conservation of energy, (81) is 0 for each term, subtracting

a zero from the each term of the law of conservation of energy will still keep it the law of conservation of energy. Here is the author's proof.

$$\begin{aligned} & \int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_i) dV \\ & = -\frac{\mu_0}{4\pi} \int_{t=-\infty}^{\infty} dt \iiint_V \iiint_{V'} \frac{\frac{\partial}{\partial t} \mathbf{J}_i(\mathbf{x}') \cdot \mathbf{J}_i(\mathbf{x})}{r} dV' dV \\ & = -\frac{\mu_0}{4\pi} \int_{t=-\infty}^{\infty} dt \frac{1}{2} \left(\iiint_V \iiint_{V'} \frac{\frac{\partial}{\partial t} \mathbf{J}_i(\mathbf{x}') \cdot \mathbf{J}_i(\mathbf{x})}{r} dV' dV \right. \\ & \quad \left. + \iiint_V \iiint_{V'} \frac{\mathbf{J}_i(\mathbf{x}') \cdot \frac{\partial}{\partial t} \mathbf{J}_i(\mathbf{x})}{r} dV' dV \right) \\ & = -\frac{\mu_0}{4\pi} \int_{t=-\infty}^{\infty} dt \frac{1}{2} \iiint_V \iiint_{V'} \frac{\frac{\partial}{\partial t} (\mathbf{J}_i(\mathbf{x}') \cdot \mathbf{J}_i(\mathbf{x}))}{r} dV' dV \\ & = \int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} \left(-\frac{\mu_0}{4\pi} \iiint_V \iiint_{V'} \frac{(\frac{1}{2} \mathbf{J}_i(\mathbf{x}') \cdot \mathbf{J}_i(\mathbf{x}))}{r} dV' dV \right) \\ & = \int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} U = U(\infty) - U(-\infty) = 0 \end{aligned}$$

We have considered the following,

$$\begin{aligned} \mathbf{E}_i & = -\frac{\partial}{\partial t} \mathbf{A}_i = -\frac{\partial}{\partial t} \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \mathbf{J}_i dV \\ U & = -\frac{\mu_0}{4\pi} \iiint_V \iiint_{V'} \frac{(\frac{1}{2} \mathbf{J}_i(\mathbf{x}') \cdot \mathbf{J}_i(\mathbf{x}))}{r} dV' dV \end{aligned}$$

Is the energy corresponding to the current. $U(\infty)$ is the energy at the end of the process, which is zero. The energy at the beginning of the $U(-\infty)$ process is zero. The above authors have demonstrated that,

$$\int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_i) dV = 0 \quad (90)$$

Consider the following,

$$\begin{aligned} & \int_{t=-\infty}^{\infty} dt \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i \right) dV \\ & = \int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} \iiint_V \left(\frac{\mu_0}{2} \mathbf{H}_i \mathbf{H}_i \right) dV \\ & = \int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} U = U(\infty) - U(-\infty) = 0 \end{aligned}$$

Among them,

$$U = \iiint_V \left(\frac{\mu_0}{2} \mathbf{H}_i \mathbf{H}_i \right) dV$$

For the same reason as above, both $U(\infty)$ and $U(-\infty)$ are 0. So we get,

$$\int_{t=-\infty}^{\infty} dt \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i \right) dV = 0 \quad (91)$$

For

$$\int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma$$

Consider that Γ is a sphere with an infinite radius, and the current J_i . They are all near the center of the ball. For magnetic quasi static conditions, the electric and magnetic fields on a sphere are

$$\begin{aligned} E_i &\sim \frac{1}{R^2} \\ H_i &\sim \frac{1}{R^2} \\ E_i \times H_i &\sim \frac{1}{R^4} \\ \oint_{\Gamma} (E_i \times H_i) \cdot \hat{n} d\Gamma &\sim \frac{1}{R^4} 4\pi R^2 \sim \frac{1}{R^2} \rightarrow 0 \end{aligned}$$

So there are

$$\lim_{R \rightarrow \infty} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (E_i \times H_i) \cdot \hat{n} d\Gamma = 0 \quad (92)$$

The author proves that the formula (82) is the law of conservation of energy, and he has already obtained the formula (87-89) so that he can get the law of conservation of energy (86), that is

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \iiint_V (E_i \cdot J_j) dV = 0 \quad (93)$$

(93) is the law of conservation of energy. Note that the author has shown that it is a law of conservation of energy, not just a theorem of energy. This proof is done under magnetic quasi-static conditions.

See (73), we have proven this energy conservation law under the condition of transformer system. So we have prove the energy conservation law two situations. It is really an energy conservation law.

5. Energy Formula of Radiated Electromagnetic Field

We have previously demonstrated that the formula (86) is a law of conservation of energy under quasi-static conditions. In this section we will see if we can get this Law of conservation of energy in the case of radiating electromagnetic fields. The radiated electromagnetic field satisfies the Maxwell equation (57-60)

5.1. Poynting Theorem

Poynting's theorem can be proved by Maxwell's equation (57-60) [13].

$$\begin{aligned} & -\oint_{\Gamma} (E \times H) \cdot \hat{n} d\Gamma \\ & = \iiint_V (E \cdot \frac{\partial}{\partial t} D + H \cdot \frac{\partial}{\partial t} B + E \cdot J) dV \end{aligned} \quad (94)$$

This formula tells us the increment of the electric field energy of the system.

$$\iiint_V (E \cdot \frac{\partial}{\partial t} D) dV = \frac{\partial}{\partial t} \frac{1}{2} \iiint_V (E \cdot D) dV$$

So the electric field energy of the system is,

$$\begin{aligned} U &= \frac{1}{2} \iiint_V (E \cdot D) dV \\ &= \frac{1}{2} \iiint_V \epsilon_0 (E_s + E_i) \cdot (E_s + E_i) dV \\ &= \frac{1}{2} \iiint_V \epsilon_0 (E_s \cdot E_s + E_i \cdot E_i + 2E_s \cdot E_i) dV \end{aligned} \quad (95)$$

among them

$$E_s = -\nabla \phi$$

$$E_i = -\frac{\partial}{\partial t} A$$

In this formula we know that there is an energy term of the induced electric field in the system.

$$\frac{1}{2} \iiint_V \epsilon_0 (E_i \cdot E_i) dV \quad (96)$$

Even if the above energy is true under the conditions of radiated electromagnetic fields, it does not exist under the conditions of magnetic quasi-static, see the formula (75). If the Maxwell equation is a more accurate physical equation, then the energy in the above equation should also occur under the conditions of magnetic quasi-static

because $\frac{\partial}{\partial t} A$ is not zero under the conditions of magnetic

quasi-static. In another paper, the author has discussed that the energy of this part of the induced electric field caused by Maxwell's equation is completely fictitious [21], at least under quasi-static conditions.

This indicates that the system described by the Maxwell equation (57-60) about the radiated electromagnetic field and the system described by the magnetic quasi-static state (61-64) are completely two different systems. It cannot be said that the radiated electromagnetic field system of Maxwell equation is more accurate, and the quasi-static or quasi-static electromagnetic field equation is inaccurate.

5.2. Failure to Prove the Law of Conservation of Energy by Maxwell's Equation

In the previous chapter, we have successfully demonstrated the law of conservation of energy from the quasi-static electromagnetic field equation. Let's start with the Maxwell equation and do the same. Considering the principle of superposition (76) we have obtained that,

$$\begin{aligned} & -\sum_{i=1}^N \sum_{j=1}^N \oint_{\Gamma} (E_i \times H_j) \cdot \hat{n} d\Gamma \\ & = \sum_{i=1}^N \sum_{j=1}^N \iiint_V (\frac{\partial}{\partial t} B_i \cdot H_j + \frac{\partial}{\partial t} D_i \cdot E_j + E_i \cdot J_j) dV \end{aligned} \quad (97)$$

The above formula can be written in two parts.

$$\begin{aligned} & -\sum_{i=1}^N \sum_{j=1, j \neq i}^N \oint_{\Gamma} (E_i \times H_j) \cdot \hat{n} d\Gamma \\ & = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \iiint_V (\frac{\partial}{\partial t} B_i \cdot H_j + \frac{\partial}{\partial t} D_i \cdot E_j + E_i \cdot J_j) dV \end{aligned} \quad (98)$$

$$\begin{aligned}
& -\sum_{i=1}^N \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma \\
& = \sum_{i=1}^N \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i + \frac{\partial}{\partial t} \mathbf{D}_i \cdot \mathbf{E}_i + \mathbf{E}_i \cdot \mathbf{J}_i \right) dV
\end{aligned} \quad (99)$$

We have considered the following relationships above.

$$\sum_{i=1}^N \sum_{j=1}^N (\dots) = \sum_{i=1}^N \sum_{j=1, j \neq i}^N (\dots) + \sum_{i=1}^N (\dots)$$

To prove the law of conservation of energy from the above two formulas (86), we must prove that,

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_j + \frac{\partial}{\partial t} \mathbf{D}_i \cdot \mathbf{E}_j \right) dV = 0 \quad (100)$$

$$\int_{t=-\infty}^{\infty} dt \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i + \frac{\partial}{\partial t} \mathbf{D}_i \cdot \mathbf{E}_i \right) dV = 0 \quad (101)$$

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma = 0 \quad (102)$$

$$\int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma = 0 \quad (103)$$

$$\int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_i) dV = 0 \quad (104)$$

As with the quasi-static case, we can prove (100,101).

Now proven (102), here we need to assume that there are two electromagnetic fields corresponding to i , j , one is the retarded wave and the other is the advanced wave, so that they do not arrive at the same time on the infinite sphere Γ . The advanced wave arrives at Γ at some point in the past, and the retarded wave arrives at some point in the future. So they are not zero at the same time on Γ . So this integral is zero. The formula (102) is proved.

As for the formula (103), this is the surface integral of the Poynting vector over the surface Γ , and the far field formula of any antenna knows that its Poynting vector is not zero. Nor can we prove (104) to be zero. So we cannot prove the law of conservation of energy from the Maxwell equation (86). However, we can still prove that the formula (86) is an energy theorem. The method to prove (86) that the energy theorem is the same as that under quasi-static conditions, and hence it is not discussed here.

5.3. Problems of Maxwell's Equation

The law of conservation of energy (86) is self-explanatory and can be determined to be correct without proof. Assuming there are N current elements, N includes all the charges in the universe very much. This formula (86) represents the total energy of the N current elements in the system. If the current element $\mathbf{J}_i$ provides energy to the current element $\mathbf{J}_j$, then The energy of of the current element $\mathbf{J}_j$ increases while the energy of of the current element $\mathbf{J}_i$ decreases. So there's always,

$$\int_{t=-\infty}^{\infty} dt \iiint_{V_j} \mathbf{J}_j \cdot \mathbf{E}_i dV = - \int_{t=-\infty}^{\infty} dt \iiint_{V_i} \mathbf{J}_i \cdot \mathbf{E}_j dV$$

among them,

$$\int_{t=-\infty}^{\infty} dt \iiint_{V_j} \mathbf{J}_j \cdot \mathbf{E}_i dV$$

is the energy of $\mathbf{J}_j$ provided by $\mathbf{J}_i$ current element.

$$\int_{t=-\infty}^{\infty} dt \iiint_{V_i} \mathbf{J}_i \cdot \mathbf{E}_j dV$$

is the energy of $\mathbf{J}_j$ provided by $\mathbf{J}_j$ current element. So there's always something for both,

$$\int_{t=-\infty}^{\infty} dt \iiint_{V_j} \mathbf{J}_j \cdot \mathbf{E}_i dV + \int_{t=-\infty}^{\infty} dt \iiint_{V_i} \mathbf{J}_i \cdot \mathbf{E}_j dV = 0$$

Or,

$$\sum_{i=1}^2 \sum_{j=1, j < i} \left(\int_{t=-\infty}^{\infty} dt \iiint_{V_j} \mathbf{J}_j \cdot \mathbf{E}_i dV + \int_{t=-\infty}^{\infty} dt \iiint_{V_i} \mathbf{J}_i \cdot \mathbf{E}_j dV \right) = 0$$

Can be extended from 2 to N

$$\sum_{i=1}^N \sum_{j=1, j < i} \left(\int_{t=-\infty}^{\infty} dt \iiint_{V_j} \mathbf{J}_j \cdot \mathbf{E}_i dV + \int_{t=-\infty}^{\infty} dt \iiint_{V_i} \mathbf{J}_i \cdot \mathbf{E}_j dV \right) = 0$$

That is,

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \left(\int_{t=-\infty}^{\infty} dt \iiint_{V_j} \mathbf{J}_j \cdot \mathbf{E}_i dV \right) = 0$$

Now Maxwell's equation can't prove this apparent law of conservation of energy. The problem with Maxwell's equation is well understood. Although Maxwell's equation can deduce the solution of the electromagnetic wave and it has been experimentally proven by Hertz, this does not mean that Maxwell's electromagnetic theory is perfect. A philosopher once pointed out that all laws of physics are wrong, but there are a few fewer errors. It's not surprising that Maxwell's electromagnetic theory is wrong. Maxwell's theory of electromagnetism does not correctly explain the problem of photon and wave-particle duality, which is really due to this error.

5.4. Correction of Maxwell's Electromagnetic Theory by the Author

To correct the error in Maxwell's electromagnetic theory, the author first gives a new law, or axiom, of the electromagnetic field. That is, electromagnetic radiation must not spill over the universe. The mathematical expression of this Law is,

$$\lim_{R \rightarrow \infty} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E} \times \mathbf{H}) \cdot \hat{n} d\Gamma = 0 \quad (105)$$

Γ is a sphere with an infinite radius R . This law is also self-evident, in fact, nothing should spill out of the universe, including electromagnetic waves. It is not allowed for electromagnetic waves to overflow the universe. This problem exists in classical electromagnetic

theory. But people don't seem to care. When Maxwell proposed his theory, he particularly emphasized that it was ether theoretical. Therefore, the nature of electromagnetic wave propagation is the same as that of sound wave in the medium. Sound waves travel through the medium, regardless of whether they will overflow the universe. So it seems that electromagnetic waves don't have to take this into account either. As the electromagnetic wave propagates, the energy density of the electromagnetic wave decreases and eventually disappears automatically. But this viewpoint cannot be accepted in quantum mechanics because although the energy density of waves decreases, the energy passing through a sphere with a radius of R remains constant. Therefore, the collapse of waves is considered in quantum mechanics. Electromagnetic waves emitted from a light source collapse onto an absorber charge that forms a photon. According to the explanation of wave collapse in quantum mechanics, electromagnetic waves do not really spill out of the universe. So when it comes to quantum mechanics, this problem is avoided. There is no concept of wave collapse in classical electromagnetic field theory, so electromagnetic waves for classical electromagnetic theory will overflow the universe. The authors believe that it is an error in classical electromagnetic theory that electromagnetic waves can overflow the universe. As for the collapse of waves in quantum mechanics, the author proposed the concept of reverse collapse. Reverse collapse is accomplished by time reversal waves, and the authors find that the process of mutual energy flow, coupled with the reverse collapse of waves, is equivalent to that of wave collapse. So the author complements the classical electromagnetic theory with the concept of backward (or reverse) collapse of waves [15].

For an interpretation of quantum mechanics, it is understandable to suggest time reversal waves and reverse collapse. But few people believe in adding a new time reversal wave to classical electromagnetic theory. Recently (2021-2022) authors began to consider another scheme, which is that electromagnetic waves are reactive power. If the electric field and the magnetic field of an electromagnetic wave are kept in 90 degree phase difference, the Poynting vector of such an electromagnetic wave is a pure imaginary number and therefore reactive power. We know that electromagnetic waves have active power according to the solution of Maxwell's equation. The electric field and magnetic field of the electromagnetic wave solved according to Maxwell's equation keep the same phase. The author slowly figured out that there are something went wrong in the Maxwell's electromagnetic field theory.

Therefore, the author proposes this new axiom to make a preliminary revision to Maxwell's classical electromagnetic theory. Assume that Γ surrounds all current $\mathbf{J}$. The radius of Γ is R .

$$\mathbf{J} = \sum_{i=1}^N \mathbf{J}_i$$

$\mathbf{J}_i$ Within volume V , the surface Γ is the boundary of volume V . Assume that there are no current elements outside Γ . $\mathbf{E}$, $\mathbf{H}$ is an electromagnetic field produced

by the current element $\mathbf{J}$. Considering the superposition principle,

$$\mathbf{E} = \sum_{i=1}^N \mathbf{E}_i, \quad \mathbf{H} = \sum_{i=1}^N \mathbf{H}_i, \quad \mathbf{J} = \sum_{i=1}^N \mathbf{J}_i \quad (106)$$

According to this superposition principle and the author's new axiom (105),

$$\lim_{R \rightarrow \infty} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} \left(\sum_{i=1}^N \sum_{j=1, j \neq i}^N \mathbf{E}_i \times \mathbf{H}_j \right) \cdot \hat{n} d\Gamma = 0 \quad (107)$$

The above formula can be replaced by the following two formulas.

$$\lim_{R \rightarrow \infty} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} \sum_{i=1}^N \sum_{j=1, j \neq i}^N (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma = 0 \quad (108)$$

$$\lim_{R \rightarrow \infty} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} \left(\sum_{i=1}^N \mathbf{E}_i \times \mathbf{H}_i \right) \cdot \hat{n} d\Gamma = 0 \quad (109)$$

The formula (108) indicates whether the mutual energy flow can overflow the universe. The formula (109) indicates whether the self-energy flow can overflow the universe. We know that nothing can spill out of the universe, requiring self-energy flow and mutual-energy flow not to spill out of the universe is obviously correct. Therefore, the law that radiation does not overflow the universe further requires that neither self- nor mutual energy flows overflow the universe. These two formulas can be seen as two theorems.

Mutual energy does not spill over into the universe, as we have known before, when the electromagnetic waves in the mutual energy flow are retarded waves, advanced waves. retarded waves and advanced waves, which reach the sphere Γ at some point in the past and one at some point in the future. So the two waves do not reach the big sphere Γ at the same time. So they are not zero at the same time. Therefore the mutual energy flow must be zero on Γ . If both quantities in the mutual energy flow are retarded or advanced, they can be combined to form an electromagnetic wave, which can then make use of the formula (109) that the self-energy flow does not overflow the universe.

This contradicts the Maxwell equation's conclusion that self-energy flows do not overflow the universe. The author believes that Maxwell's theory is wrong or wrong in this place. In addition, the Poynting's theorem of the current element $\mathbf{J}_i$ is,

$$\begin{aligned} & - \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma \\ & = \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i + \frac{\partial}{\partial t} \mathbf{D}_i \cdot \mathbf{E}_i + \mathbf{E}_i \cdot \mathbf{J}_i \right) dV \end{aligned}$$

We have already proven that,

$$\int_{t=-\infty}^{\infty} dt \iiint_V \left(\frac{\partial}{\partial t} \mathbf{B}_i \cdot \mathbf{H}_i + \frac{\partial}{\partial t} \mathbf{D}_i \cdot \mathbf{E}_i \right) = 0$$

The law that the flow of self-energy does not overflow the universe tells us,

$$\int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma = 0$$

So we have,

$$\int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_i) dV = 0 \quad (110)$$

In fact, this formula can also be directly proven, so that we can omit the subscript (i). First the body current is converted to the line current, so there is,

$$\int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E} \cdot \mathbf{J}) dV \doteq \Re \iiint_V (\mathbf{E} \cdot \mathbf{J}^*) dV$$

The symbol “ $\doteq$ ” indicates permutation to the frequency domain. “ $\Re$ ” means taking a real number. Consider

$$\mathbf{E} = -\frac{\partial}{\partial t} \mathbf{A} = -j\omega \mathbf{A}$$

$$\mathbf{A} = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J} dV}{r}$$

When it comes to radiating electromagnetic fields, retarded and advanced effects should be considered, so there are,

$$\mathbf{A}^{(r)} = \frac{\mu_0}{4\pi} \iiint_V \exp(-jkr) \frac{\mathbf{J} dV}{r}$$

$$\mathbf{A}^{(a)} = \frac{\mu_0}{4\pi} \iiint_V \exp(+jkr) \frac{\mathbf{J} dV}{r}$$

Merge together,

$$\mathbf{A} = \frac{\mu_0}{4\pi} \iiint_V \exp(\mp jkr) \frac{\mathbf{J} dV}{r}$$

$$\iiint_V (\mathbf{E} \cdot \mathbf{J}^*) dV$$

$$= -j\omega \frac{\mu_0}{4\pi} \iiint_{V_2} \iiint_{V_1} \exp(\mp jkr) \frac{\mathbf{J}_1 \cdot \mathbf{J}_2}{r} dV_1 dV_2$$

$$\simeq -j\omega \frac{\mu_0}{4\pi} \sum_{i=1}^N \sum_{j=1, j \neq i}^N \exp(\mp jkr_{ij}) \frac{\mathbf{J}_{1i} \cdot \mathbf{J}_{2j}}{r_{ij}} \Delta V_{1i} \cdot \Delta V_{2j}$$

For the above formula, consider if $\mathbf{J}_{1i}$ towards $\mathbf{J}_{2j}$ emits a retarded wave, $\mathbf{J}_{2j}$ will towards $\mathbf{J}_{1i}$ emits an advanced wave so that the two effects can be synchronized, and therefore,

$$\exp(\mp jkr_{ij}) \frac{\mathbf{J}_{1i} \cdot \mathbf{J}_{2j}}{r_{ij}}$$

corresponding to,

$$\frac{\mathbf{J}_{1i} \exp(-jkr_{ij}) \cdot \mathbf{J}_{2j}}{r_{ij}}$$

$$\frac{\mathbf{J}_{1i} \exp(+jkr_{ij}) \cdot \mathbf{J}_{2j}}{r_{ij}}$$

we can replace the following,

$$\exp(\mp jkr_{ij}) \frac{\mathbf{J}_{1i} \cdot \mathbf{J}_{2j}}{r_{ij}}$$

as

$$\begin{aligned} & \frac{\mathbf{J}_{1i} \cdot \mathbf{J}_{2j}}{r_{ij}} (\exp(-jkr_{ij}) + \exp(+jkr_{ij})) \\ &= \frac{\mathbf{J}_{1i} \cdot \mathbf{J}_{2j}}{r_{ij}} 2\Re \exp(-jkr_{ij}) \end{aligned}$$

hence,

$$\begin{aligned} & \sum_{i=1}^N \sum_{j=1, j \neq i}^N \exp(\mp jkr_{ij}) \frac{\mathbf{J}_{1i} \cdot \mathbf{J}_{2j}}{r_{ij}} \Delta V_{1i} \cdot \Delta V_{2j} \\ & \rightarrow \sum_{i=1}^N \sum_{j=1, j < i}^N 2\Re \exp(-jkr_{ij}) \frac{\mathbf{J}_{1i} \cdot \mathbf{J}_{2j}}{r_{ij}} \Delta V_{1i} \cdot \Delta V_{2j} \end{aligned}$$

The above formula is a real number, so

$$\iiint_V (\mathbf{E} \cdot \mathbf{J}^*) dV$$

is a pure imaginary number. In this way,

$$\Re \iiint_V (\mathbf{E} \cdot \mathbf{J}^*) dV = 0$$

In switching back from the frequency domain to the time domain, there are

$$\int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E} \cdot \mathbf{J}) dV = 0$$

It is worth noting that in the above proof, we consider not only the retarded wave but also the advanced wave, but also a synchronization relationship if the current $\mathbf{J}_{1i}$ toward $\mathbf{J}_{2j}$ produce a retarded effect, then there must be $\mathbf{J}_{2j}$ toward $\mathbf{J}_{1i}$ produces a advanced effect. This is also the result of the author's electromagnetic theory. If only the retarded effect is considered according to Maxwell's classical electromagnetic theory, the above conclusion cannot be reached.

After considering the above three equations (108,109,110), we can prove the law of conservation of energy (86). It is worth noting, however, that we have now corrected the Maxwell equation. In particular, corrections have been made to the magnetic field derived from Maxwell's equation. (109,110) is impossible to satisfy the Maxwell equation. The time integral for the formula (98) is obtained by substituting (100) for (98).

$$\begin{aligned} & -\sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma \\ &= \sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_j) dV \end{aligned} \quad (111)$$

Magnetic field $\mathbf{H}_j$ in the formula above can be either a magnetic field derived from Maxwell's equation or a magnetic field defined by the author's law of radiation not overflowing the universe. The area integral on the left side of the above formular is generally not zero, only if the surface Γ is infinite and all the current elements are surrounded by it. The author calls the above formula a relaxed mutual energy principle. Note that the author refers to the formula (98) as the mutual energy principle.

Relaxation is caused by the time integral in the formula above. Because of this time integral, the electromagnetic field in this formula has obtained one more degree of freedom, hence, either from the Maxwell equation or from a new electromagnetic field defined by the author according to the law that radiated energy does not overflow the universe are all Ok for (111).

If the electromagnetic field defined by the author does not radiate out of the universe, the law of conservation of energy can be deduced from the relaxed mutual energy principle.

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{E}_i \cdot \mathbf{J}_j) dV = 0 \quad (112)$$

Hence there is,

$$\begin{aligned} & -\int_{t=-\infty}^{\infty} dt \iiint_{V_i} (\mathbf{E}_j \cdot \mathbf{J}_i) dV \\ & = (\xi_1, \xi_2) \\ & = \int_{t=-\infty}^{\infty} dt \iiint_{V_j} (\mathbf{E}_i \cdot \mathbf{J}_j) dV \end{aligned} \quad (113)$$

where,

$$(\xi_1, \xi_2) = \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j + \mathbf{E}_j \times \mathbf{H}_i) \cdot \hat{n} d\Gamma \quad (114)$$

The magnetic field in the three formulas above can be calculated according to Maxwell's equation, so the three formulas above are the theorems of mutual energy and mutual energy flow.

If the above magnetic field is the one calculated by the author's method, i.e. (109), then the formula (112) is the law of conservation of energy.

The formula (113, 114) states that the formula (112) is a localized law of conservation of energy. Localization here is because these energies can be accomplished

through mutual energy flows. The magnetic field defined by the author's method ensures that the electric field and the magnetic field correspond specifically to the far field and maintain a 90 degree phase difference. The electromagnetic waves that maintain a 90 degree phase difference are reactive power electromagnetic waves. This wave does not convey energy on average because it conveys energy forward and backward. So the average energy transferred is zero.

Although this wave averages no energy transfer, energy can be transmitted by mutual energy flow, which is composed of retarded waves from the source and advance waves from the sink. The formula (114) is an expression of mutual energy flow. For this mutual energy flow theorem the author will no longer prove it, because he has given it several times before [15]. In this chapter, we may be confused about the magnetic field defined by the author because it is all formula derivation and it is too abstract. However, it will be clearer when we wait for the following specific examples.

6. Emission and Reception of Electromagnetic Wave

6.1. Retarded Radiation of Infinite Flat Current

Assume that the flat current is, see Figure 1

$$J = \delta(x) J_0 \exp(j\omega t) \hat{z}$$

The magnetic field produced by this current follows the ampere circuital law in Maxwell's equation

$$\oint \mathbf{H} \cdot d\mathbf{l} = I$$

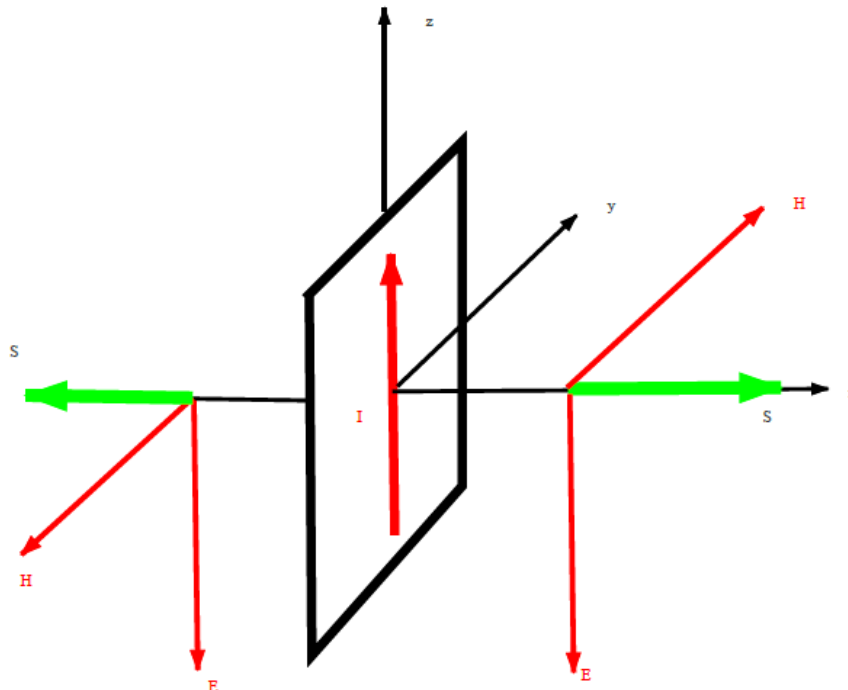


Figure 1. Retarded electromagnetic field of infinite flat current

Obtain,

$$\mathbf{H} = \begin{cases} \frac{1}{2} J_0 \exp(-jkx) \hat{y} & x > 0 \\ \frac{1}{2} J_0 \exp(+jkx)(-\hat{y}) & x < 0 \end{cases}$$

In accordance with Maxwell's equation, the current is zero when $x \neq 0$ is considered, hence,

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t} \epsilon_0 \mathbf{E}$$

Consider $x > 0$,

$$-jk\hat{x} \times \mathbf{H} = j\omega\epsilon_0 \mathbf{E}$$

$$-\frac{k}{\omega\epsilon_0} \hat{x} \times \mathbf{H} = \mathbf{E}$$

Consider,

$$k = \omega\sqrt{\epsilon_0\mu_0}$$

$$-\sqrt{\frac{\mu_0}{\epsilon_0}} \hat{x} \times \mathbf{H} = \mathbf{E}$$

$$\mathbf{E} = -\eta_0 \hat{x} \times \mathbf{H} = -\hat{z} \eta_0 \frac{1}{2} J_0 \exp(-jkx)$$

where

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

For electric fields, the same method can be used to prove that the above formula also holds true for $x < 0$. So you have $x > 0$.

$$\begin{aligned} \mathbf{S} &= \mathbf{E} \times \mathbf{H}^* = (-\hat{z} \eta_0 \frac{1}{2} J_0 \exp(-jkx)) \times (\frac{1}{2} J_0 \exp(-jkx) \hat{y})^* \\ &= \frac{\eta_0}{4} J_0 J_0^* \hat{x} \end{aligned}$$

It can also be proven for $x < 0$, there is,

$$\mathbf{S} = \frac{\eta_0}{4} J_0 J_0^* (-\hat{x})$$

We can calculate,

$$\mathbf{E}(x=0) \cdot \mathbf{J}^* = (-\hat{z} \eta_0 \frac{1}{2} J_0 \exp(-jk0)) \cdot (J_0 \hat{z})^* = -\eta_0 \frac{1}{2} J_0 J_0^*$$

So we verify that,

$$-\mathbf{E}(x=0) \cdot \mathbf{J}^* = S(x > 0) \cdot \hat{x} + S(x < 0) \cdot (-\hat{x})$$

The above two formulas verifies Poynting's theorem,

$$-\iiint \mathbf{E} \cdot \mathbf{J}^* dV = \oint \mathbf{S} \cdot \hat{n} d\Gamma$$

In this example, it seems normal. The green arrow in Figure 1 gives the direction of the Poynting energy flow. The radiation of an infinite current $\oint \mathbf{S} \cdot \hat{n} d\Gamma$ is equal to the output power of the flat current $-\iint \mathbf{E} \cdot \mathbf{J}^* dV$.

However, the author does not think the above is all right, because the author believes that both transmission and reception must be considered instead only consider the transmission.

6.2. Advanced-wave Radiation of Infinite Flat Current

Assume that the flat current is still,

$$\mathbf{J} = \delta(x) J_0 \exp(j\omega t) \hat{z}$$

Based on Maxwell's electromagnetic theory, the advanced wave of the magnetic field is obtained.

$$\mathbf{H} = \begin{cases} \frac{1}{2} J_0 \exp(+jkx) \hat{y} & x > 0 \\ \frac{1}{2} J_0 \exp(-jkx)(-\hat{y}) & x < 0 \end{cases}$$

In accordance with Maxwell's equation, the current is zero when $x \neq 0$ is considered, hence,

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t} \epsilon_0 \mathbf{E}$$

$$jk \times \mathbf{H} = j\omega\epsilon_0 \mathbf{E}$$

$$\mathbf{E} = \eta_0 \hat{x} \times \mathbf{H} = \hat{z} \eta_0 \frac{1}{2} J_0 \exp(+jkx)$$

In the author's electromagnetic theory, the advance wave is physically objective. The effect of the advance wave is to absorb the energy of the electromagnetic wave.

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}^*$$

$$\begin{aligned} &= (\hat{z} \eta_0 \frac{1}{2} J_0 \exp(+jkx)) \times (\frac{1}{2} J_0 \exp(+jkx) \hat{y})^* \\ &= \begin{cases} (\hat{z} \eta_0 \frac{1}{2} J_0 \exp(+jkx)) \times (\frac{1}{2} J_0 \exp(+jkx) \hat{y})^* & x > 0 \\ (\hat{z} \eta_0 \frac{1}{2} J_0 \exp(-jkx)) \times (\frac{1}{2} J_0 \exp(-jkx)(-\hat{y}))^* & x < 0 \end{cases} \\ &= \begin{cases} -\frac{\eta_0}{4} J_0 J_0^* \hat{x} & x > 0 \\ \frac{\eta_0}{4} J_0 J_0^* \hat{x} & x < 0 \end{cases} \end{aligned}$$

We can calculate,

$$\mathbf{E}(x=0) \cdot \mathbf{J}^* = (\hat{z} \eta_0 \frac{1}{2} J_0 \exp(jk0)) \cdot (J_0 \hat{z})^* = \eta_0 \frac{1}{2} J_0 J_0^*$$

The previous formula validates Poynting's theorem,

$$-\mathbf{E}(x=0) \cdot \mathbf{J}^* = S(x > 0) \cdot \hat{x} + S(x < 0) \cdot (-\hat{x})$$

The above two formulas verifies Poynting's theorem,

$$-\iiint \mathbf{E} \cdot \mathbf{J}^* dV = \oint \mathbf{S} \cdot \hat{n} d\Gamma$$

This indicates that this plate absorbs the energy flow, as shown in Figure 2. The green arrows indicate the direction of the Poynting energy flow. Next let's consider the case of two current plates. One transmits, one receives.

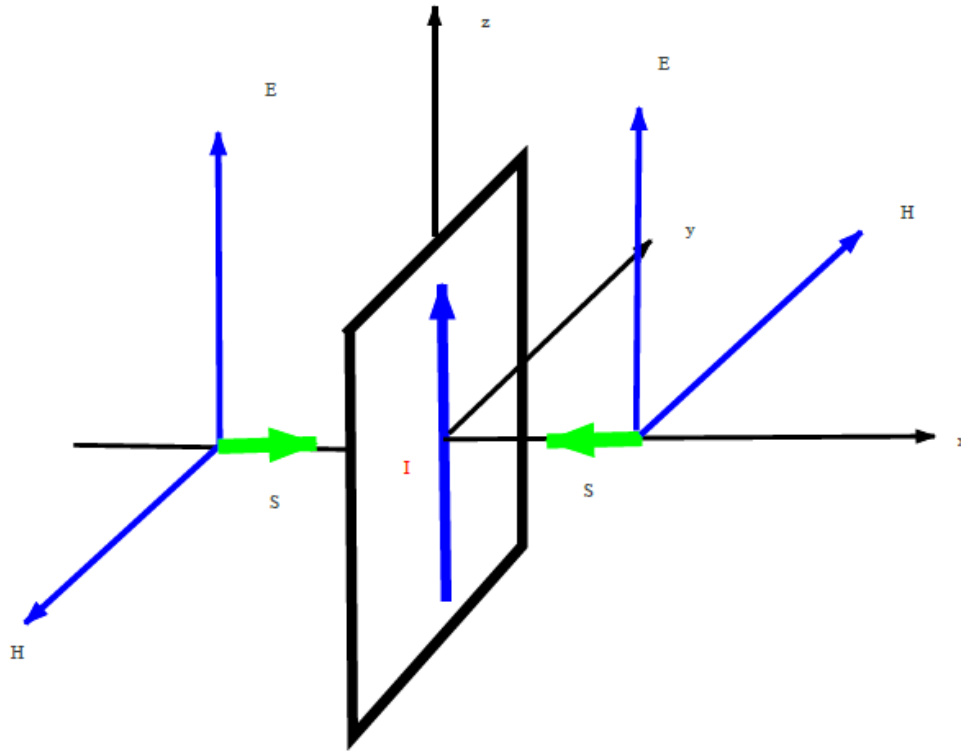


Figure 2. Advanced-wave radiation of electromagnetic field of infinite flat current

6.3. Radiation of Dual Infinite Plate Current

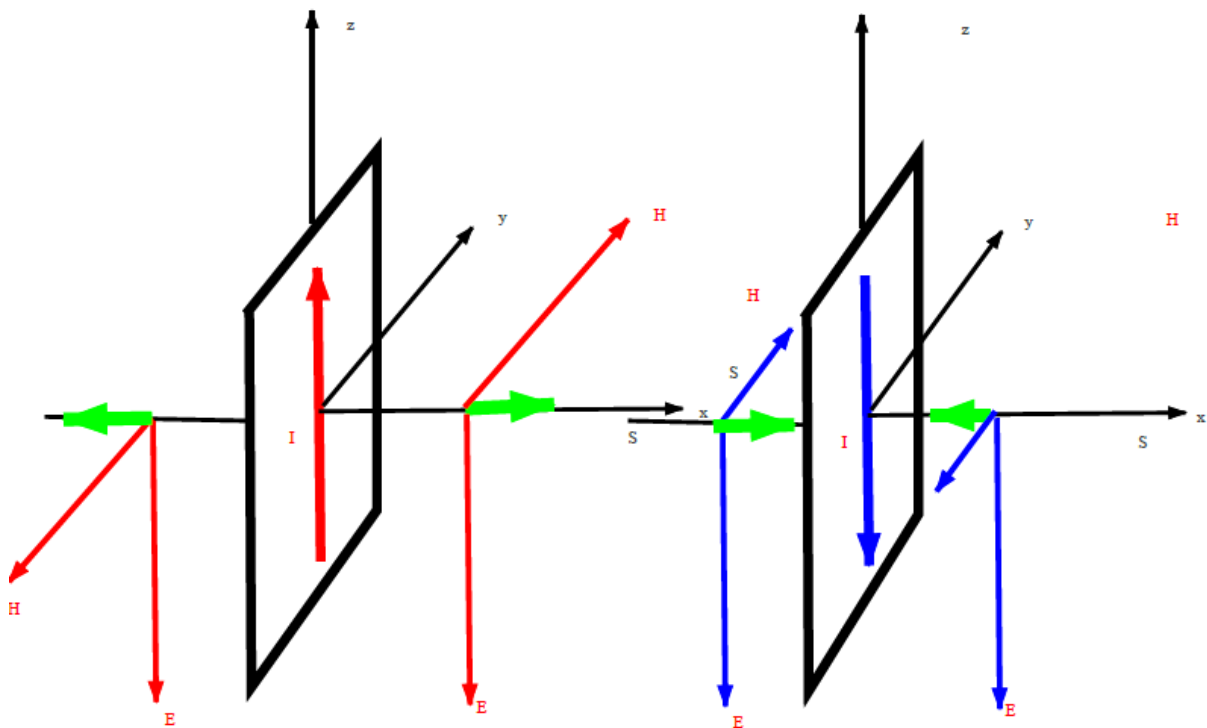


Figure 3. retarded electromagnetic field of infinite flat current

Assuming that current 1 is at $x = 0$,

$$J_1 = \delta(x) J_{10} \exp(j\omega t) \hat{z}$$

It emits retarded waves, so it has,

$$H_1 = \frac{1}{2} J_{10} \exp(-jkx) \hat{y} \quad x > 0$$

$$E_1 = \eta_0 \frac{1}{2} J_{10} \exp(-jkx) (-\hat{z}) \quad x > 0$$

For convenience, suppose the second current plate is at position $x = +L$ and the current is,

$$J_2 = \delta(x) J_{20} \exp(j\omega t)(-\hat{z})$$

In the formula above, we consider the current J_2 . The direction of is $(-\hat{z})$. It's just for convenience, and eventually we'll have to decide J_{20} , the phase of J_{20} will also adjust the true direction of the current J_2 . The second plate should emit an advanced wave, so there is,

$$H_2 = \frac{1}{2} J_{20} \exp(+jk|x-L|)\hat{y} \quad x < L$$

According to Maxwell's electromagnetic theory, electric and magnetic fields have the same phase,

$$E_2 = (-\hat{z})\eta_0 \frac{1}{2} J_{20} \exp(+jk|x-L|) \quad x < L$$

The above formula can be written as,

$$H_2 = \frac{1}{2} J_{20} \exp(-jk(x-L))\hat{y} \quad x < L$$

$$E_2 = (-\hat{z})\eta_0 \frac{1}{2} J_{20} \exp(-jk(x-L)) \quad x < L$$

Consider that load resistance R_2 of Loop 2 is relatively large, so the impedance $Z_2 = j\omega L_2 + R_2$ is close to the resistance value $Z_2 \approx R_2 \sim 1$.

$$I_2 = \frac{\mathcal{E}_{2,1}}{Z_2} \sim \frac{E_1 \cdot (-\hat{z})}{R_2} \sim \eta_0 \frac{1}{2} J_{10} \exp(-jkL)$$

$$J_{20} \sim I_2 \sim \frac{1}{2} J_{10} \exp(-jkL)$$

Hence,

$$H_2 \sim \frac{1}{2} (\eta_0 \frac{1}{2} J_{10} \exp(-jkL)) \exp(-jk(x-L))\hat{y} \quad x < L$$

$$E_2 \sim (-\hat{z})\eta_0 \frac{1}{2} (\eta_0 \frac{1}{2} J_{10} \exp(-jkL)) \exp(-jk(x-L)) \quad x < L$$

Or,

$$H_2 \sim \frac{\eta_0}{4} J_{10} \exp(-jkx)\hat{y} \quad x < L$$

$$E_2 \sim (-\hat{z})\frac{\eta_0^2}{4} J_{10} \exp(-jkx) \quad x < L$$

In the range $0 < x < L$,

$$\begin{aligned} S_m &= E_1 \times H_2^* + E_2^* \times H_1 \\ &\sim (-\hat{z})\eta_0 \frac{1}{2} J_{10} \exp(-jkx) \times (\frac{\eta_0}{4} J_{10} \exp(-jkx)\hat{y})^* \\ &\quad + ((-\hat{z})\frac{\eta_0^2}{4} J_{10} \exp(-jkx))^* \times (\frac{1}{2} J_{10} \exp(-jkx)\hat{y}) \\ &= \frac{1}{4} \eta_0^2 J_{10} J_{10}^* \hat{x} \end{aligned}$$

We see that this mutual energy current is from the first current plate to the second one, which is correct. The problem, however, is that there are still two boards with their own energy flows for this system.

$$S_{11} = E_1 \times H_1^* = \frac{\eta_0}{4} J_{10} J_{10}^* \hat{x}$$

This energy flow is coming from the right side of the first board.

$$S_{22} = E_2 \times H_2^* = \frac{\eta_0}{4} J_{20} J_{20}^* (\hat{x})$$

Above is the energy the second panel receives from the left side of the second panel. We see that this is incorrect. If energy is transmitted by mutual energy flow, then the energy flow transmitted by self-energy flow is redundant. If the energy is transmitted by the self-energy flow, then the mutual energy flow is redundant. Both mutual- and self-energy flows transfer energy, which ultimately leads to more energy being transferred. This is not trustworthy.

6.4. Calculating the Radiation of a Double-infinite Plate Current According to the Electric And Magnetic Fields Defined by the Author

It can be seen from the above that if a double current plate is considered, then the self-energy flow is redundant. According to the author's method, radiation does not overflow the universe, so the electric field and the magnetic field produced by the plane current should have a phase difference of 90 degrees. Therefore, the author assumes that the electromagnetic field of a single current plate is,

$$H = \frac{1}{2} J_0 \exp(-jkx)\hat{y} \quad x > 0$$

$$E = -j\hat{z}\eta_0 \frac{1}{2} J_0 \exp(-jkx) \quad x > 0$$

Note that the electric and magnetic fields in this section should have been superscripted "new", but the formula becomes too complex with the superscript. Therefore, it is still omitted. However, it is important to understand that the electric and magnetic fields are not calculated as defined by the Maxwell equation. It is calculated as defined by the author. The phase factor $(-j)$ above refers to the quasi-static electromagnetic field condition obtained,

$$E = -j\omega A \sim -jJ_0 \hat{z}$$

For retarded waves of $x < 0$,

$$H = -\frac{1}{2} J_0 \exp(+jkx)\hat{y} \quad x < 0$$

$$E = -j\hat{z}\eta_0 \frac{1}{2} J_0 \exp(+jkx) \quad x < 0$$

the advanced wave of $(x > 0)$ is,

$$H = \frac{1}{2} J_0 \exp(+jkx)\hat{y} \quad x > 0$$

$$E = -j\hat{z}\eta_0 \frac{1}{2} J_0 \exp(+jkx) \quad x > 0$$

the advanced wave of ($x < 0$) is,

$$H = -\frac{1}{2} J_0 \exp(-jkx) \hat{y} \quad x < 0$$

$$E = -j\hat{z}\eta_0 \frac{1}{2} J_0 \exp(-jkx) \quad x < 0$$

For electromagnetic waves defined by the authors above, they are reactive power waves, which do not transfer energy themselves. Or the average energy flow they transmit is 0. For such waves, only the mutual energy flow will have an impact. For self-energy flows, since they do not convey energy anyway, they can be ignored. We assume that a current can generate both a retarded wave and an advanced wave at the same time. But some of them can be ignored.

Assume that the first plate produces electromagnetic waves and the second plate receives them. It is assumed that there is a second current plate on the right side of the first current plate. Therefore, for the right side of the two plates, the retarded wave is taken. On the left side, the advanced wave is taken.

$$H_1 = \begin{cases} \frac{1}{2} J_{10} \exp(-jkx) \hat{y} & x > 0 \\ -\frac{1}{2} J_{10} \exp(-jkx) \hat{y} & x < 0 \end{cases}$$

$$E_1 = -j\hat{z}\eta_0 \frac{1}{2} J_{10} \exp(-jkx)$$

$$H_2 = \begin{cases} -\frac{1}{2} J_{20} \exp(-jk(x-L)) \hat{y} & x > L \\ \frac{1}{2} J_{20} \exp(-jk(x-L)) \hat{y} & x < L \end{cases}$$

$$E_2 = -j\hat{z}\eta_0 \frac{1}{2} J_{20} \exp(-jk(x-L))$$

For convenience, consider J_2 . The direction of is also in the direction of $\hat{z}$,

$$J_{20} \sim \frac{1}{\eta_0} E_1(x=L) \cdot (-\hat{z}) = j \frac{1}{2} J_{10} \exp(-jkL)$$

therefore,

$$H_2 = \begin{cases} -\frac{1}{2} (j \frac{1}{2} J_{10} \exp(-jkL)) \exp(-jk(x-L)) \hat{y} & x > L \\ \frac{1}{2} (j \frac{1}{2} J_{10} \exp(-jkL)) \exp(-jk(x-L)) \hat{y} & x < L \end{cases}$$

$$E_2 = -j\hat{z}\eta_0 \frac{1}{2} (-j \frac{1}{2} J_{10} \exp(-jkL)) \exp(-jk(x-L))$$

Or,

$$H_2 = \begin{cases} -\frac{1}{4} (jJ_{10}) \exp(-jkx) \hat{y} & x > L \\ \frac{1}{4} (jJ_{10}) \exp(-jkx) \hat{y} & x < L \end{cases}$$

$$E_2 = -j\eta_0 \frac{1}{4} (jJ_{10}) \exp(-jkx) (-\hat{z})$$

This has a value of $0 < x < L$

$$\begin{aligned} S_m &= S_{12} + S_{21} = E_1 \times H_2^* + E_2^* \times H_1 \\ &= (j\eta_0 \frac{1}{2} J_{10} \exp(-jkx) (-\hat{z})) \\ &\quad \times (\frac{1}{4} (jJ_{10}) \exp(-jkx) \hat{y})^* \\ &\quad + (-j\eta_0 \frac{1}{4} (jJ_{10}) \exp(-jkx) (-\hat{z}))^* \\ &\quad \times (\frac{1}{2} J_{10} \exp(-jkx) \hat{y}) \\ &= ((j\eta_0 \frac{1}{2} J_{10}) (\frac{1}{4} (jJ_{10}))^* \\ &\quad + (-j\eta_0 \frac{1}{4} (jJ_{10}))^* (\frac{1}{2} J_{10})) (\hat{x}) \\ &= \frac{1}{8} \eta_0 J_{10} J_{10}^* ((j)(j)^* + (-jj)^* (1)) (\hat{x}) \\ &\sim \hat{x} \end{aligned}$$

hence,

$$S_m \sim \hat{x} \quad 0 < x < L$$

Further available,

$$S_m \sim \hat{x} \begin{cases} 0 & x < 0 \\ 1 & 0 < x < L \\ 0 & L < x \end{cases}$$

The above formula takes into account that the magnetic field just turns around when it crosses the current plate. So the superposition of S_{12} and S_{21} becomes cancellation. So outside the two current plates, mixed Poynting vector corresponding to mutual energy flow is exactly zero. From above it can be seen that the mutual energy flow or mixed Poynting vector is generated on the first current plate and annihilated on the second one.

We see that in the inner brackets of the above formula,

$$(j)(j)^* + (-jj)^* (1) \sim 1$$

This is a golden structure for receiving electromagnetic waves. This structure was discovered by the author when studying the transformer. The energy flow from the primary coil to the secondary coil of the transformer will appear this structure [24]. According to this structure, the electromagnetic fields of the transformer are synchronized. Now electromagnetic waves are synchronized according to this structure. This synchronization is performed by,

$$E_1 \sim j(-\hat{z})$$

$$H_2 \sim j(\hat{y})$$

$$E_2 \sim -j\hat{z}$$

$$H_1 \sim 1(\hat{y})$$

It can be seen that the electromagnetic waves of two plate currents are exactly synchronized. In this synchronization case, all self-energy flows have a phase difference of 90 degrees, and hence,

$$\begin{aligned} S_{11} &= \mathbf{E}_1 \times \mathbf{H}_1^* \\ &= (j(-\hat{z})\eta_0 \frac{1}{2} J_{10} \exp(-jkx)) \\ &\quad \times (\frac{1}{2} J_{10} \exp(-jkx)\hat{y})^* \\ &\sim j \\ S_{22} &= \mathbf{E}_2 \times \mathbf{H}_2^* \\ &= (-j\eta_0 \frac{1}{4} (jJ_{10}) \exp(-jkx)(-\hat{z})) \\ &\quad \times (\frac{1}{4} (jJ_{10}) \exp(-jkx)\hat{y})^* \\ &\sim j^* \sim -j \end{aligned}$$

Description of self-energy flow S_{11} , S_{22} is a pure imaginary number, so it is a reactive power wave and does not transfer energy in time average. Energy is transmitted by mutual energy flow.

Please note that the electromagnetic field in this section should be marked with the subscript "new". However, it is omitted for convenience. But this subscript should be kept in mind.

6.5. Antenna Reception of Electromagnetic Wave

Now we will discuss the reception of electromagnetic waves by the receiving antenna.

It is assumed that there are plane electromagnetic waves in space, see Figure 4. Let's study how the antenna can

receive this electromagnetic wave. It is assumed that the electromagnetic wave is,

$$\xi_1 = [\mathbf{E}_1, \mathbf{H}_1]^T \quad (115)$$

The electric and magnetic fields are first calculated according to Maxwell's equations, so the electric and magnetic fields are in phase.

$$\mathbf{E}_1 = -\frac{\partial}{\partial t} \mathbf{A}_1 - \nabla \phi_1 \quad (116)$$

For planar electromagnetic waves, it can be seen that only $\mathbf{A}_1$ Generated. Namely

$$\nabla \phi_1 = 0$$

Consider

$$\mathbf{A}_1 = A_{10} \exp(-jkx)\hat{z}$$

$$\mathbf{E}_1 = -j\omega \mathbf{A}_1 = j\omega A_{10} \exp(-jkx)(-\hat{z})$$

$$\mathbf{E}_1 = E_1(-\hat{z})$$

In the above formula, we assume that $\mathbf{E}_1$ The direction of is $-\hat{z}$ where

$$E_1(x=l) = j\omega A_{10} \exp(-jkx) \sim j \exp(-jkl)$$

$x=l$ is the position of the receiving antenna. According to Maxwell's equation

$$H_1 = \frac{1}{\eta_0} E_1$$

Considering that the above electromagnetic fields are defined according to Maxwell's electromagnetic theory, we specially subscript them,

$$\mathbf{E}_{1\text{maxwell}}(x=l) \sim j \exp(-jkl)(-\hat{z})$$

$$\mathbf{H}_{1\text{maxwell}} = H_{1\text{maxwell}}\hat{y}$$

$$= \frac{1}{\eta_0} j\omega A_{10} \exp(-jkx)\hat{y} \sim j \exp(-jkx)\hat{y}$$

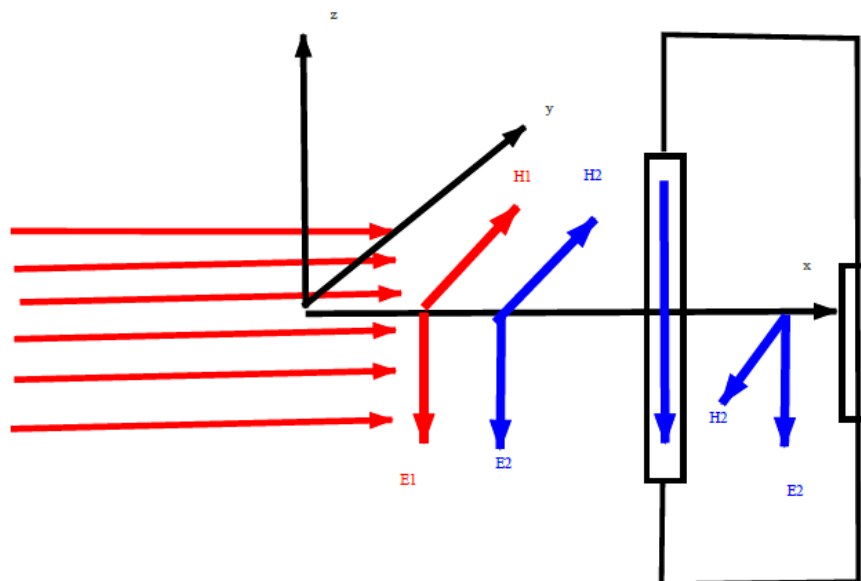


Figure 4. Consider a straight wire antenna that receives electromagnetic waves. The antenna is connected to a load resistor and is placed under the irradiation of electromagnetic waves

We know that the current at the receiving antenna is,

$$I_2 = \frac{\mathcal{E}_{2,1}}{j\omega L_2 + R_2} \quad (117)$$

among them,

$$\mathcal{E}_{2,1} = \int_{C_2} \mathbf{E}_1 \cdot d\mathbf{l} \quad (118)$$

Is the inductive electromotive force, $j\omega L_2 + R_2$ is the impedance of the antenna circuit. Where L_2 is the receiving antenna circuit inductance, R_2 is its load resistance. For the receiving antenna, because it is very close to the antenna, near the antenna, the main body of the electromagnetic and magnetic fields is the quasi-static electromagnetic field. The radiated electromagnetic field is negligible.

$$\begin{aligned} \mathbf{E}_2 &= -j\omega \mathbf{A}_2 = -j\omega \frac{\mu_0}{4\pi} \int_{C_2} \frac{1}{r} d\mathbf{l} I_2 \\ &= -j\omega \frac{\mu_0}{4\pi} \int_{C_2} \frac{1}{r} d\mathbf{l} (-\hat{z}) I_2 \end{aligned}$$

In the formula above we assume the current I_2 . The direction of is $-\hat{z}$, so there is $d\mathbf{l} = -d\mathbf{l}\hat{z}$.

$$\mathbf{E}_2 = E_2(-\hat{z})$$

$$\mathbf{E}_2 = -j\omega \frac{\mu_0}{4\pi} \int_{C_2} \frac{1}{r} d\mathbf{l} I_2$$

consider,

$$\begin{aligned} I_2 &= \frac{\mathcal{E}_{2,1}}{j\omega L_2 + R_2} \approx \frac{\mathcal{E}_{2,1}}{R_2} \sim \mathcal{E}_{2,1} \sim E_1 \\ &= j\omega A_{10} \exp(-jkl) \sim j \exp(-jkl) \end{aligned}$$

Notice the above formula the current I_2 direction and electric field $\mathbf{E}_1$ are the same direction $(-\hat{z})$,

$$\begin{aligned} \mathbf{E}_2 &= -j\omega \frac{\mu_0}{4\pi} \int_{C_2} \frac{1}{r} d\mathbf{l} I_2 (-\hat{z}) \\ &\sim -jI_2(-\hat{z}) \sim (-j)(j \exp(-jkl))(-\hat{z}) \end{aligned}$$

We have assumed the receiving antenna is in the place,

$$x = l$$

According to the quasi-static electromagnetic field theory, the magnetic field and the current have the same phase.

$$H_2 \sim I_2 \hat{y} \sim j \exp(-jkl) \hat{y}$$

Consider the mutual energy flow theorem [15],

$$\begin{aligned} &\oint_{\Gamma} \mathbf{S}_m \cdot \hat{n} d\Gamma \\ &= \oint_{\Gamma} (\mathbf{E}_{1\maxwell} \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_{1\maxwell}) \cdot \hat{n} d\Gamma \\ &= \iiint (\mathbf{E}_1 \cdot \mathbf{J}_2^*) dV \end{aligned}$$

Among them,

$$\begin{aligned} \mathbf{S}_m &= \mathbf{E}_{1\maxwell} \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_{1\maxwell} \\ &= (j \exp(-jkl)(-\hat{z})) \times (j \exp(-jkl) \hat{y})^* \\ &\quad + ((-j)(j \exp(-jkl)(-\hat{z})))^* \times (j \exp(-jkl) \hat{y}) \\ &\sim ((j)(j)^* + (-jj)^*(j)) \hat{x} \\ &= (1+j) \hat{x} \end{aligned}$$

We see that the second term of the above formula is imaginary, so the second term is reactive power and does not transfer energy, which is not what we expected. We expect it to be real. This indicates that Maxwell's definition of the magnetic field of electromagnetic waves is questionable. So we need to calibrate the electric and magnetic fields defined by Maxwell, that is, redefine them.

$$\mathbf{E}_{1new} \triangleq \mathbf{E}_{1\maxwell}$$

$$\mathbf{H}_{1new} \triangleq -j\mathbf{H}_{1\maxwell}$$

This way,

$$\begin{aligned} \mathbf{S}_m &= \mathbf{E}_{1new} \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_{1new} \\ &\sim \mathbf{E}_{1\maxwell} \times \mathbf{H}_2^* + \mathbf{E}_2^* \times (-j)\mathbf{H}_{1\maxwell} \\ &\sim ((j)(j)^* + (-jj)^*(j)) \hat{x} \sim \hat{x} \end{aligned}$$

According to the newly defined electromagnetic field of the electromagnetic wave, the receiving antenna can also receive the electromagnetic wave perfectly when it is a quasi-static field. Because we once encountered the golden structure $(j)(j)^* + (-jj)^*$ of electromagnetic wave propagation. So our newly defined magnetic field is the correct one. The magnetic field defined by Maxwell's electromagnetic theory is incorrect.

7. Definition and Measurement of Magnetic Field

This chapter considers the measurement and definition of magnetic fields. First, the measurement of the quasi-static electric field is considered, then the measurement of the magnetic field of the electromagnetic wave is considered.

7.1. Testing of Magnetostatic Magnetic Field (1)

For the test of the magnetic field of a quasi-static field, the direction of the magnetic field is assumed to be $\hat{y}$. The magnetic field is alternating current and uniform, that is,

$$\mathbf{H} = H_0 \hat{y} \exp(j\omega t) \quad (119)$$

Our goal is to measure the size of H_0 . Only in vacuum, so $B_0 = \mu_0 H_0$. The direction of the magnetic field is known to be $\hat{y}$. The easiest thing to think of is using coils

to consider a small square coil, as shown in Figure 5. The area of the coil is S . Suppose the coil has only one circle, a square shape, and an L side. We can use the following Faraday's Law relations.

$$\mathcal{E}_O \triangleq \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \oint \mathbf{B} \cdot \hat{n} d\Gamma$$

$\triangleq$ is the meaning of the definition. $\mathcal{E}_O$ is an inductive EMF measured on a closed circuit. Rewrite the above style to,

$$\mathcal{E}_O = -j\omega\mu_0 \mathbf{H} \cdot \hat{n} S \quad (120)$$

Or,

$$\mathbf{H} \cdot \hat{n} = \frac{\mathcal{E}_O}{-\mu_0 j\omega S} \quad (121)$$

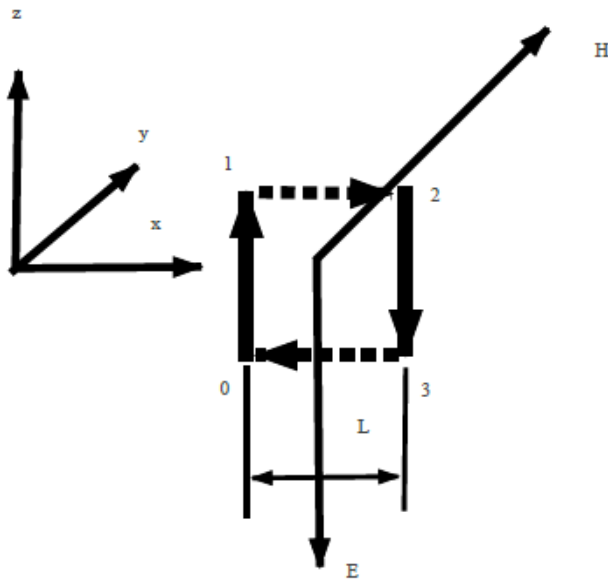


Figure 5. Suppose the magnetic field points to the $\hat{y}$ axis. The magnetic field is measured with a square ring measuring coil with an L -side length

We have assumed that the coil is square and the side length is L , so $S = L^2$, the direction of the coil is $\hat{n} = \hat{y}$, in which case we measure that,

$$\mathbf{H} \cdot \hat{n} = (H_0 \hat{y} \exp(j\omega t)) \cdot \hat{y} = H_0 \exp(j\omega t) \quad (122)$$

Therefore,

$$H_0 \exp(j\omega t) = \mathbf{H} \cdot \hat{n} = \frac{\mathcal{E}_O}{-j\mu_0 \omega S} \quad (123)$$

$$H_0 = \frac{\mathcal{E}_O}{-\mu_0 j\omega L^2 \exp(j\omega t)}$$

If we measure the EMF $\mathcal{E}_O$, we know the magnetic field H_0 . There is a time factor $\exp(j\omega t)$ in the upper denominator. This factor will end up with the measured $\mathcal{E}_O$. The same time factor in is eliminated. The above formula can be rewritten as

$$H_0 = \frac{\mathcal{E}_O}{-\mu_0 j\omega L^2} \quad (124)$$

The phase of the magnetic field and the electromotive potential is determined by the following formula.

$$\mathcal{E}_O \sim -jH_0 \quad (125)$$

$$H_0 \sim j\mathcal{E}_O \quad (126)$$

7.2. Testing of Magnetostatic Magnetic Field (2)

The magnetic field is consistent with the previous section. That is (119).

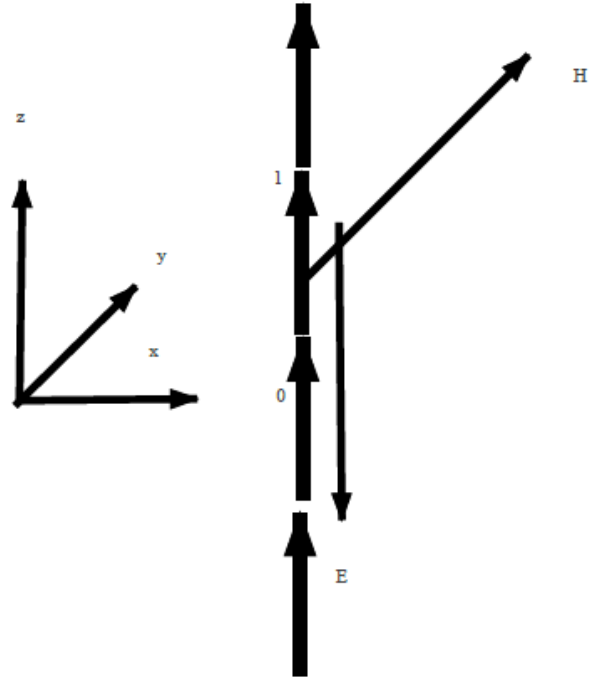


Figure 6. Measure the magnetic field by considering an electric dipole antenna or a current element

To test with a small current element, which is four times as long as one side of the first four square coils. Considering that in this case we know that the four sides contribute the same to the EMF, we have,

$$\mathcal{E}_O = \mathcal{E}_L$$

As with the meet formula (124),

$$H_0 = \frac{\mathcal{E}_L}{-j\omega L^2 \exp(j\omega t)} \quad (127)$$

Obviously, there is no difference between the two measurement methods in the above case. The phase difference between the magnetic field and the measured electromotive potential is j .

$$\mathcal{E}_L \sim -jH_0 \quad (128)$$

$$H_0 \sim j\mathcal{E}_L \quad (129)$$

Thus, for a quasi-static magnetic field, the phase of the magnetic field can be exactly the same by both measurements. Therefore, the two methods for measuring magnetic fields are identical. That is, (124, 127), the measurements for the two methods in magnetic quasi-static electromagnetic field situation are exactly the same.

7.3. Measurements of Magnetic Field of Radiation Field (1)

Now look at the electromagnetic wave. Suppose an infinite planar current produces an electromagnetic field of a planar wave that we want to measure. From the previous discussion, it is known that electromagnetic waves and quasi-static electromagnetic fields are two completely different electromagnetic fields, and their definitions of magnetic fields need not be exactly the same. Assuming the electromagnetic wave is,

$$\mathbf{E} = E_0(-\hat{z})\exp(j\omega t - jkx) \quad (130)$$

We already know that the electric field of the electromagnetic wave is in the $-\hat{z}$ direction, so the direction of the magnetic field should be in the $\hat{y}$ axis.

$$\mathbf{H} = H_0\hat{y}\exp(j\omega t - jkx) \quad (131)$$

The phase of H_0 is what we want to measure. Consider Faraday law,

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B}$$

or

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B}$$

or

$$\mathcal{E}_L = -j\omega B_y S$$

or

$$\mathcal{E}_O = -j\omega\mu_0 H_y S$$

$$H_y = \frac{\mathcal{E}_O}{-j\omega\mu_0 S} \quad (132)$$

therefore,

$$\mathbf{E} = E_0(-\hat{z})\exp(j\omega t - jkx) \quad (133)$$

$$\mathcal{E}_O = \mathcal{E}_{0,1} + \mathcal{E}_{1,2} + \mathcal{E}_{2,3} + \mathcal{E}_{3,0} = \mathcal{E}_{0,1} + \mathcal{E}_{2,3} \quad (134)$$

$$\begin{aligned} \mathcal{E}_{0,1} &= \mathbf{E}(x = -\frac{L}{2}) \cdot L\hat{z} \\ &= E_0(-\hat{z})\exp(j\omega t - jk(-\frac{L}{2})) \cdot L\hat{z} \end{aligned} \quad (135)$$

$$\begin{aligned} &= -E_0 \exp(j\omega t + jk\frac{L}{2})L \\ \mathcal{E}_{2,3} &= \mathbf{E}(x = \frac{L}{2}) \cdot L(-\hat{z}) \\ &= E_0(-\hat{z})\exp(j\omega t - jk\frac{L}{2}) \cdot L(-\hat{z}) \\ &= E_0 \exp(j\omega t - jk\frac{L}{2})L \end{aligned} \quad (136)$$

It is clear there is,

$$\mathcal{E}_{1,2} + \mathcal{E}_{3,0} = 0$$

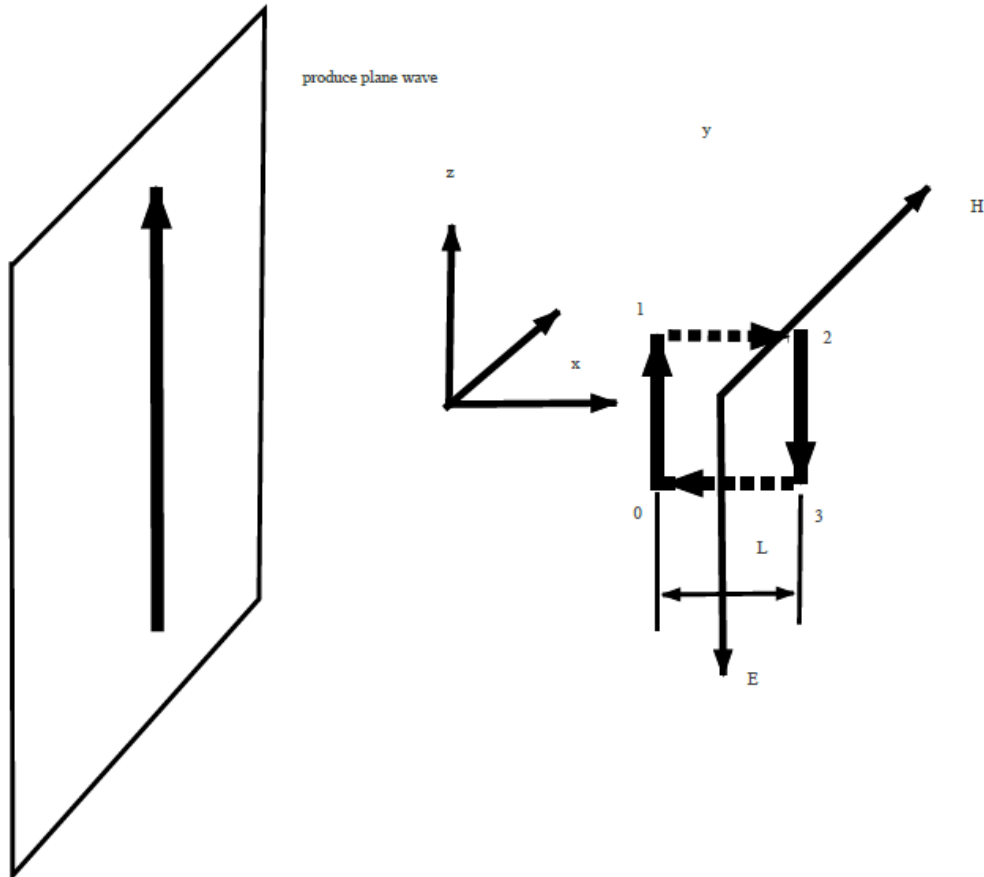


Figure 7. The magnetic field points to the y-axis. Square ring measuring coil. Magnetic field receiving electromagnetic wave (retarded electromagnetic field)

Hence,

$$\begin{aligned}
 \mathcal{E}_O &= \mathcal{E}_{0,1} + \mathcal{E}_{2,3} \\
 &= -E_0 \exp(j\omega t + jk \frac{L}{2})L \\
 &\quad + E_0 \exp(j\omega t - jk \frac{L}{2})L \\
 &= -E_0 L \exp(j\omega t) (\exp(jk \frac{L}{2}) - \exp(-jk \frac{L}{2})) \\
 &= -E_0 L \exp(j\omega t) 2j \frac{1}{2j} (\exp(jk \frac{L}{2}) - \exp(-jk \frac{L}{2})) \\
 &= -j2E_0 L \exp(j\omega t) \sin(k \frac{L}{2}) \\
 \mathcal{E}_O &= -j2E_0 L \sin(k \frac{L}{2}) \exp(j\omega t) \quad (137)
 \end{aligned}$$

$\mathcal{E}_O$ denotes an inductive electromotive force that is a radiated electromagnetic field on a closed loop O.

$$\begin{aligned}
 H_y &= \frac{\mathcal{E}_O}{-j\omega\mu_0 S} \\
 &= \frac{-j2E_0 L \sin(k \frac{L}{2}) \exp(j\omega t)}{-j\omega\mu_0 S} \quad (138) \\
 &= \frac{2E_0 L \sin(k \frac{L}{2}) \exp(j\omega t)}{\omega\mu_0 S}
 \end{aligned}$$

E_0 is the electric field strength of the electromagnetic wave. H_y is the measured magnetic field of this method.

$$H_y \sim E_0 \quad (139)$$

This method tell us the measured magnetic field has the same phase as the electric field E_0 .

7.4. Magnetic Field Testing for Retarded Fields (2)

Now consider using only one current element or electric dipole antenna to measure the magnetic field of electromagnetic waves. The current element is placed at the position of $x = 0$, see Figure 8,

$$\begin{aligned}
 \mathcal{E}_L &= 4\mathcal{E}_{0,1} = E(x=0) \cdot 4L\hat{z} \\
 &= E_0(-\hat{z}) \exp(j\omega t - jk(0)) \cdot 4L\hat{z} \quad (140) \\
 &= -E_0 \exp(j\omega t) 4L
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 \mathcal{E}_L &= 4\mathcal{E}_{0,1} = -E_0 4L \exp(j\omega t) \\
 H_y &= \frac{\mathcal{E}_L}{-j\omega\mu_0 S} \quad (141) \\
 &= j \frac{\mathcal{E}_L}{\omega\mu_0 S} = -j \frac{E_0 \exp(j\omega t) 4L}{\omega\mu_0 S}
 \end{aligned}$$

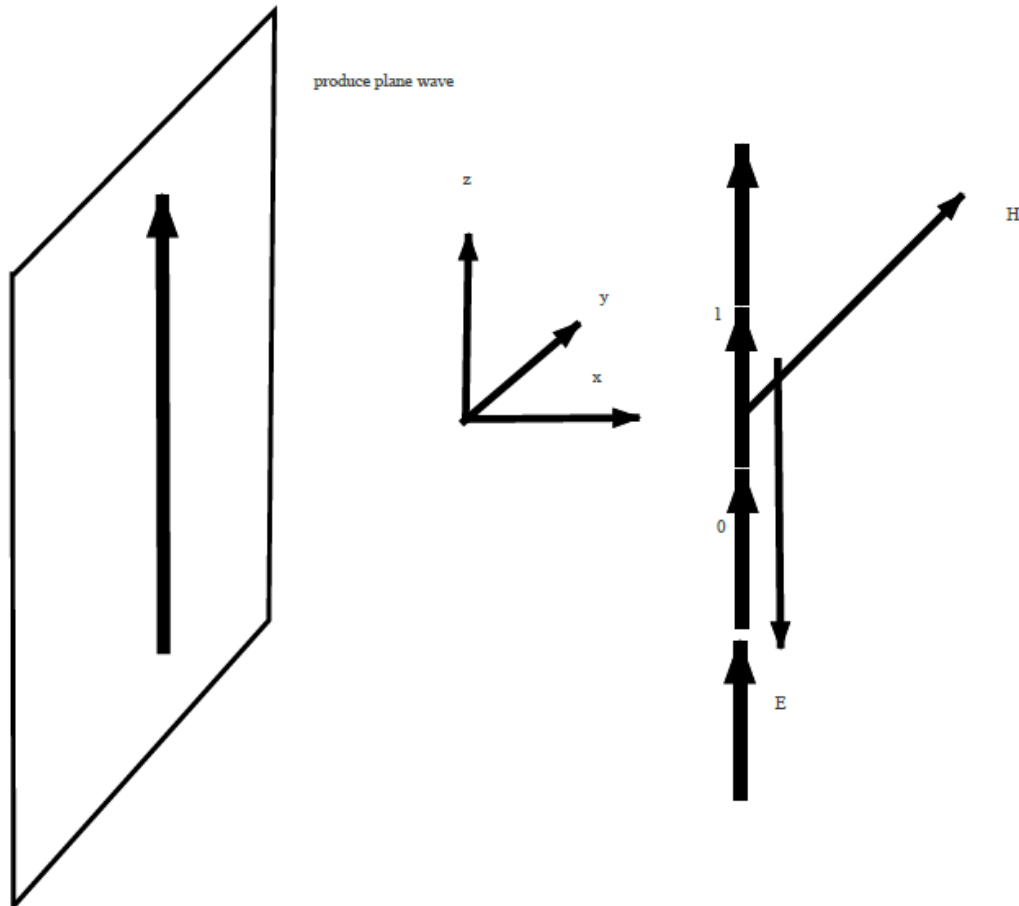


Figure 8. Consider using a current element or an electric dipole antenna to measure the magnetic field of electromagnetic waves

At this time, it was found that if a wire was used to measure the magnetic field in a retarded electromagnetic wave, the measured magnetic field (141) and the results (129) under magnetic quasi static state unchanged!

According to Maxwell's classical electromagnetic field theory, the magnetic field is defined as,

$$\mathbf{B} = \frac{1}{-j\omega S} \oint_C \mathbf{E} \cdot d\mathbf{l} \quad (142)$$

Therefore, if we take a loop to measure the magnetic field, the above formula holds true for both magnetic quasi static and retarded fields. We can obtain the above magnetic field from the loop measurement, which is defined by Maxwell's electromagnetic theory. The question is, is this definition reasonable? For magnetic quasi static fields, this definition is reasonable and has no problems. The question is, is the definition of the magnetic field of electromagnetic waves, or retarded waves, still reasonable? The author doesn't think so. He feels that using a short current element antenna to measure magnetic fields may be a more correct method. Therefore, the author needs to redefine a magnetic field and compare it with the definition of magnetic field in Maxwell's theory to see if the author's new definition of magnetic field is more appropriate.

7.5. A New Definition of Electromagnetic Wave Magnetic Field

Suppose there is an electromagnetic wave whose magnetic vector potential is known,

$$\mathbf{A} = A_0 \exp(j\omega t - kx)\hat{z} \quad (143)$$

According to Maxwell's electromagnetic theory,

$$\mathbf{H}_{maxwell} \triangleq \frac{1}{\mu_0} \nabla \times \mathbf{A} \quad (144)$$

To distinguish it from the new electromagnetic field theory defined in the next step, we use $\mathbf{H}_{maxwell}$ represents the magnetic field of an electromagnetic wave defined by Maxwell. Because this magnetic field is calculated according to Maxwell's electromagnetic theory. The author does not believe that the definition of this magnetic field is reasonable. The above equation can be written as,

$$\begin{aligned} \mathbf{H}_{maxwell} &= -\frac{1}{\mu_0} jk\hat{x} \times \mathbf{A} \\ &= -\frac{1}{\mu_0} jk\hat{x} \times (A_0 \exp(j\omega t - kx)\hat{z}) \\ &= -\frac{1}{\mu_0} jk(A_0 \exp(j\omega t - kx))(-\hat{y}) \\ &= j \frac{1}{\mu_0} kA_0 \exp(j\omega t - kx)\hat{y} \end{aligned} \quad (145)$$

Assume that the magnetic field amplitude of the electromagnetic wave is H_0 . So the Maxwell magnetic field can be written as,

$$\mathbf{H}_{maxwell} = jH_0 \exp(j\omega t - kx)\hat{y} \quad (146)$$

Among them,

$$H_0 = \frac{1}{\mu_0} kA_0 \quad (147)$$

The author defines a new magnetic field,

$$\begin{aligned} \mathbf{H}_{new} &\triangleq -j\mathbf{H}_{maxwell} \\ &= -j(jH_0 \exp(j\omega t - kx)\hat{y}) \\ &= H_0 \exp(j\omega t - kx)\hat{y} \end{aligned} \quad (148)$$

Therefore, the author uses Maxwell's magnetic field $\mathbf{H}_{maxwell}$ multiplied by $(-j)$ to define the new magnetic field.

7.6. New Definitions for All Magnetic Fields

Assuming that in general, the magnetic vector is,

$$\begin{aligned} \mathbf{A} &= \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \mathbf{J} \exp(-\mathbf{k} \cdot \mathbf{r}) dV \\ \mathbf{B}_{maxwell} &\triangleq \nabla \times \mathbf{A} \\ &= \nabla \times \frac{\mu_0}{4\pi} \iiint_V \frac{1}{r} \exp(-\mathbf{k} \cdot \mathbf{r}) \mathbf{J} dV \\ &= \frac{\mu_0}{4\pi} \iiint_V \nabla \left(\frac{1}{r} \exp(-\mathbf{k} \cdot \mathbf{r}) \right) \times \mathbf{J} dV \\ &= \frac{\mu_0}{4\pi} \iiint_V \left(-\frac{\hat{r}}{r^2} - jk\hat{r} \frac{1}{r} \right) \exp(-\mathbf{k} \cdot \mathbf{r}) \times \mathbf{J} dV \\ &= \frac{\mu_0}{4\pi} \iiint_V \mathbf{J} \times \hat{r} \left(\frac{1}{r^2} + \frac{jk}{r} \right) \exp(-\mathbf{k} \cdot \mathbf{r}) dV \end{aligned}$$

The author defines a new magnetic field,

$$\begin{aligned} \mathbf{B}_{new} &\triangleq \frac{\mu_0}{4\pi} \iiint_V \mathbf{J} \times \hat{r} \left(\frac{1}{r^2} + (-j) \frac{jk}{r} \right) \exp(-\mathbf{k} \cdot \mathbf{r}) dV \\ &= \frac{\mu_0}{4\pi} \iiint_V \mathbf{J} \times \hat{r} \left(\frac{1}{r^2} + \frac{k}{r} \right) \exp(-\mathbf{k} \cdot \mathbf{r}) dV \end{aligned}$$

As can be seen from the above, when defining $\mathbf{B}_{new}$, we have added a factor $(-j)$ to the electromagnetic wave term. If we assume that the propagation direction of the electromagnetic wave is $\hat{x}$ and the direction of the current is $\hat{z}$,

$$\mathbf{J} = J_0 \hat{z}$$

$$\hat{r} = \hat{x}$$

$$\begin{aligned} \mathbf{B}_{new} &= \frac{\mu_0}{4\pi} \iiint_V J_0 \hat{z} \times \hat{x} \left(\frac{1}{r^2} + (-j) \frac{jk}{r} \right) \exp(-kx) dV \\ &= \frac{\mu_0}{4\pi} \iiint_V J_0 \hat{y} \left(\frac{1}{r^2} + \frac{k}{r} \right) \exp(-kx) dV \end{aligned}$$

The direction of magnetic field is $\hat{y}$. In this way, the electromagnetic wave term and the static magnetic field term of the newly defined magnetic field maintain the same initial phase.

7.7. Relationship between the Newly Defined Magnetic Field and the Magnetic Field Measurement Method Defined by the Author

Assuming that the magnetic vector potential of an electromagnetic wave is,

$$\mathbf{A} = A_0 \exp(j\omega t - kx)\hat{z} \quad (149)$$

Therefore, the electric field of the electromagnetic wave is,

$$\mathbf{E} = -j\omega\mathbf{A} = j\omega A_0 \exp(j\omega t - kx)(-\hat{z}) \quad (150)$$

Consider,

$$H_0 = \frac{1}{\mu_0} k A_0 \quad (151)$$

$$A_0 = \frac{1}{k} \mu_0 H_0 \quad (152)$$

Hence,

$$\begin{aligned} \mathbf{E} &= j\omega\left(\frac{1}{k} \mu_0 H_0\right) \exp(j\omega t - kx)(-\hat{z}) \\ &= j\omega\left(\frac{1}{\omega\sqrt{\mu_0\epsilon_0}} \mu_0 H_0\right) \exp(j\omega t - kx)(-\hat{z}) \\ &= j\left(\frac{1}{\sqrt{\mu_0\epsilon_0}} \mu_0 H_0\right) \exp(j\omega t - kx)(-\hat{z}) \\ &= j\eta_0 H_0 \exp(j\omega t - kx)(-\hat{z}) \end{aligned} \quad (153)$$

If a straight wire electric dipole antenna is used to measure the electromotive force,

$$\begin{aligned} \mathcal{E}_L &= 4\mathbf{E} \cdot (L\hat{z}) \\ &= 4j\eta_0 H_0 \exp(j\omega t - kx)(-\hat{z}) \cdot (L\hat{z}) \\ &= -4j\eta_0 H_0 \exp(j\omega t - kx)L \end{aligned} \quad (154)$$

We use Faraday's law in Maxwell's equation for the measurement of electromagnetic waves, but $\mathcal{E}_O$ is replaced with $\mathcal{E}_L$, that is,

$$B_{\text{maxwell}} \triangleq \frac{1}{-j\omega S} \mathcal{E}_O \rightarrow B_{\text{new}} \triangleq \frac{1}{-j\omega S} \mathcal{E}_L$$

In this way, we can measure the magnetic field of electromagnetic waves using the following methods,

$$B_{\text{measure}} = B_{\text{new}} = \frac{1}{-j\omega S} \mathcal{E}_L$$

B_{measure} is a method defined by the author to measure magnetic fields, which allows us to measure electromagnetic waves using a straight line instead of a ring coil.

$$\begin{aligned} B_{\text{measure}} &= \frac{-4j\eta_0 H_0 \exp(j\omega t - kx)L}{-j\omega S} \\ &= \frac{4\eta_0 H_0 \exp(j\omega t - kx)L}{\omega L^2} \\ &= \frac{4\eta_0 H_0 \exp(j\omega t - kx)}{\omega L} \\ &= \frac{4\eta_0 H_0 \exp(j\omega t - kx)}{kL} \sqrt{\mu_0\epsilon_0} \end{aligned} \quad (155)$$

$$k = \omega\sqrt{\mu_0\epsilon_0}$$

$$\omega = \frac{k}{\sqrt{\mu_0\epsilon_0}}$$

$$\begin{aligned} H_{\text{measure}} &= \frac{1}{\mu_0} B \\ &= \frac{4\eta_0 H_0 \exp(j\omega t - kx)}{kL} \sqrt{\mu_0\epsilon_0} \frac{\epsilon_0}{\mu_0} \\ &= \frac{4H_0 \exp(j\omega t - kx)}{kL} \\ &= H_0 \exp(j\omega t - kx) \frac{4\lambda}{2\pi L} \end{aligned} \quad (156)$$

If we choose

$$\frac{4\lambda}{2\pi L} = 1 \quad (157)$$

that is,

$$L = \frac{4\lambda}{2\pi} \quad (158)$$

$$H_{\text{measure}} = H_0 \exp(j\omega t - kx) \quad (159)$$

Where x tends to take a fixed value such as $x = 0$, so,

$$H_{\text{measure}} = H_0 \exp(j\omega t)$$

If we measure the electric field,

$$\begin{aligned} \mathbf{E}_{\text{measure}} &= -j\omega\mathbf{A} = j\omega A_0 \exp(j\omega t - kx)(-\hat{z}) \\ &= j\omega\left(\frac{1}{k} \mu_0 H_0\right) \exp(j\omega t - kx)(-\hat{z}) \\ &= j\omega\left(\frac{1}{\omega\sqrt{\mu_0\epsilon_0}} \mu_0 H_0\right) \exp(j\omega t - kx)(-\hat{z}) \\ &= j(\eta_0 H_0) \exp(j\omega t - kx)(-\hat{z}) \\ &= j(\eta_0 H_0) \exp(j\omega t)(-\hat{z}) \end{aligned} \quad (160)$$

In which,

$$A_0 = \frac{1}{k} \mu_0 H_0 \quad (161)$$

The measured electric field and magnetic field maintain the phase difference j . The comparison formulas (160) and (148) show that the new magnetic field defined by the author is consistent with the electromagnetic wave measured by the author.

7.8. Why Redefine Magnetic Fields?

First of all, the receiving antenna often uses a straight wire antenna rather than a ring antenna, so it seems more reasonable to measure the magnetic field with a short straight antenna than a current loop. For measurement, we are measuring magnetic fields, not so-called "retarded" magnetic fields or magnetic fields defined by Maxwell.

We assume that we do not know whether the external magnetic field is a so-called retarded magnetic field transmitted by electromagnetic waves or a quasi static magnetic field generated by a nearby current. It is

assumed that the direction of this magnetic field is along the $\hat{y}$ axis. Then the magnetic field was measured using a prepared dipole antenna. It is assumed that two measurements result in the same electromotive force. The first measurement is to measure the magnetic quasi-static electromagnetic field, and the second is to measure the retarded magnetic field. We think that the magnetic field is the same in both cases. What's wrong with that? But according to Maxwell's electromagnetic theory, this is indeed wrong. Because Maxwell's electromagnetic theory stipulates that magnetic fields must be measured using a circular coil. Instead, we used a straight dipole antenna, or current element, to measure it. So it violates Maxwell's definition of a radiated magnetic field.

This seems to be just a question of definition, and it doesn't matter. But the definition of magnetic field should be reasonable. The author believes that the definition of the magnetic field of electromagnetic waves in Maxwell's theory is unreasonable.

There are two main irrationalities. According to Maxwell's definition of a magnetic field, the electric and magnetic fields of electromagnetic waves are in phase, resulting in the Poynting vector being a real number. Therefore, there is an active power propagating outside the universe. In this way, electromagnetic waves will overflow the universe. This violates the principle that electromagnetic waves do not overflow the universe. So it's wrong. The author believes that the overflow of electromagnetic waves into the universe is a bug of Maxwell's electromagnetic theory, so it is wrong. Therefore, it is necessary to redefine the magnetic field of electromagnetic waves.

The author emphasizes that Maxwell's electromagnetic theory, which includes displacement current and is applied to electromagnetic wave, is completely different from that of quasi-static electromagnetic field. Therefore, the electric and magnetic fields of the two systems may have different meanings. Therefore, we need to calibrate the magnetic field of the electromagnetic wave. This means that the factor " $-j$ " plays a calibration role. So for electromagnetic waves (retarded),

$$B_{new} = (-j)B_{maxwell}$$

The author found that the main body of Maxwell's theory is still correct, but for the magnetic field of electromagnetic waves, a calibration factor ($-j$) needs to be added. It is similar for the advanced waves, in that case the calibration factor is $(+j)$.

8. Conclusion

The author found that the definition of magnetic field of electromagnetic wave in Maxwell's electromagnetic theory is problematic or defective. The author comprehensively discusses the reasons for this problem. The reason is that the quasi-static electromagnetic field theory and Maxwell's radiation electromagnetic wave theory are two different systems. Therefore, the meaning of electric field and magnetic field is different. In order to keep their meanings consistent, it is necessary to calibrate the electric and magnetic fields. This calibration process

is to multiply the magnetic field of Maxwell's electromagnetic theory by " $-j$ ". In this way, the phase difference between the magnetic field and the electric field of the electromagnetic wave is 90 degrees rather than in phase. This electromagnetic wave is actually a wave of reactive power. The average value of electromagnetic energy transmitted by this wave is zero. This satisfies the new axiom of electromagnetic theory proposed by the author, that electromagnetic waves do not overflow the universe.

This article introduces a new method for measuring the magnetic field of electromagnetic waves and quasi-static electromagnetic fields. This method is to use a straight wire current element instead of a small coil. The author believes that for magnetic fields related to electromagnetic waves, straight wire current elements should be used instead of small coils. According to Maxwell's theory, the measurement of a magnetic field is defined as a small coil. Therefore, the author's new magnetic field measurement method is different from Maxwell's theory. The magnetic field measured using this method should maintain a 90 degree phase difference from the electric field of the electromagnetic wave.

The correction of this defect in Maxwell's electromagnetic theory may have a very important impact on the whole modern physics. In this way, waves no longer carry energy. At least the wave does not carry energy in average. Then there is no need for the wave to collapse. The mutual energy flow can transfer energy. In the author's electromagnetic theory, the mutual energy flow transfers energy from point to point, so the shape and properties of the mutual energy flow are the same as those of particles. Therefore, we can think that the mutual energy flow corresponding to electromagnetic wave is photon. This provides a good solution to the problem of wave-particle duality.

The conclusion of this paper may be extended to other particles. The wave satisfying Schrodinger equation or Dirac equation is probably also reactive power! So these waves need not collapse. Energy is transmitted by mutual energy flow. All particles are composed of mutual energy flow. Mutual energy flow involves retarded wave and advanced wave. Therefore, the advanced wave must be the objective existence of physics. The existence of advanced waves means that causality can be violated.

The next step is to compare this new measurement method with the Hall effect method through experiments.

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