

A Cross-Sectional Study Investigating the Effects of Learning-Based Activities, Algebra Tiles and Problem-Solving Teaching Techniques on Students' Conceptual Understanding and Problem-Solving Skills in the Jamaican Mathematics Classroom

Akeba Collins, Aljay Mattocks, Antonette Lewis, Shaneille Samuels*, Troy Williams, Marsha King, Shaniqua Willis

Mathematics Department, Shortwood Teachers' College, St. Andrew, Jamaica

*Corresponding author: shaneille.samuels@gmail.com

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Abstract This action research was conducted across six (6) high schools in Jamaica. This study aimed to investigate the effects of learning-based activities, algebra tiles and problem-solving teaching techniques on students' conceptual understanding and problem-solving skills in the Jamaican Mathematics Classroom. The target population was Grades 8 and 9 students from which thirty-six ($n = 36$) were selected using the purposive sampling method. The method of data collection included observations, pre-tests, post-tests, questionnaires and interviews. A Modified Abbreviated Math Anxiety Scales instrument (mAMAS) and student journals were also used in the data collection process over a period of twelve weeks between September and December 2022. The data was analyzed using item analysis, coding and thematic analysis and descriptive statistics. The findings revealed that prior to the intervention approximately 44% of the participants lacked interest towards mathematics, 25% had negative perceptions of taking mathematics tests, at least 75% of the students lacked understanding in different mathematics concepts. Thirty-six percent (36%) of the participants seemed unable to employ problem solving skills in relation to different mathematical areas which include algebra, geometry and measurement. After implementing the intervention, it was found that some participants excellently mastered 7 of the 17 mathematical concepts such as performing addition on integers with opposing signs and obtaining the rules for geometric patterns using problem-solving steps; 1 in 5 participants who were exposed to geometric concepts showed full mastery. There were minimal comparative differences among the strategies used in the intervention process. The study concluded that the Algebra tiles, learning-based activities and problem-solving teaching techniques showed positive results in addressing the participants' lack of conceptual understanding and problem-solving skills. It is recommended that more of the above-mentioned strategies be implemented and more frequently used in the Jamaican Mathematics Classroom.

Keywords: *conceptual understanding, problem solving skills, learning-based activities*

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1. Introduction

In today's society, Mathematics has historically been considered a difficult subject at all levels of education. Over the years Caribbean students have generally underperformed on the Mathematics Caribbean Secondary Examination Council (CSEC) exam mainly due to a lack of conceptual comprehension, and their inability to connect their prior knowledge and problem-solving skills

to successfully complete questions given on the mathematics exam [1]. This means that students are unable to apply their mathematical knowledge to real world situations through critical thinking and problem solving. According to the Caribbean Examination Council (CXC) annual report on the 2018 exams, there has been a decrease in the exam's comprehension and reasoning profiles, vital skills which account for 70% of the exam [1]. This indicates that the students are not understanding the problems in order to provide rationales for the questions being posed. The Jamaican public high schools

have been faced with the unreasonably high student to teacher ratio, coupled with inadequate class time given for practice questions and the application of real life scenarios add to the students' lack of conceptual understanding in the mathematics classroom [2].

This study was conducted by a group of student-teachers in their Final Year Practicum from September to December 2022, across six high schools in Jamaica. The study examined the effects of Algebra tiles, learning-based activities and problem solving teaching techniques on Grade 8 and 9 students' conceptual understanding and problem-solving skills. The schools used in this study referred to by pseudonym are: Singers High, Monk Town High, XYZ High, Runners High, AJ High and Hilltop High. Singers High, XYZ High and Hilltop High can be described as non-traditional which focuses on students' learning abilities considering three pathways, namely Pathway 1, Pathway 2 and Pathway 3. Pathway 1 students usually have little to no evidence of learning disability that create hindrances in learning, these students can normally be accepted into the traditional high schools, while Pathway 2 students usually are in need of additional instructional support by teachers. Pathway 3 students, however, are those who have not yet mastered the grade 4 literacy and numeracy skills coming from primary school who are taught using personal empowerment, performing and creative arts, technical and vocational instructions [3]. Monk Town High, AJ High and Runners' High, however, can be described as traditional high schools which are usually classified as Pathway 1.

The Grade 8 students from Singers High who participated in the study and are described as Pathway 2, and at times needed instructional support. While Grade 9 students in Pathway 2 from XYZ high who participated displayed an academic performance of 50% based on their test results where they possessed average procedural knowledge, but fair [hardly any] conceptual knowledge of Mathematics. The students at Hilltop High, on the other hand, had conceptual understanding issues in the mathematics classroom. The other three schools, Monk Town High, Runners High and AJ High displayed similar issues of conceptual understanding and lacking problem-solving skills. Consequently, the teacher-researchers collaborated after noticing that all the students they interfaced with had problems with conceptual understanding of general mathematics concepts including integers, solving simple linear equations, geometric patterns and illustrations, perimeter and area, and other concepts. Coupled with issues related to conceptual understanding are the Grades 8 and 9 students lacked the problem-solving skills needed to help provide solutions related to these concepts.

A trap door is held in a horizontal position by a 2-meter length of chain attached to the wall above the hinges and the outside edge of the door. If the chain makes an angle of 50° with the door, find the height of the trap door.

Figure 1. Problem-based real life scenario

The teacher-researcher [she] at Monk Town High while teaching a topic related to geometric patterns, geometric

illustrations, perimeter and area gave her Grade 9 students a real-life example about finding the height of a trapdoor given the affiliated angle and length of the chain attached to the door (see Figure 1).

She noticed during the lesson that the students struggled to recognize all the terms in the scenario and as such could not proceed in answering the question. The students could not provide definition for the words “hinges”, “trapdoor” and “horizontal”. The teacher-researcher proceeded to create depictions for the scenario in the classroom using physical objects to help the students to become more aware of the terms in order to formulate a solution strategy. Even with the use of visual aids, the students moved at a very slow pace to generate a solution which required their use of problem-solving skills. It was [4] shared that problem-solving is not only a crucial skill needed to comprehend mathematics but is considered to be the methodological backbone to approach mathematical problems related to real life or abstraction. Problem-solving is very important in building students with competencies such as analysis, comprehension, reasoning and applications [4]. However, the students needed to have the necessary prerequisites such as properties of geometric figures and cardinal points, cultural connection, alongside the well-needed problem-solving skills. Consequently, it was noted that the students were not able to bridge the gap between concepts and real life scenarios as well as to outline the problem solving steps needed. Grade 8 students at Runners High also had similar issues with identifying terms in a given problem and as such they struggled. The teacher-researcher [she] noticed that students could not provide meaning of mathematical terms such as “products” and “times” when given the question “Half the product of two numbers ‘a’ and ‘b’ subtract three times a third number ‘c’ ” to translate symbolically. She shared that even when given the symbolic representations to write worded problems they could not identify words such as cube, difference nor sum to complete the task. She expressed the delay in her lesson as well and eventually re-teaching the topic again addressing students' use and understanding of fundamental mathematics vocabulary. It was [5] postulated that there is a need for students to develop the necessary mathematical skills and concepts not just through knowledge of mathematics but knowledge gained from their own individual experiences. In other words, students need to learn things contextually and not keep isolating learnt concepts and algorithms.

The teacher-researcher at Singers High [she], shared that she encountered similar issues seen at Monk Town, however, her students were not as responsive. She stated that the students expressed their dislike of the subject, with one or two of them saying it is hard. Overall they spoke negatively about their ability to do Mathematics, and some even shared that no one in their family did well at mathematics. She noticed the disinterest, low performance, lack of confidence and absenteeism. She felt disheartened and considered that they might be struggling with mathematics anxiety. The teacher-researcher at Runners High also posited similar dispositions of her students including their unwillingness to take notes, so they often make little to no scribbles. Reference [6] shared that the signs displayed are symptoms of

mathematical anxiety, especially since these symptoms usually affect students learning and understanding mathematics. Aiden Severs [he], a Primary Leader Practitioner who writes for Third Space Blog [7], defined mathematics anxiety as “a negative emotional reaction to mathematics, leading to varying degrees of helplessness, panic and mental disorganization that arises among some people when faced with a mathematical problem”. The teacher-researcher at XYZ High [he], noticed the same challenges displayed and expressed by the students at Singers High. His host teacher was teaching Grade 9 students simple linear equations that they should have been previously exposed to in former grade levels based on the National Standards Curriculum (NSC), and gave them the question, “Solve: $5x + 2 = 4x + 6$ ”. A student was called to share the solution done outlining each step. The student proceeded to add $4x$ on both sides, then add 2 on both sides. The student then simplified the expression and solved to obtain an incorrect answer of $\frac{8}{9}$. It was evident that students did not yet grasp proper transposition and understanding of integers (inverse operations). However, he shared that the host teacher continued with the lesson and it further frustrated the students who were not aware of where they went wrong and how to address it. A study titled “Supporting students’ understanding of linear equations with one variable using algebra tiles” [8] shared that when teachers only know one method (purely procedural) in teaching students how to solve simple linear equations, they cannot address issues that surface in a classroom where students are not getting the answer. This type of teacher is usually in a self-conflicting battle wondering what more can be done to get the answer and end up just moving on from question to question to get the solution. Consequently, leaving students irritable and not learning anything.

The integer problem was also seen in the Grade 9 classroom at Hilltop High while the host teacher taught the students distributive property. The teacher-researcher realized that the students struggled to add and subtract integers during the lesson; and while speaking to the host teacher it was shared they already learnt the distributive property and they mastered it then. It appears that the students are trying to memorize the procedure of expanding but lacking in simplifying because they have not grasped a full understanding of integers. The teacher-researcher pointed out that students mostly struggled with expressions involving negative numbers. After speaking with the host teacher, it was revealed that these students belonged to Pathway 2. A Professor in Didactics of Mathematics [9] stated that teachers usually find it difficult to teach integers because of the different complexities that need to be considered. Consequently, students in turn will show signs of not understanding integers. The teacher-researcher at AJ High [he] expressed that class size is one of his main concerns along with classroom space. He expressed that he had 43 students who needed individual attention because they had very poor conceptual understanding. He expressed his frustration in trying to attend to each of them to facilitate quality learning. Also, because of the demanding curriculum and testing ordeals at his host school, he had to move on to another concept before the students had

concretized the one taught. Over the years, CXC reports religiously indicate the need for students to be exposed to real life problems especially for geometry, measurement and algebra but this teacher-researcher struggled to do so at AJ High due to insufficient resources such as calculators, rental textbooks and required grade level workbook; or the lack thereof to support the class size. A retired Jamaican university mathematics education lecturer Dr. Camella Buddo [2], has also shared similar issues expressing that for years classroom teachers have been talking about these issues and the students have been suffering. She [2] expressed that along with large class size, teachers have to contend with students' attitudes and behaviors, accommodating their learning styles and mixed abilities. So, the question being asked is, how can students be given the attention that they need in order to engage properly in the teaching and learning process?

The teacher-researchers also expressed the need for students to develop attributes to become independent learners such as reviewing what was done in class, dedicating adequate time to practice, completing homework and properly taking notes in class. They went further to say that students have not yet grasped the importance of knowing mathematics concepts and developing problem-solving skills. The reality is that if students do not have conceptual understanding they really cannot master solving real world problems. Both go hand in hand - if they have real world experience and the teacher helps them to make connections then that is what builds their conceptual understanding. Many of these students did not attempt the assigned homework with them making excuses that they did not understand or that they did not get the time to attempt it.

1.1. Purpose of the Study

The aim of the study was to determine the challenges students had with conceptual understanding and their inability to problem solve in six Jamaican mathematics classrooms at the Grade 8 and 9 levels in the parishes of Kingston, St. Andrew and St. Catherine. The study further sought to ascertain the impact of the use of learning-based activities, algebra tiles and the Polya’s problem-solving approach to address the challenges students' faced.

1.2. Research Questions

The study sought answers for the following research questions:

1. What are some of the challenges students generally encounter with conceptual understanding and their problem-solving skills in mathematics?
2. How effective are the use of Algebra tiles, learning-based activities and Polya’s problem solving approach in helping students to improve their understanding of mathematics concepts and problem-solving skills?
3. Is there a comparative difference among the effects of the Algebra tiles, the learning-based activities and Polya’s problem-solving approach on students’ conceptual understanding and their problem-solving skills?

1.3. Definition of Terms

Algebra tiles - these are rectangular visual aids used in the mathematics classroom to teach students various concepts in algebra.

Learning-based activities - these are subject orientated activities aimed at helping students learn content while engaging them, so that they are involved with fun moments but are able to accomplish productive tasks such as open-ended problems.

Problem-based learning - this is the process by which students are taught in groups to provide solutions to open-ended problems.

Activity-based learning - this is referred to as an active learning process involving mathematics activities such as manipulatives and games where students participate enthusiastically in the learning experience.

Polya's Problem-Solving Approach - this approach is a logical method of solving problems which usually involves four steps.

Trapdoor - a door which has removable panels usually found in the floor or ceiling.

2. Literature Review

The literature has been organized into four (4) sub-headings.

2.1. Constructivism to Support Conceptual Understanding and Problem-Solving

According to Radford's [10] analysis of the constructivist theory, the teachers' and students' roles must be clearly defined in the classroom from the very beginning. Brousseau, a Canadian Didactics mathematician [he] [11] as cited in [12] stated that a teacher's role is to act as a facilitator where students are not shown how to solve the problems given but instead allowing students to work through each problem at hand to provoke the expected learning. Conversely, he [11] shared that a student's role is to "produce, formulate, prove and construct models, languages and theories" [10]. It is believed that once students and teachers recognize and work closely together according to their roles, then students will develop conceptual understanding. He [11] has encouraged teachers to avoid depriving students of true learning by giving into their every little whims [10]. Conceptual understanding in mathematics is defined as one's comprehension of mathematical ideas being able to give meanings to mathematics concepts, recognizing relationships amongst concepts and making linkages in an integrated and organized manner [13]. Conceptual understanding of mathematics is needed in the classroom and should be encouraged so that students can become independent learners who are confident and willing to take on challenges and solve problems at hand.

Conceptual understanding is believed to be closely linked to problem-solving skills, which requires students to become intimate with concepts to be able to solve problems inside and outside the classroom. Now, Schoenfeld, a mathematics educator from The University of California, Berkeley [14] expressed that in learning and teaching mathematics problem-solving is very important, allowing the students to become actively involved in their own learning as they seek solutions to problems [15]. It was [15] shared that using a student-centered approach should then be encouraged along with problem-based, activity-based and open-ended learning. The educators [15] went further to say that both problem-solving and critical thinking should be encouraged in the teaching and learning process but using effective instructional approaches. The idea is that each of the teacher's instructional approaches must appeal to the students so they can think critically and problem solve well [15]. It was [16] shared that problem solving is encouraged because it is fundamental for career choices such as medicine and engineering. Consequently, hands-on activities are encouraged in the classroom along with different activity types and models.

A longstanding discussion in the Jamaican mathematics classrooms, as had been among different theorists, is the need for conceptual understanding, problem-solving, and learning based activities to facilitate a student-centered classroom, so that students can participate in productive struggle, identify and design their solution pathways and connect mathematical ideas using their own experiences. However, for both the students and the teachers, the idea of a student-centered classroom is still in development because of the concerns and challenges experienced. Jamaica has taken initiatives to have mathematics and Alternative Pathways to Secondary Education (APSE) coaches across the island, however, the missing link of making sure that they have everything needed to create that classroom proposed by the government is still yet to be addressed. Teachers have long been feeling like they are being given "baskets to carry water".

Prior to the APSE Programme, learners were simply being passed through the system, once exiting the primary schools. However, with the introduction of the Programme, high school teachers have welcomed the move as the learners have finally been considered based on their constant struggle to keep up or their mastery of the mathematics skills and concepts. The idea too is that APSE is not only for weak students, but also to identify those who are advanced as they too can become a problem in the classroom if they are overlooked. So generally it is to determine students' level of competence wherever they fall on the continuum and provide the needed degree of support. The Ministry of Education and Youth (MOEY) after assessing students at the primary level transitioning to high school noticed that some students were not at the same level as others as they were lacking in the basic reading and computational skills leaving the primary school. With this in mind, MOEY made the decision to provide for the multiple intelligences and the diverse needs of students to maximize their capabilities. Consequently, MOEY created alternative pathways at the secondary level to best help the students [17]. MOEY [17] shared that students are placed in different pathways based

on their performance on the Primary Exit Profile (PEP) exam where they would be monitored from Grades 7-9 and then they 'float' into the system there after; pathway coaches as well as have been provided at these grade levels to help phase this programme at each of the schools having the APSE programme. MOEYI shared that "APSE is an initiative that is based on tailored curricula that enables learners to perform at his or her fullest potential based on aptitude, interest and ability" [17].

However, even with the good intention of this programme, the schools have seen experienced teachers resigning either because of migration overseas or transitioning to schools across the island. For instance, the principal at Yallahs High School in 2019 stated that only 50% of his mathematics teachers were present to teach, to help organize and facilitate the APSE programme initiative especially at the beginning of a new school year [18]. The principal expressed his frustration of the challenge he experienced to get quality teachers at the beginning of the school year [18]. Likewise, Mignott, a Jamaican APSE teacher [19] [she] has also shared her experience as an APSE coach declaring that there is a great need to support the APSE programme. She expressed that the biggest obstacle is that students being sent to the non-traditional high schools are "drastically lacking the basic prerequisites that they should have gotten from basic and primary schools" [19]. It in turn causes teachers to be re-teaching the basic prerequisites at the high school level which is very time consuming especially for Mathematics and English, delaying teaching the prescribed curriculum creating a seemingly unending cycle when they move to the next grade all the way up to Grade 11 [19]. Therefore, students are not adequately prepared for CSEC by the time they are in Grade 11 [19]. She [19] also added that students are also lacking the necessary finances, social background and resources needed coupled with the Ministry of Education's reluctance to truly address the issues in the non-traditional schools.

2.2. Teacher Migration Erodes Student-Teacher Relationship

Teacher migration is more than a quantitative issue; it has presently been denting the education sector of Jamaica. A lot of teachers have taken advantage of their freedom to leave Jamaica to work in more lucrative employment abroad [20]. It has been recorded that more than 500 secondary mathematics and science teachers abandoned their jobs to travel abroad between 2014 and 2015, according to a 2016 report from the former Minister of Education Maxine Henry-Wilson [21]. More seasoned educators, the ones who have the caliber of experience, have been and are departing from the Jamaican classrooms to work overseas; consequently the MOEY has been seeking out teacher trainees who have not yet graduated who are either in their 3rd or 4th year of teachers' college to help fill the gap. These teacher trainers would be needing guidance when they eventually graduate and as such would have been given the responsibility to work without adequate supervision and even any supervisor in some cases because the senior teachers at times are the ones who left. Prior to migration, having rich and priceless

experiences of educators who see the students in a holistic way have helped many students achieve high levels of performance in the past since they have allowed teachers to develop strategies and techniques for teaching that are not taught in institutions [22]. It was further explained that the upheaval in these teachers' departures damages the foundations of schools [22]. Thus, creating a need for new teachers to be groomed as they require assistance to successfully transition into the classroom as no expert teacher is there so students are not able to create strong bonds with teachers so they are frequently overlooked because of the excessive number of students and the shortage of teachers [22, 23]. As a result, vacancy affects the quality of experience and the students' performance.

With the rapid increase in teacher migration in Jamaica and the fact there are too many students in the classrooms, there is not enough time being given to the teacher to fully cater to his or her students' learning styles and needs. So some students are being robbed of the opportunity to explore with the mathematical materials and different solutions to problems impacting their ability to think critically and problem solve. Consequently, it was [18] emphasized that it would have been difficult for students to apply their concepts to projects, current events, or other new settings if we do not allow them time to construct understanding. Otherwise, students' conceptual understanding and problem-solving skills will be impaired. Additionally, according to the National Mathematics Coordinator Dr. Tamika Benjamin, teachers must ask students thoughtful questions that encourage them to explain and reflect which leads to relevant discussions and ideas "because thorough examination fosters both a greater conceptual comprehension of mathematics as well as more robust general critical thinking abilities" [24]. According to a study [12] titled 'An investigation into challenges faced by secondary school teachers and pupils in Algebraic Linear equations', students encountered several difficulties when it comes to conceptual understanding. It is stated in their results and findings that the students' inability to solve linear equations was caused by a lack of prerequisite knowledge. Since algebra requires conceptual understanding to build upon it becomes a problem when it is evident that students are lacking the necessary foundation. A study [25] on pedagogical content knowledge of algebra among mathematics teachers, shared that students who usually show excellent conceptual understanding are not usually taught by inexperienced teachers but rather those who bring creative and diverse forms of learning activities that are geared towards enhancing students' conceptual understanding. Otherwise, she shared that students will only be taught to demonstrate the procedural skills in problem solving which will not drive the understanding of concepts to explain the need for the procedural processes involved.

2.3. Manipulatives to Enhance Conceptual Understanding and Problem-Solving

A study [26] in the United States cited in Wingett's journal [27] which looked at the use of manipulative and its algebraic thinking revealed that 96% of the students spent most of their time on routine procedures and

problems, while 1% focused on problem-based activities where they problem solve, create new solutions and engage in meaningful activities. It was also shared that since teachers and mathematicians across the states worked together to find solutions to help students move away from looking only at routine problems and completing worksheets, students have now been exposed to problem-based learning activities and using concrete models to bring meaning to their learning and create new solution paths [27]. Algebra tiles serve as concrete or virtual models to help students formulate algebraic thinking by linking abstract with concrete and pictorial representations [27]. For instance, algebra tiles have been used in teaching topics such operations on integers, modeling and simplifying algebraic expressions, solving linear equations, factoring amongst other algebraic topics. Similarly, a study on mathematical operations of integers in primary schools [28] revealed that it is better to have a teacher with a strong content background who should be teaching operations on integers to reduce misconceptions as much as possible. So when students are exposed to the use of the tiles, they can use the concrete models related to each operation in order to form their own schemas and hence generate rules that would best be needed to carry out these operations [28].

2.4. Problem-based Learning to Enhance Conceptual Understanding and Problem-Solving

Vygotsky, however, has written about exposing students to problem-based learning. A study titled “Problem-Based Learning” [29] referred to Vygotsky’s work and shared that students need to be able to solve real world (authentic) problems inside and outside the classroom where the teacher is their facilitator. In other words, overtime as a teacher guides the students to make discoveries, eventually they will become independent thinkers, being able to share and collaborate with their peers. In a journal article titled, “The importance of instructional scaffolding” [30, 31], it was revealed that the role of the teacher should change as students become independent learners learning to problem solve. With this in mind Jamaica has made changes in recent years at the primary level to do away with the Grade Six Achievement Test (GSAT) replacing it with Primary Exit Profile (PEP) instead. PEP now consists of performance tasks which provide authentic assessment developed using problem-based questions. This decision is to focus on enhancing students’ critical thinking skills and facilitating their creativity [31]. However, since the pandemic, the students have less preparation time to spend on the performance tasks to ensure that the tasks were properly learnt and understood in a timely manner [32]. Teachers have also proposed, out of frustration, a consideration to get rid of the PEP because the exams are most suited for high school students [33]. Sonia Bennett, a publisher from Carlong Publisher encouraged the government to provide the necessary training for teachers and provide them with appropriate resources to minimize their anxiety because students are in need of these assessments [34].

While there is a need for problem-based learning, problem-solving skills are very important to help students

master problem-based tasks in and outside the classroom. It has been stated by Dr. Camella Buddo [she] [2], that as Jamaica seeks to transform the educational system, problem-solving skills must be a driving factor in the teaching and learning of mathematics as described by the NSC. She affirmed that teachers need to facilitate problem solving teaching strategies to drive the cognitive processes of the learners where they become independent learners through self-reflection. This will in turn allow students to move away from the drill and practice method that they have become acclimatized to over the years as they develop varying levels of thinking [35]. In other words, students needed to be taught what they needed to be aware of when they attempted a problem and not sure of what they should do to obtain the solution [35]. Whenever this happens students can be taught about Polya’s problem solving approach which is one of a few approaches to successful problem solving.

3. Methodology

3.1. The Design

This study is an action research design using a qualitative approach to explore the challenges students faced with conceptual understanding and their lack of problem-solving skills in six Jamaican high school classrooms at the Grade 8 and 9 levels.

3.2. The Participants

A sample of thirty-six Grade 8 and 9 students, ages 14-15 years, participated in the study. Purposive sampling was utilized to capture the classroom experiences of students along with responses from the host teachers and/or head of the mathematics departments. The participants were taken from across six high schools in Jamaica, namely using pseudonyms: Singers High, Monk Town High, XYZ High, Runners High, AJ High and Hilltop High. Twenty five of the thirty-six students were male and eleven were female. Five students were selected from Singers High, Monk High, AJ High and Hilltop High respectively; while eight were equally selected from XYZ High and Runners High to participate in the study.

3.3. Data Collection and Data Analysis

Observation, interviews, pre-tests and post-tests were used to gather qualitative data over a twelve weeks period, between September and December 2022. The collected data was analyzed using coding, thematic and item analysis, descriptive statistics, and Microsoft Excel. These were used to explain experiences and make interpretations.

The teacher-researcher [she] at Singers’ High noticed that when the students’ subjects (including mathematics) grades were compared, they seemed to perform better at the other subjects excluding Mathematics. Also, the students seemed to enjoy the non-mathematics classes and are unwilling to participate in mathematics lessons expressing that they cannot do Mathematics. With this in mind, she used the Modified Abbreviated Math Anxiety Scales instrument, (mAMAS) to identify students’ math

anxiety scores on a scale from 9 to 45, indicating low to very high math anxiety. The Modified Abbreviated Math Anxiety Scale [36] is a popular and psychometrically validated measure of mathematics anxiety [37]. Table 1 indicates the questions found on the Modified Abbreviated math anxiety scale [37].

The mAMAS provides a valid and reliable scale for measuring math anxiety in children and adolescents. Here is the key to interpret the rating scale for each item:

1. Low Anxiety (low feeling of fear and sense of unease towards Maths)
2. Some Anxiety (sense of unease when working with some basic concepts, and fear in the moment they have to perform some task they are unclear of)
3. Moderate Anxiety (feeling of frustration when faced with new concepts or math task they are unsure of)
4. Quite a bit of Anxiety (feeling of fear and some sense of unease when performing math tasks)
5. High Anxiety (high feeling of fear and sense of unease towards Math)

Some signs of anxiety are sweaty palms, students zoning out, distracting others, otherwise having little group discussions; sense of defeat.

Table 1. Items On The Abbreviated Modified Anxiety Scale [37]

Items
Having to use the tables in the back of a mathematics book
Thinking about an upcoming mathematics test one day before
Watching a teacher work an algebraic equation on the board
Taking an examination in a mathematics course
Being given a homework assignment of many difficult problems which is due the next class meeting
Listening to a lecture in mathematics class
Listening to another student explain a mathematics formula
Being given a "pop" quiz in a mathematics class

4. Findings

The findings are sequentially organized according to the three research questions.

4.1. Results based on Research Question 1

What are some of the challenges students generally encounter with conceptual understanding and their problem-solving skills in mathematics?

This research question focuses on two main areas: the factors and difficulties students are faced with in understanding mathematics concepts and problem solving.

4.1.1. Factors that affect students' conceptual understanding and problem solving skills

It was observed at Singers High that when the participants responded to the item on taking an examination in a Mathematics course, 80% of them (4 of 5) showed moderate anxiety while 20% displayed quite a bit of anxiety (see Table 1 and Figure 2). Signs of moderate anxiety were evidently displayed where the participants who sat exams expressed they felt a sense of fear and unease periodically or ever so often while working through the paper. One (20%) student, however, frequently struggled to keep it together in order to focus

on the test. Data collected from AJ High indicated that 1 of 5 expressed that worded problems were not difficult; however, once that student knew of an impending math test or was in a test, the student struggled to remember what he or she had studied. Another one of the students at AJ High expressed the need to remain focused while completing the test. This was also revealed at Singers High where participants often become uneasy once a test is involved which negatively affects the outcome of their solutions. Table 2, which summaries the students' perceptions of mathematics across all six schools, concurred with Figure 2. It showed that 25% (9) of the Grade 8 and 9 participants across all schools demonstrated a negative attitude towards taking Mathematics tests, 11% of the 25% (4) from Singers High displayed signs of moderate anxiety through their expressed frustration. The other 14% (5) were from Hilltop High who however had a negative perception whenever their teacher exposed them to operations on integers. Five percent (5%) of those participants from Hilltop High expressed that they found the test to be difficult; 2.7% equally shared that performing the operations on the test seemed challenging, puts him or her in a state of shock, and was considered nerve racking to figure out the answers. Figure 2 also revealed that the participants at Singers High displayed anxiety issues when called to work questions on the board. The findings in Figure 2 showed that 1 Grade 8 participant from Singers High displayed signs of moderate anxiety where the participant was often frustrated when faced with new math concepts when called to the front of the class to work questions. The study revealed, on the other hand, that 2 participants displayed signs of low anxiety where they expressed a low feeling of fear and sense of unease towards math; while 1 displayed some anxiety in going to the board. The 2 students with moderate and some anxiety were reluctant in working questions on the board and even stated when interviewed that they disliked being asked to do so (see Table 2). The study discovered that the students who showed signs of some anxiety are usually the ones who panics whenever they lack clarity of the mathematics task given to them. It was revealed that those participants believed they would get wrong answers so they refused to answer questions on the board. One of the 2 participants who showed low anxiety eventually worked a question on the board after being called for some time.

Students' anxiety issues identified based on the mAMAS

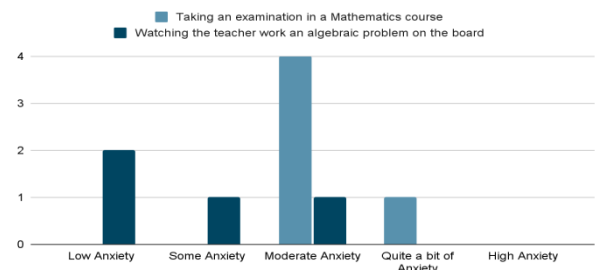


Figure 2. Grade 8 student's anxiety issues identified based on the Modified Abbreviated Maths Anxiety Scale (mAMAS) at Singers High

Table 2 shows that of the total number of participants across the six high schools, 44% expressed their disinterest in learning mathematics from all of the high

schools. All 8 participants from Runners High indicated a lack of interest in mathematics; it was shared that 4 of them, in responding to the questionnaire, wrote they were not interested in class. The study further revealed that 3 of them could not see how mathematics related to real life while 1 indicated that he or she had no interest in participating in the learning process. Data collected at Singers High discovered that 60% of the participants showed lack of interest as they kept zoning out during classes and 1 from Monk Town High clearly shared a lack of interest. Reference [7] [he] revealed that whenever students begin to feel helpless in the mathematics classroom they often zone out. He [7] shared that this is usually related to a learning problem which often is

triggered by something that happened in the classroom and tends to begin at the primary school level. One of the participants [he] at Singers High expressed boredom and often rested on the desk. The participants at AJ High, also, shared similar sentiments where 1 student stated that “the challenges that I am facing is that I don’t like math one bit and no matter how hard I try, I just can’t understand”. The study showed that the students at both AJ High and Runners High were unwilling to participate in the class. Interestingly, the participants at Singers’ High, however, in an interview expressed that even though they disliked mathematics they liked their teacher; 1 even stated, “Miss jus[t] cool, she talks to us and make[s] us feel ok”.

Table 2. Students’ Perceptions Toward The Teaching And Learning Of Mathematics Across All Six Schools

Negative Perceptions	Indicators	Number of Participants
Lack of interest towards Mathematics	Mathematics is too hard; Free fire is more interesting than math class; We don’t understand the math; not involved in the learning process; unable to see how Math relates to real life; I tend to zone out in math class; I don’t need mathematics to ‘chop the line’, a slang that means to scam.	16
Feelings towards the host teacher	Miss cannot teach; Miss not teaching them to understand; Miss is too boring; Miss don’t care; Miss has her favourite student.	3
Attitude towards Mathematics test	Fear of taking a Mathematics test more than other tests; integer test is difficult; integer test makes me nervous; integer test is challenging; when I see the integer test I am in shock.	9
Attitude towards going to the board to work problems	Reluctant to work on the board; lack of confidence in their ability to get the answer correct; fear of getting the answer wrong.	3
Answering worded problems in Mathematics	Not sure. But maybe I just need to be more focused; I don’t have any challenges but when I get a test I forget most of the things I studied; the challenges that I am facing is that I don’t like math one bit and no matter how hard I try, I just can’t understand.	4

The study revealed that the classroom climate at XYZ High seemed hostile. The data found that 80% of the participants at XYZ High when interviewed stated that they do not have any interest in mathematics because it is too hard, playing the Free Fire game is more interesting than mathematics class, they ‘do not understand the math’ and as such they have no need to learn mathematics because it is better to ‘chop the line’ (scam people for money) (see Table 2). It was revealed in the interview that 75% of them expressed that ‘Miss cannot teach’, ‘Miss not teaching them to understand’, ‘Miss is too boring’, ‘Miss don’t care’ and that she has her favourite student (see Table 2). It was also found that their host teacher [she] when interviewed said they were lazy and did not spend the necessary time to practice; they also failed to complete classwork and homework. She further explained that they disliked mathematics and none of them have ever expressed that they did not understand the mathematics concepts taught. She also stated that they were also frequently absent from classes; and refused to participate the few times they were present, often spending time on their phone playing games and/ or browsing social media such as Tiktok and Instagram. The study discovered that the relationship between the host teacher and her students was disengaged which has helped to create disconnect between students’ understanding mathematical concepts. The study also uncovered that 100% of the participants played the Free Fire game which kept them up through the nights so they were often late for their classes.

The study provided findings related to students' content and skills gaps across the schools. It was found that 100%

of the participants at Hilltop High lacked the knowledge necessary in applying the rules of operation when performing addition, subtraction and multiplication of integers (see Table 3). They lack integer skills, the host teacher confirms it was not taught (due to the pandemic), students are also in Pathway 2 which means they needed more support with the challenging topic. These participants’ negative perceptions of taking the pre-test on integers have gravely impacted how they answered the questions since they perceived the test to be difficult, challenging and nerve racking (see Table 2); they expressed that they did poorly on the pre-test. The study conveyed that the participants’ negative perceptions while completing the pre-test is closely tied to the results from the test indicating their inability to provide and use the rules to perform operations on integers (see Table 2 and Table 3). Moreover these students are in Pathway 2 so some of them learn at a slower pace than their other Grade 9 counterparts. The study also revealed that 100% (8) of the participants at XYZ High who were described as Pathway 2 showed signs of conceptual errors while 50% (4) of them had minimal computational errors based on the pre-test on solving simple linear equations in one variable. Consequently, it was evident that the participants were not able to effectively solve simple linear equations. For example in Figure 3, a completed solution was provided by one of the participants when given the question $x + 6 = 11$. The study revealed that this student lacked the prior knowledge of directed numbers and transposition which facilitated the erroneous steps in the solution process (see Figure 3). Thus the participants from XYZ High were not able to generate all required solution

steps for the simple linear equations in one variable. Figure 4 revealed that when the participants were given $y + 6 = -8$, one made an incorrect computation in obtaining the final answer even after correctly applying the transposition steps. The study showed that the participants at Runners High also had issues understanding math concepts when they were given the pre-test because they had difficulty interpreting worded problems. Table 3 revealed that 1 of them in particular, struggled immensely to provide meaning for terms used in the worded problems

in order to write the expressions and equations required. The study also showed that 87.5% (7) of the participants at Runners High could not correctly group like terms in given expressions in order to simplify them correctly. Additionally, 25% (2) of the participants struggled to recall the rules of operations on integers in order to simplify expressions.

4.1.2. Difficulties Students Have With Conceptual Understanding And Problem-Solving Skills

Table 3. Difficulty Understanding Mathematics Concepts across All Schools

Challenging Factors	Identified Errors	Number of Students
Lack of knowledge: integer addition rule	Students ignore the negative sign completely which shows a lack of conceptual understanding.	5
Lack of knowledge: integer subtraction rule	Students also show a lack of conceptual knowledge of operation on integers where they apply the wrong computation.	7
Lack of knowledge: integer multiplication rule	Still again students were able to multiply two integers but lacked knowledge on what sign the product should take.	5
Conceptual errors	Incorrect transposition: lack of knowledge of directed numbers	8
Computational errors	Minimal calculation problems	4
Difficulty identifying mathematical terms in word problems	Could not correctly identify mathematical concepts for example, "square" they did not understand that this meant to the second power	1
Answering worded problems in mathematics	The working out part; I am facing the problem of knowing and figuring out what to do with the information given in the problem	1
Grouping like terms	Could not process that variables with different powers were not like terms, for example, x^2 was not a like term with x^3	7
Expand brackets fully	Did not understand that when expanding brackets the term outside the bracket was supposed to be multiplied with the terms inside the bracket	3

$$x + 6 - 6 = 11 + 6 = 17$$

$$x = 17$$

Figure 3. An Unsuccessful Attempt made by a Student at XYZ High to Solve the Simple Linear Equation in one Variable

A similar issue of students' inability to comprehend operations on integers that occurred at XYZ High was found amongst these participants at Runners High. The study showed however that the participants at

$$y = -8 - 6$$

$$y = 2$$

Figure 4. Shows that a Student at XYZ High Incorrectly Computed the Integer Values

Runners High were challenged with simplifying algebraic expressions while those at XYZ High struggled to solve linear equations in one variable. It was discovered that the participants when simplifying the expression $-7a - 4a$ provided the incorrect solution $-3a$ rather than $-11a$. This revealed that these students have not yet

mastered how to subtract a negative and positive integer; especially based on their explanation provided. Therefore, it is evident that the participants need to work more efficiently with negative numbers in order to transfer that knowledge to simplifying algebraic expressions. The study also uncovered that 37.5% (3) of the participants at Runners High were not able to fully expand brackets which involved the employment of the distributive property. For example, when given $7(-2a + 3b)$, these students wrote down $-9a + 10b$ rather than the correct solution $-14a + 21b$. Their solution $-9a + 10b$ showed that they did not understand the distributive property and how to apply it in the simplification process. The participants in this case added 7 to $-2a$ to get $-9a$, treating the 7 as -7 and they applied the same misconception to obtain $10b$. It was found that all 3 participants showed a similar workout of the incorrect solution. The study found 1 participant at AJ High just like the participant at Runners also struggled in translating worded problems to mathematical (symbolic) statements (see Table 3). It became evident as a result of the participants at AJ High refusing to participate in class activities such as discussions; and they also have not taken the time to complete the tasks given to them in class and for homework.

Table 4. Challenges Students at Monk Town High and XYZ High Have With Problem-solving Skills

Challenges	Indicators	Number of Participants
Lack of prior knowledge	Difficulty recalling and using knowledge of the cardinal points to generate illustration required; inability to recall the properties of geometric figures	5
Lack of critical thinking	pattern was not properly drawn	3
Lack of mastery in solving problems involving patterns outlining the problem-solving steps	clear problem solving steps in the solution are needed	5
Lack of mastery in solving linear equations	clear problem-solving steps in the solution process are needed.	6

Problem-solving skills seem to be poorly developed across the student participants at Monk Town High and XYZ High. It was revealed that 100% (5) of them from Monk Town High lacked the prior knowledge needed to understand each problem given coupled with their inability to critically think (see Table 4). It was also found that these participants at Monk Town High lacked mastery in providing clear steps in the derivation process to obtain geometric patterns through diagrams and explanations. This was evident where the participants provided the diagram required for the patterns requested but the explanations and computations needed were not done. When the students were asked to explain and show the process they said they knew the generated patterns but did not know how to explain it and show the necessary calculations. Their host teacher [she], when interviewed, shared that they were never taught problem-solving steps because it requires a lot of time to teach, especially with the stringent curriculum timeline it was impossible to do so. Consequently, the students' attitude, and their lack of prior knowledge and suitable problem solving strategies, gravely impacted their ability to interpret geometric patterns in order to develop rules for each.

It was discovered that 100% of the participants from XYZ High lacked the mastery of solving linear equations because they are not able to correctly outline the problem-solving steps needed to generate a solution. These students could verbally state the steps but struggled to correctly represent them on paper (see Figure 3 and Figure 4). A similar issue was revealed at AJ High where 1 student could not even write down the first step in the problem-solving process given worded problems because of the inability to comprehend the problem given in order to translate to symbolic statements.

4.2. Results from Research Question 2

How effective are the use of Algebra tiles, learning-based activities and Polya's problem solving approach in helping students to improve their understanding of concepts and problem-solving skills?

The participants across the six schools were given post-tests after the implemented strategy. Eighty percent (80%) of the participants at Monk Town High were able to clearly provide the rules for a given geometric pattern; but only 40% (2) were able to use the rules when given to obtain a geometric pattern (see Table 7). It was noticed that the 80% showed all the steps required clearly outlining the Polya's problem-solving steps in the solution process, however, the 40% could not show the steps

needed to interpret the rules in order to obtain the geometric patterns (see Table 5, Table 6 and Table 7).

Table 5. Key for the Rating of the Items (Objectives) On the Pre-test and Post-test

Rating	Notation	Interpretation
Excellent	E	Mastery: All steps achieved
Good	G	Mastery: All steps achieved but could be explained better
Satisfactory	S	Near mastery: One step missing
Average	A	Fair: Two steps missing
Needs Improvement	NI	NI: Three steps missing
Weak	W	Non-mastery: Nothing achieved

Table 6. Polya's Steps To Problem Solving [38]

Steps	Outline of steps
1	Comprehend the issue
2	Make a plan
3	Execute the plan
4	Check and Interpret

This can be attributed to the need for the participants to spend time to think through problems and figure out a suitable strategy needed to provide the solution. The students at Monk Town High were also given a worded problem requiring them to generate geometric illustration, calculate the missing length and area for the generated illustration (see Figure 5). Figure 6 shows the required geometric illustration for item 2a) on the post-test as indicated in Figure 5. It was noticed that 4 in 5 participants were able to produce the diagram after the intervention. However, the other 1 participant displayed an error where ladder B and ladder A were placed on the two opposite sides of the wall (see Figure 7). The incorrect triangle provided in Figure 7 resembles an equilateral triangle but the dimensions of AC and CB are different which is not possible; so the information given in Figure 5 does not fit to obtain this diagram. Moreover, the length AB in Figure 7 was the length of the ground from the wall is impossible since the ground was found to be on one side of the wall; and this length is not of the same dimension as AB in Figure 6.

Elisa has two ladders, ladder A and ladder B. Ladder A measures 15 meters and ladder B 17 meters. Elisa leans **ladder A** against a wall forming an angle of 35 degrees with the ground and then leans **ladder B** against the same wall where the top of the ladders meet.

- Provide a diagram for Elisa to illustrate the graphical representation of the placement of her ladders against the wall.
- State the measurement for the foot of **ladder B** to the ground and the height of the ground to the top of the ladders. Give reasons for your answer.
- Find the area of the triangle made with **ladder B**? Show all working.

Figure 5. Question 2 Obtained From Post-test

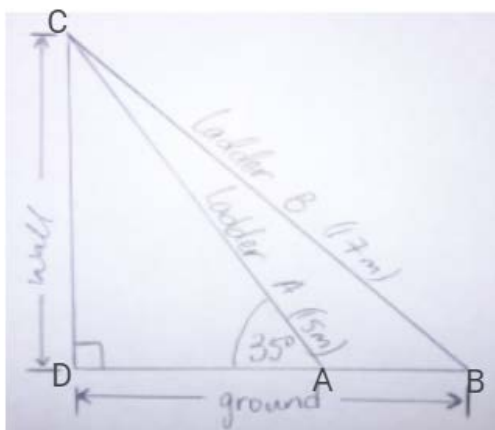


Figure 6. Solution for question 2 part a in Figure 5



Figure 7. Incorrect solution for question 2 part a in Figure 5

On the other hand, only 1 participant was able to calculate the missing length of the geometric illustration obtained while 2 could not provide the solution process in order to obtain the answer (see Figure 5, Figure 6 and Figure 8). These 2 participants seemed to lack prior knowledge in finding missing lengths of geometric figures because they were unable to correctly identify the trigonometric ratio most suited for the solution and apply it. The other 2, however, did not make any attempt to provide a solution.

Using Trigonometric Ratios to find the height of the wall.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 35^\circ = \frac{\text{opposite}}{15}$$

$$\sin 35^\circ \times 15 = \text{opposite}$$

$$0.574 \times 15 = \text{opposite}$$

$$8.61 = \text{opposite}$$

Height of wall $\approx 8.61\text{m}$

Using Pythagorean Theorem to find length of ground from ladder B to wall

$$(17)^2 = (8.61)^2 + (\text{ground})^2$$

$$289 = 74.1321 + (\text{ground})^2$$

$$289 - 74.1321 = (\text{ground})^2$$

$$\sqrt{214.8679} = (\text{ground})^2$$

$$14.66 = \text{ground}$$

$\therefore$ length of ground from foot of ladder B to wall $\approx 14.66\text{m}$

Figure 8. Solution for question 2 part b in Figure 5

Also, that very same participant who correctly found the missing length was the only student able to compute the area using the area formula for the geometric illustration obtained with the correct problem-solving steps (see Figure 9). It was discovered that the 4 participants did not understand trigonometric ratios along with Pythagoras Theorem in obtaining lengths, and area concepts to apply that knowledge to the scenarios given.

Area of triangle $= \frac{1}{2} b \times h$

$$b = 14.66\text{m}$$

$$h = 8.61\text{m}$$

$$\text{Area of triangle} = \frac{1}{2} \times 14.66 \times 8.61$$

$$= \frac{1}{2} \times 126.2226$$

$$\text{Area of triangle} \approx 63.11\text{m}^2$$

Figure 9. Solution for question 2 part c in Figure 5

It was revealed that just like Monk High, the participants at XYZ High were made aware that they should work through every problem given using Polya's problem-solving approach as outlined in Table 6. It was found based on the multiple choice items on the post-test requiring them to identify the correct solution from the list of options A, B, C and D for a given simple linear equation in one variable, that 100% (8) of the participants chose the correct option. When they were asked to show and explain using the problem-solving steps how and why the selected option satisfied the equation, they outlined all the steps; but their explanations needed clarity at a few instances (see Table 7). The study found that 87.5% (7) of them also selected the correct option with the first problem-solving step and provided explanations of how that step was generated, even though the explanations could be clearer. Additionally, 75% (6) of the students

were able to satisfactorily identify the correct option showing the first problem solving step involving negative numbers for item 3 on the post-test (see Table 5, Table 6 and Table 7). It was evident that they could now better balance the equation using correct operations but needed to clearly explain how they could obtain the required step. Also, the data showed that 62.5% (5) of the participants at XYZ High could identify the first two steps of the problem-solving process for the final item but made minimal mistakes in explaining. It was noticed, however, as the difficulty level increased for the items given, more of the participants could not comprehend and solve simple linear equations.

The participants at AJ High were also exposed to Polya's steps of problem solving (see Table 6). The researcher explained to participants the paramount importance of using these steps when attempting to solve math problems because their problem-solving skills would be developed which will help them to use practical mathematics in real life as well as to be equipped to solve real life issues. Based on post-intervention, 40% (2) of the participants were able to satisfactorily express numbers in exponential form based on a scenario given (see Table 7). It was evident that these participants were able to achieve the first three steps of Polya's process but were not able to check if the answer they produced successfully satisfied or solved the problem given. On the other hand, 60% (3) of the participants did an excellent job in evaluating numbers in exponential form, where they were able to successfully express the exponential number in the standard form (regular form) of writing numbers. However, it was noticed that the 40% (2) were unable to do so because they believed that exponential numbers represent repeated addition rather than repeated multiplication. When given worded problems involving matrices 80% (4) of the students at AJ High could translate them to matrix forms. Just like the topic of exponents, when students were given matrix worded problems, 40% (2) of them were able to successfully solve the problems involving addition and multiplication of matrices using the first 3 steps of Polya's process but did not check their answers (step 4) (see Table 5, Table 6 and Table 7). Also, 100% of them could solve problems involving scalar multiplication using the first 3 steps but saw the 4th step as extra work that was not relevant.

Eleven out of a total of 13 participants from Hilltop High (5) and Runners High (6) were now fully able to apply the rule of adding integers to obtain the correct answer (see Table 7). Similarly, the data showed that all 5 from Hilltop High could apply the rules in adding negative integers. For example, when given $-1 + (-6)$ on the post-test, the participants at Hilltop High shared the correct answer, -7 using the algebra tiles. It was evident that the participants were able to show the process in obtaining the solution as they worked collaboratively however they could elaborate more on the addition process. It was found that the participants used red coloured tiles to depict negative numbers, where one of the tiles represented -1 , and 6 of them represented -6 ; in order to convey the addition process they combined the tiles to obtain 7 tiles which represented -7 (see Figure 10). Moreover, it was noticed that all 5 participants could provide satisfactory solutions when applying multiplication on integers with

opposite signs using the Algebra tiles; but they needed to clearly convey the steps in the solution process (see Figure 11). However, the study revealed that none of them could provide solutions and steps when multiplying two negative numbers (see Figure 12). It was uncovered that the students could not recall and apply the rule when multiplying two negative numbers. These participants evidently could correctly identify the red tiles to represent -5 and -6 but could not show that they needed to have 30 red tiles while applying the multiplication rule using the tiles of turning the tiles over twice (indicating opposite operation) to obtain positive 30 since both numbers being multiplied are negative. Mrs. Thomas 4 Math, a teacher [39] explained on her YouTube channel that students usually lack comprehension of the multiplication of two negative integers because they do not understand that multiplication of numbers will always involve grouping regardless of whether they are inclusive of positive and/ or negative numbers; and how to interpret the signs of the numbers recognizing that $-$ and $+$ are opposites of each other.

At Runners High, before the implementation of the activity-based learning strategy, participants were asked in an interview if they would engage in learning activities to help them understand mathematics concepts. It was found that 50% (4) of the participants said that they would engage in learning activities because it would make understanding mathematics easier for them, while the other 50% (4) of the participants stated they did not think that learning activities would make math easier for them, however all 8 (100%) participants expressed their willingness to engage in the learning activities in class. As a follow-up question, the participants were asked to identify the activities (group activities, interactive whiteboard activities and games) that they would engage in. Fifty percent (50%) said that they would prefer to engage in games, 36% (3) preferred doing group activities, and 14% (2) preferred to be engaged using the interactive whiteboard. The data collected showed these learning-based activities were relevant in engaging the participants in the teaching and learning process to learn concepts. Evidently, the use of activity-based learning had a positive impact on their conceptual knowledge, since 62.5% (5) of the participants were now able to comprehend the worded problems and provide meanings though minimal clarifications were needed. It was also discovered that all participants could now correctly group like terms and expand brackets for given expressions but did not write clear explanations of what was done. The participants at Singers High were also engaged in learning-based activities, however, they were instructed to complete tasks in groups. Twenty percent (20%) of the participants prior to the intervention were unwilling to work with anyone, however, it was revealed that they eventually met. It was found that despite the fact that all the participants still had great issues with understanding mathematics ideas or were uncertain about the correctness of their answers, they eventually participated willingly and shared what they understood and did not. Shamali Eriyagama [she], a researcher at Amarasuriya Teachers College, Unawatuna, Sri Lanka, [40] found that 87% of the sample used shared that activity-based learning facilitates collaborative learning and give students a sense of participation; while

84.4% found the strategy helpful in improving interactions in the classroom. Thus, the study showed that the participants became active learners as they sought to understand the concepts taught.

Table 7. Students Understanding After The Strategy Was Implemented And Given A Post-test

Students Understanding of Mathematical Concepts	Positive Changes Identified	Ratings	Number of Students
Improved understanding of the rule that applies when adding negative and positive integers	Students were able to arrive at the correct answer when performing addition on integers with opposing signs by applying the correct process	E	11
Improved understanding of the rules that apply when adding two negative integers	Students were able to arrive at the correct answer when performing addition on two negative integers by applying the correct process	G	5
Improved understanding of the rules that apply when multiplying two integers	Students were able to arrive at the correct answer when performing multiplication on negative and positive integers (or positive integers only) by applying the correct process after the strategy was implemented	S	5
Correct substitution of solution for simple linear equations with explanations	Students mastered the problem-solving steps to outline the solution for a simple linear equation in order to show why the chosen substituted value from the list of options on a multiple choice paper worked	G	8
Correct identification the first step of the problem-solving process in solving simple linear equations	Students were able to select the first problem-solving step for a given question on a multiple choice paper and showed why the chosen step was correct	G	7
Correct use of knowledge of negative numbers to identify the first step in the problem-solving process of simple linear equations	Students were able to correctly choose and explain the first problem-solving steps involving the use of negative numbers	S	6
Correctly indicated the first two steps in the problem-solving steps for simple linear equations	Students were able to identify the correct option with the first two steps and explained the process needed to generate those steps by illustration	G	5
Mastery of generating rules for geometric pattern using the problem-solving steps	Using the problem-solving steps, students were able to generate a rule for the geometric pattern associated with the series	E	4
Mastery of providing geometric patterns given rules	Students were able to use the rules to provide a geometric pattern	E	2
Correctly calculated the missing lengths of the geometric illustration obtained using formula	Students were able to calculate the missing lengths of the geometric illustration using the correct formula	S	1
Correctly computed the area of the geometric illustration obtained using formula	Students were able to calculate the area of the geometric illustration using the correct formula	E	1
Correctly expressed the numbers in exponential form given worded problems	Students were able to express numbers in exponential form based on a scenario given	S	2
Correctly evaluated the numbers in exponential form	Students were able to correctly express numbers in exponential form to standard form	E	3
Correctly wrote down matrices using matrix characterization given worded problems	Students were able to correctly take information from worded questions and represent it in matrix form	E	4
Correctly added and subtracted matrices	Students were able to correctly add and subtract matrices given in a worded problem	G	2
Correctly applied scalar multiplication	Students were able to correctly apply the concept of scalar multiplication based on the problem given	G	5
Correctly identified mathematical terms in worded problems	Students were able to identify key terms in worded problems and provide contextual meanings for each	G	5
Mastery of grouping like terms	Students were able to recognize and correctly group terms in given expressions using their knowledge of operations on integers	E	8
Correct expansion of brackets	Students were not able to fully expand brackets using the distributive property	G	8



Figure 10. Sample of participants' solution of adding two negative integers using Algebra tiles at Hilltop High

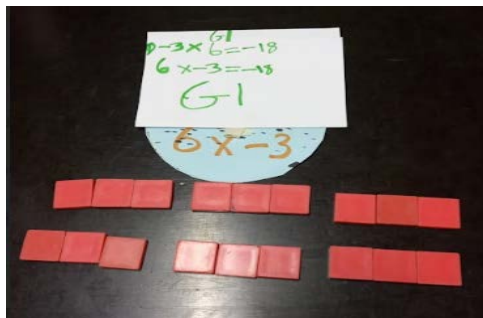


Figure 11. Sample of participants' solution of multiplying positive and negative integers using Algebra tiles at Hilltop High

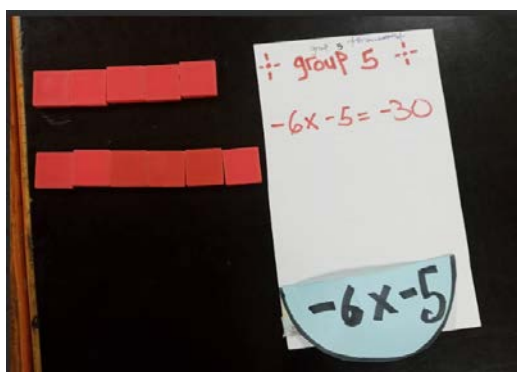


Figure 12. Sample of participants' solution of multiplying negative integers using Algebra tiles at Hilltop High

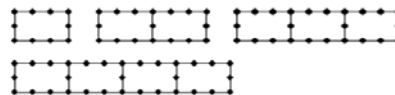
4.3. Results from Research Question 3

Is there a comparative difference among the effects of the Algebra tiles, the learning-based activities and Polya's problem solving approach on students' conceptual understanding and their problem-solving skills?

In this study, the strategies were used to relate to different concepts in the teaching and learning process to help students improve their problem-solving skills and their ability to understand concepts. These strategies were: algebra tiles, learning-based activities including both problem-based learning and activity-based learning; and Polya's Problem-Solving Approach. The Algebra tiles were used to teach participants how to perform operations on integers at Hilltop High, the problem-based learning strategy coupled with Polya's Problem-Solving Approach was used to allow participants to explore geometric patterns, perimeter and area concepts at Monk Town High. Polya's problem-solving approach was also used to help the participants at XYZ High understand how to solve simple linear equations with one variable, as well as to improve the participants at AJ High understanding of the

concepts related to law of indices and basic matrices. While activity-based learning was used to help participants understand algebraic concepts at both Runners High and Singers High.

The figure shows a series of rectangles where each rectangle is bounded by 10 dots.



- Provide a diagram for the 6th rectangle and explain how you arrive at your answer.
- How many dots are required for 7 rectangles?. **Show or explain how you arrive at your answer.**
- If the figure has 73 dots, how many rectangles would there be?

Figure 13. Question adapted by Monk Town High from an activity sheet with questions on geometric patterns [41]

Table 8 shows the intervention strategies used implicitly or explicitly in the classroom at the six schools indicated. The strategies were considered based on the appropriateness of the topic(s) taught; however, alternative strategies also could have been employed where suitable. The study revealed that the researchers each thought about using one another's strategy while considering an intervention plan they would have undertaken prior to the development of the study. It was uncovered that the Algebra tiles which was used to help participants at Hilltop High understand how to perform operations on integers could have been adapted and tailored at XYZ High for the participants to employ to guide the solution process in solving simple linear equations; as well as at Runners High in teaching algebraic concepts such as distributive property. It was discovered that since algebra tiles are originally created to work with polynomials of different forms, then it could have been used alongside the problem-solving approach and activity-based learning.

On the contrary, the data collected at Monk Town High revealed that the algebra tiles were very limited in teaching geometric patterns. The study found that these tiles could assist to some extent in obtaining simple rules for geometric patterns that have little to no complexity which are usually easily generated using algebra. However, as the difficulty level of these geometric patterns intensifies, obtaining algebraic expressions becomes even harder to obtain. Moreover, algebra tiles are usually used to help understand algebraic structures (expressions) to facilitate abstract operations in a pictorial way; but not to provide opportunities to form the algebraic expressions [42]. Thus, their suitability to obtain geometric patterns seemed to be too far-fetched. Consequently, using the tiles proved to be too complicated to teach certain geometric concepts; especially in the derivation process of the problem-solving steps for some geometric patterns (see Figure 13). It was also revealed at Hilltop High that problem-based learning came to mind while teaching participants the rules required to multiply negative integers, since they were not able to comprehend the process to get the correct answer using the algebra tiles

(see Figure 12), however, there was insufficient time to adapt that strategy. The teacher-researchers at Singers High and Runners High pointed out that activity-based learning was used to help the participants understand algebraic concepts (see Table 8).

Table 8. Strategies Implemented In The Teaching And Learning Process Across The Six Schools

	XYZ High	Monk Town High	Hilltop High	Singers High	AJ High	Runners High
Algebra tiles	-	-	√	-	-	-
Activity-based learning	-	-	-	√	-	√
Problem-based learning	-	√	-	-	-	-
Polya's problem-solving approach	√	√	-	-	√	-

Participants at Singers High were allowed to work as pairs to help them to think pair-share as they worked through the activities given and discussed with the class. This study showed that participants at Singers High and Runners High were engaged in learning-based activities that facilitated the development of their problem-solving skills even though they did not use Polya's problem-solving approach. The participants at XYZ High, Monk Town High and AJ High were exposed to Polya's problem-solving approach. Problem-solving skills were considered in order to get participants to think about the concepts, in order for them to break up the knowledge they have and use each piece to fill into multiple topic areas where appropriate and create connections. It was noticed that in the second step in the problem-solving approach, which required the participants to make a plan; they could have made the decision to use a tool or strategy such as algebra tiles. It was revealed at AJ High that Polya's problem-solving steps were important in ensuring that participants could show the steps needed to explore solutions related to worded problems on exponential forms, generating matrices and performing operations on matrices. It was evident that at AJ High learning-based activities coupled with Polya's problem-solving approach because the participants were required to identify and explain each problem-solving step to ensure they comprehend all concepts. So, even though algebra tiles could be used to help with exponential forms, it was not the most appropriate based on the designed intervention plan. Also for the teaching of matrices, algebra tiles would not be appropriate at all since algebraic representations were not required; however, the study revealed that problem-based learning was effective since it allowed the participants to think critically in order to derive the exponential forms and matrices stepwise from given open-ended problems.

Similarly, the participants at Monk High were given problem-based learning activities (see Figures 5 and 13) to solve using Polya's problem-solving approach. This approach was found to be effective at Monk High since the students were better able to think critically, design a plan of action and carry it out which would develop their

problem solving skills. Reference [23] shared that problem-based learning usually helps students to develop a deeper understanding of mathematical concepts; especially since it facilitates them to be active and independent learners in deriving their solution pathways.

4.4. Further Discussions and Implications

There is a need to reshape the environment where learning takes place by seriously considering students' learning abilities and experiences in the teaching and learning process. The study revealed the participants' spirits were seemingly dampened as they kept echoing their disinterest for various reasons, sharing their fear of doing mathematics tests and that mathematics is difficult to understand prior to the intervention (see Table 2). Some of these issues, as professed by the student participants at Singers High, were generational where their own family members had told them from their younger years that they did not like math and found it to be difficult. Rev Ronald Thwaites in 2015, shared in an article titled "That worrying math phobia", that students across the schools for generations now have constantly expressed their fear of mathematics [43]. This family fear has already crippled the students before they have even entered the school walls and thus creates a tug-o-war between the teachers and students. Just as 1 participant from XYZ High shared that he or she did not see the relevance of mathematics, Rev Thwaites also shared a similar comment implying that their phobia mainly impacts the Algebraic and Geometric concepts [43]. Consequently, the utilization of the strategies provided participants opportunities to explore activities related to real life and having meaningful impact. It was jointly shared across the schools that the participants' expressed anxieties are mostly related to the teaching methods used, students' lack of understanding mathematical concepts coupled with their inability to problem solve as outlined in Table 2, Table 3, Table 5, Table 6, Figure 3 and Figure 4.

From the study, it was noticed that of the 17 mathematics areas relating to integers, simplifying algebraic expressions, solving simple linear equations, exponential forms, matrices, geometric patterns, area and perimeter covered during the teaching Practicum, only 7 of them were mastered after the intervention. Participants were able to add integers with opposite signs, outline clear problem solving steps in generating the rules for geometric patterns and vice versa, calculating area of Geometric illustrations obtained from worded problems, express exponential forms as standard forms, develop matrices from worded problems and group like terms. Based on the findings with the students involved, all students at Hilltop High excellently mastered adding integers with opposite signs; so too did most of the students (75%) at Runners' High. The teacher-researcher at Monk High, on the other hand, expressed that even though most (80%) of her students excellently mastered generating the rules for the given geometric patterns, only 40% excellently provided the geometric patterns for the given rules; and only 20% could also calculate the area of the obtained geometric illustrations from worded problems. However, it was discovered at AJ High that 60% of the students excellently wrote standard forms for exponential

forms while 80% wrote matrix representations for given worded problems with great ease. While all participants over at Runners High can excellently group like terms. Even though levels of mastery were seen in different areas, there were still quite a few areas that needed to be addressed. It was also found that different strategies used in this study would have been suited for most of the mathematical concepts discussed in this study; however looking at geometric and matrix concepts the choice of strategy became very stringent. It was revealed that more time was needed to work with the participants because the period of intervention was not enough to truly effectuate the strategies employed. It was also found that all teacher-researchers used activity-based learning throughout their intervention.

5. Conclusion

From the study it was found that before the implementation of the strategies, the students across the 6 schools had similar challenges in the classroom even though most were exposed to different mathematics topics (concepts) namely, operations on integers, solving simple equations, algebraic expressions and geometric patterns, trigonometric ratios along with Pythagoras Theorem in obtaining lengths and area concepts. The challenges were found to be mainly attributed to factors such as their disinterest and the dreaded math anxiety. The study revealed that 100% of the participants had math anxiety though seen in diverse ways; also 44% displayed signs of disinterest prior to the intervention. Coupled with these factors were the difficulties that students initially had such as their inability to comprehend concepts and problem solve. The data collected revealed that 7 of the 13 students from both Hilltop High and Runners High lacked conceptual understanding of operations on integers; while at least 50% of the students from XYZ High showed signs of both conceptual and computational errors in different forms. It was also found that 100% of the participants at XYZ High and Monk Town High struggled with problem-solving. There was a resounding echo across the schools that the participants lacked prior knowledge which was the main hindrance to students understanding concepts and problem solving at the Grade 8 and 9 levels. After the intervention, it was evident that the strategies, Algebra tiles, learning-based activities and Polya's problem solving approach, were effective because the study showed that the students were all able to improve both their conceptual understanding and their problem-solving skills in 7 of 17 mathematics concepts from September to December 2022. It was also found that the strategies implemented could have been used by the six teacher-researchers contextually to address the identified issues since some of the challenges were similar. However there was a slight comparative difference when considering geometric concepts and matrix representations, and the use of Algebra tiles. It was revealed that only specific strategies designed to focus on matrices could be used since Algebra tiles were not suitable for those concepts; as well as for some geometric concepts. Consequently, it is safe to conclude that these strategies can be employed in the classroom to help students with the issues they may

face relating to disinterest, inability to comprehend concepts, difficulty to problem solve and math anxiety.

6. Recommendations

It was recommended that the present idea of a student-centered Jamaican classroom needs to be re-addressed in order to identify the shortfalls and re-defined to strategically consider tasks that match with the tools, teaching methods and strategies based on topics. Also, greater emphasis should be made on the integration across topics so that students desist from compartmentalizing their knowledge and creating disconnections of related concepts.

In light of the great deficiencies found prior to the intervention, which revealed that the participants' lack of conceptual understanding adversely affects their ability to effectively problem-solve in the classroom, the MOEY should prioritize the need for teacher training and personal development to address the surging mathematical crisis that the country now faces, especially with the ongoing teaching migration.

The MOEY needs to place emphasis not only on improving students' mathematics skills but more on helping teachers to improve their mathematics skills in using different strategies to effect change in problem-solving and conceptual understanding; and conduct follow up programmes to ensure the training is being utilized. Thus, the teachers will in turn be properly equipped to help change the students' attitude toward mathematics and by extension improve the students' performance.

There is a need to consider tailored learning to suit learners' needs so that the correct strategy is utilized to address the correct challenges faced. In addition, teachers should be willing to research and use different strategies that can be tailored to meet the learners' needs of students and in turn help students improve their interest, understanding and performance in mathematics.

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