

Assessing the Impact of Algebra Tiles as Visual and Manipulative Aids on Students' Algebraic Understanding

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Abstract This action research study aims to explore the effectiveness of algebra tiles as a visual and manipulative tool in improving Junior High School students' understanding of algebraic concepts. The research was conducted over a period of six weeks and involved a total of 30 grade 7 students. The study utilized pre- and post-assessments of students' algebraic knowledge to record changes in students' understanding of basic algebra. The Statistical Package for Social Sciences (SPSS) version 12 software was used for the data analysis, and the data were analysed using percentages and paired sample tests. The results revealed a significant enhancement in students' algebra understanding and achievement ($X=7.4667$; $p<0.05$) after the intervention activity using algebra tiles. The Junior High School students exhibited a favorable perception of algebra tiles as tangible tools for mastering algebraic concepts. These findings emphasize the importance of mathematics educators improving their pedagogical knowledge in algebra instruction and incorporating visual and tactile aids like algebra tiles to foster a deeper understanding of abstract mathematical concepts.

Keywords: action research, algebra tiles, algebraic understanding, junior high school, manipulative aid

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1. Introduction

Mathematics is taught and learned across all educational levels in both Ghana and the global context. It constitutes an essential component of Ghana's educational curriculum. Ghana emphasizes that a proficient Mathematics education, essential for sustainable development, should adopt an inquiry-based approach. Consequently, Mathematics education must furnish students with opportunities to expand, transform, enrich, and revise their perspectives of the world. It should primarily revolve around student-centred methodologies in teaching and learning Mathematics, engaging students both physically and cognitively within a fertile and rigorous inquiry-oriented setting. Success and progress across numerous domains of life fundamentally rely on a solid comprehension of Mathematics and Science. A solid grasp of fundamental mathematical concepts at the lower levels is essential for a student's advancement to higher grades in secondary school [1].

Beyond basic arithmetic, mastery of algebra is indispensable for a meaningful grasp of various pivotal mathematical concepts within fields such as science, statistics, business, and contemporary technology [2]. Algebra functions as the cornerstone and language of mathematics [3,4]. Its crucial and substantial role in students' achievement and advancement in mathematics renders it nearly impracticable for students with weak algebraic foundations to excel in the subject [3]. Enhancing one's capacity for algebraic reasoning translates into a profound comprehension of algebra beyond mere procedural knowledge [5,6]. This underscores the significance of algebraic reasoning, which should be accessible to all students if they are to actively participate in society [7,8]. A robust grasp of algebra thus stands as a prerequisite for proficiency and triumph in advanced mathematical realms and in life as a whole [9].

In spite of the significant role algebra plays in human development, students often encounter conceptual challenges when dealing with algebraic concepts, necessitating consistent attention. Analysis of the Chief Examiners' Reports from the West Africa Examinations

Council (WAEC) over recent years has revealed a consistent weakness among students in solving algebraic problems, alongside other mathematical topics [10,11,12,13]. The authors argue that these struggles in algebra act as substantial obstacles to students' success in mathematics and related subjects. Consequently, there is a pressing need to explore teaching methods that can effectively address these challenges. Traditional methods of teaching mathematics often revolve around rule-based approaches, where students learn a series of algorithmic steps.

Research conducted by [14] and [15] suggests that a more engaging approach to teaching algebra involves a "hands-on minds-on approach," utilizing manipulatives that make learning enjoyable and adaptable. Manipulatives, as defined by [16], refer to real-world materials or objects that students can physically interact with to develop mathematical concepts. Incorporating manipulatives into mathematics instruction holds the promise of sparking and maintaining students' interest, fostering active participation, and enhancing learning outcomes. One effective type of manipulative for teaching algebra is algebra tiles [17]. These tiles provide a visual representation of operations involving variables and numbers.

The National Council of Teachers of Mathematics (NCTM) suggests that algebra instruction should commence in early elementary school, giving students opportunities to familiarize themselves with algebraic concepts to prepare for more advanced studies [18]. While the advantages of using manipulatives like algebra tiles are well-documented, there exists a potential research gap in evaluating the effectiveness of such tools on JHS students' comprehension of algebraic concepts in Ghana. Thus, this study aims to investigate the potential of algebra tiles as tangible aids to enhance the understanding and academic performance of JHS school students in areas related to algebraic expressions and simple linear equations. The underlying research issue stems from the need to enhance pedagogical approaches in algebra instruction, and the study's objective is to evaluate the influence of algebra tiles on students' understanding and achievement in this subject.

1.1. Problem Statement

In the course of the third author's engagement with grade 7 mathematics lessons at Boxer JHS (a pseudonym), a concerning issue emerged. It was observed that a substantial number of students struggled when confronted with relatively simple questions involving the addition and subtraction of algebraic expressions. Upon closer examination, it became apparent that this academic hurdle was not solely attributed to the students' comprehension abilities, but rather, it stemmed from the teaching approach employed in the classroom.

The primary factor contributing to this predicament was the teachers' utilization, or lack thereof, of appropriate and effective Teaching Learning Materials, particularly the omission of an essential tool known as algebra tiles. These visual aids are designed to enhance the understanding of abstract mathematical concepts, providing students with a tangible representation that aids in the visualization and manipulation of algebraic expressions.

Given the undeniable impact of this challenge on the students' mathematical proficiency and educational experience, the researchers are compelled to delve into a comprehensive investigation. The aim of this research endeavour is to not only comprehend the root causes of the issue but also to develop and propose a set of meticulously designed Teaching Learning Materials that will serve as a potent solution to the identified problem.

In essence, this research seeks to bridge the gap between traditional teaching methods and contemporary pedagogical approaches by harnessing the power of innovative visual aids such as algebra tiles. Through this proactive initiative, the researchers aspire to empower both educators and students alike, facilitating a more engaging, effective, and enriching learning environment. By addressing the dearth of suitable Teaching Learning Materials and proposing a viable alternative, this research ultimately strives to elevate the quality of mathematics education at Boxer JHS, consequently nurturing the academic growth and success of its students.

1.2. Research Questions

The following research questions guided the study.

- (1) Does the use of algebra tiles as a visual and manipulative tool enhance junior high school students' understanding of basic algebraic concepts?
- (2) Is there a statistically significant mean difference between the pre-test and post-test scores after the use of algebra tiles?

1.3. Hypothesis

H₀: There is no significant difference between pre-test and post-test scores after the use of algebra tiles.

H₁: There is a significant difference between pre-test and post-test scores after the use of algebra tiles.

2. Literature Review

Algebra, often considered a cornerstone of mathematics education, forms a bridge between arithmetic and higher-level mathematical concepts. However, mastering algebraic concepts poses challenges for many students. Prior research underscores the significance of algebraic understanding and acknowledges the hurdles that students frequently encounter in their journey to grasp these fundamental concepts [5,19]. In response to these challenges, educators have explored innovative teaching methodologies, with tangible teaching aids like algebra tiles emerging as potential solutions to bridge the gaps and foster deeper comprehension [16].

Algebra tiles, a set of square and rectangular-shaped tiles, offer a unique approach to visualizing algebraic expressions and operations. These tiles represent integers or variables and are designed to provide a concrete and tactile model for abstract algebraic ideas. In this visual, area-based model, each tile's shape corresponds to a unit square, aiding students in connecting visual patterns with algebraic concepts. The tiles are cleverly color-coded, with one hue denoting negative monomials and the other

representing positive monomials, creating a clear visual distinction (see Figure 1).

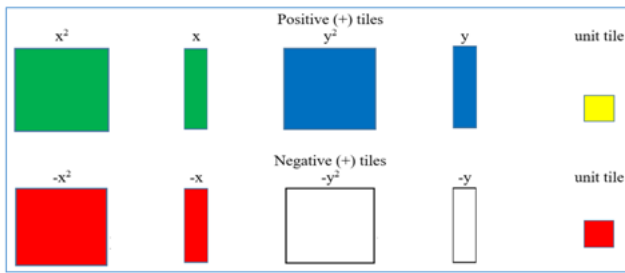


Figure 1. Algebra Tiles

The application of algebra tiles extends to a variety of mathematical processes in basic algebraic concepts, proving to be an invaluable tool for enhancing conceptual understanding. Scholars such as [20] and [15] have explored the potential of algebra tiles in modelling and comprehending algebraic operations. These manipulatives empower students to engage with algebraic ideas in a tangible manner, enabling them to visualize and intuitively understand complex processes that might otherwise remain elusive.

The benefits of incorporating algebra tiles into mathematics instruction extend beyond mere visualization. These benefits are rooted in their alignment with cognitive and constructivist principles [16], tapping into fundamental aspects of learning theory that enhance the educational experience in several key ways. Some of the key benefits of incorporating algebra tiles into mathematics instruction are outlined below:

(i) Concrete Representation of Abstract Concepts:

One of the pivotal advantages of algebra tiles lies in their ability to provide a physical representation of abstract algebraic notions. This unique feature allows students to bridge the gap between abstract ideas and tangible manipulations, facilitating a deeper understanding of algebraic concepts [21]. Through hands-on manipulation of tiles to model algebraic operations, students can connect mathematical symbols and expressions to real-world [17,24], palpable scenarios. This integration of the abstract and the concrete enhances comprehension and retention, making algebraic concepts more approachable and relatable. Furthermore, the visual cues embedded within the tiles offer students a dynamic means to grasp intricate algebraic relationships, nurturing spatial reasoning and conceptual clarity [5].

(ii) Active Engagement and Experiential Learning:

Algebra tiles invite students to actively participate in the learning process, encouraging them to explore and experiment with algebraic operations. This engagement goes beyond passive reception of information and encourages students to construct their understanding through direct interaction [17]. As students manipulate the tiles, they embark on a journey of discovery, testing hypotheses, and refining their mental models. This hands-on exploration fosters a more profound comprehension of algebraic principles, equipping students with the tools to apply their knowledge in

various contexts.

(iii) Guided Problem-Solving: The role of algebra tiles as scaffolds is another crucial facet of their effectiveness. They serve as guides, leading students through step-by-step solutions to problems, promoting a systematic approach to algebraic problem-solving [21]. This guided process helps students break down complex problems into manageable steps, reducing cognitive load and enhancing their problem-solving skills. The tiles provide a structured framework for tackling algebraic challenges, empowering students to navigate intricate equations with confidence.

(iv) Immediate Feedback and Metacognition: The tangible nature of algebra tiles introduces a valuable feedback loop into the learning process. students can promptly identify errors and misconceptions by observing how tiles interact and relate to one another. This immediate feedback mechanism cultivates metacognition, enabling students to reflect on their thought processes and identify areas for improvement [22]. Through self-correction and iterative refinement, students develop a deeper awareness of their own learning journey, fostering a growth mindset that propels them toward mastery.

(v) Transitioning from Concrete to Abstract:

Algebra tiles facilitate a smooth transition from concrete models to abstract notations, aiding students in comprehending the symbolic language of algebra [22]. As students manipulate tangible tiles to represent algebraic expressions and operations, they gradually internalize the underlying concepts. This tangible interaction paves the way for a seamless shift to symbolic representations, equipping students with the ability to fluently navigate algebraic notations and expressions.

Empirical evidence further underscores the efficacy of algebra tiles in enhancing algebraic instruction and comprehension. A multitude of studies (e.g., [14] [17,23,24] and [25]) have demonstrated the positive impact of algebra tiles on students' mathematical abilities. For instance, [14] found that students who utilized algebra tiles outperformed their peers who employed traditional methods when solving algebraic equations. This outcome points to improved understanding and heightened problem-solving skills.

Furthermore, the research of [26] revealed that incorporating manipulatives like algebra tiles enhanced algebraic understanding and retention among this demographic. A study by [15] delved into the use of manipulatives, including algebra tiles, and illuminated how these tools mitigate the abstract nature of mathematics, enabling students to grasp seemingly elusive concepts. The study unveiled that students taught the concept of distribution using algebra tiles exhibited a higher level of achievement compared to those who learned without these aids.

The empirical evidence discussed in the literature highlights the substantial role of algebra tiles in elevating the quality of algebraic instruction and comprehension.

The array of research studies, exemplified by [14,17,23,24,25], has effectively showcased the advantageous impact of algebra tiles on students' mathematical proficiency, underscoring enhanced problem-solving skills and heightened conceptual understanding. These research works have paved the way for recognizing the potential of manipulatives like algebra tiles in making abstract mathematical concepts more accessible. By acknowledging both the current achievements and potential avenues for future research, it becomes evident that algebra tiles offer a promising path towards improved algebraic learning experiences and outcomes.

3. Methodology

3.1. Research Design

This study's research design incorporates action research, a dynamic approach well-suited for investigating practical teaching strategies [27]. Action research involves identifying specific issues in a classroom or work environment and seeking potential interventions. As described by [27], this method entails collecting data to test hypotheses or answer questions about the current subject of study. The selection of action research is driven by the study's focus on a real-life problem. A descriptive analysis was employed to depict various situations.

3.2. Context and Participants

The action study was used in grade 7 classroom setting consisting of 30 students (12 boys and 18 girls) at a public JHS in the East Mamprusi municipality of the North East Region of Ghana.

3.3. Data Collection Procedure

The study's data collection procedure encompassed three phases: Pre-Intervention, Intervention, and Post-Intervention. The primary data source was tests (pre-test and post-test). These two types of assessments were carried out at different stages of the research to gather data and assess the intervention's efficacy.

The pre-test, administered in the first week of the intervention phase, established baseline data before the intervention. This initial data provided a starting point for later analysis and insight into students' algebra problem-solving abilities upon entry. The intervention, involving the use of algebra tiles, took place between the pre- and post-test (weeks two to four). Subsequently, the post-test was conducted to evaluate the intervention's impact, comparing results against the pre-test data. This comparison determined the effectiveness of the intervention devised to address the study's problem.

3.3.1. Pre-Intervention

The initial stage of the study involved a pre-intervention process designed to uncover the reasons behind students' struggles with basic algebraic problems. This was achieved through observation and testing. Basic algebra questions were developed to assess students' prior understanding of algebraic expressions. The goal was to

formally diagnose the challenges within the classroom and gauge the students' grasp of the subject matter. To achieve this, a pre-test was administered, consisting of ten algebraic questions drawn from grade 7 textbooks. These questions covered topics such as simplifying and evaluating variable expressions as well as basic algebraic equations. The entire class of 30 students took part in the test, with a time limit of twenty minutes. Once completed, their workbooks were collected, marked, and scored.

3.3.2. Intervention Phase

Following the pre-test and the subsequent identification of students' difficulties in solving fundamental algebraic problems, the intervention phase was initiated. At this stage, the researchers embarked on a process to address these challenges by introducing algebra tiles as a teaching tool.

The intervention spanned four weeks and aimed to enhance conceptual understanding. Throughout this period, students were educated on the purpose and utilization of algebra tiles in tackling algebraic problems. Over three weeks, three key objectives were met with the aid of these tiles. Students successfully learned to: employ algebra tiles to model algebraic expressions, perform addition and subtraction of algebraic expressions, and solve simple linear equations. These objectives were accomplished through a series of activities outlined below.

Activity 1: Algebraic Expression Modelling

During the first week of the intervention, students acquainted themselves with different algebra tiles, as depicted in Figure 1. Through think-paired-share sessions, the students actively examined the differences and connections existing among these various tiles. Organized into groups of five, each group was tasked with the modelling of specific algebraic expressions (such as $3x - 4y + 4 - 2x - 3 + 2y$) using algebra tiles, as illustrated in Figure 2. Subsequently, chosen representatives from each group showcased their solutions, leading to a comprehensive class-wide discussion.

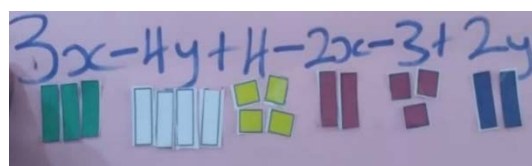


Figure 2. Modelled algebraic expression (Student sample work)

Activity 2: Addition and Subtraction of Algebraic Expressions

Week 2 began with a review of the previous lesson's content. Students were tasked with modelling the expression $2x - 4$, a challenge they successfully accomplished. Working in pairs, students collaborated to select the appropriate tiles needed to model and simplify the expression $8 + 3x - 4y + 2x + 6y - 3 - 5x$. They were encouraged to identify pairs of tiles that could cancel each other out, resulting in a null pair. Most students completed this task accurately; however, two students initially misinterpreted a positive tile as negative, leading to errors in their modelling. This confusion was addressed during a whole-class discussion, and students ultimately arrived at the correct solution of $2y - x + 5$ (see Figure 3).

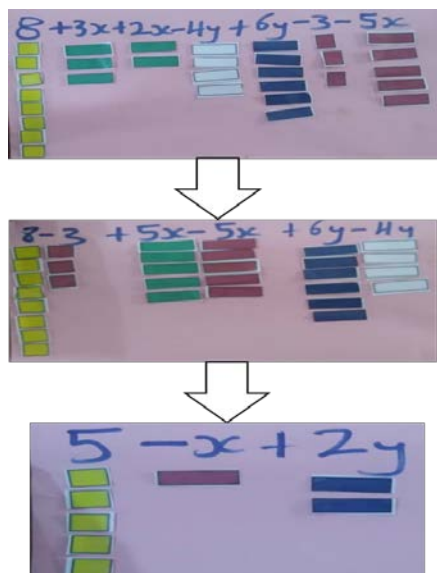


Figure 3. Modelled algebraic expression (Student sample work)

Activity 3: Solving Simple Linear Equations

The third week began with a review of the previous lesson. Students were then divided into groups of three to model and solve the equation $2x - 3 = 5$. Each group modelled the equation (see Figure 4) before exploring methods to eliminate negative unit tiles on the left side of the equation.



Figure 4. Modelled algebraic expression (Student sample work)

Through guidance, students introduced zero pairs by adding 3 negative unit tiles to both sides of the equation (see Figure 5).

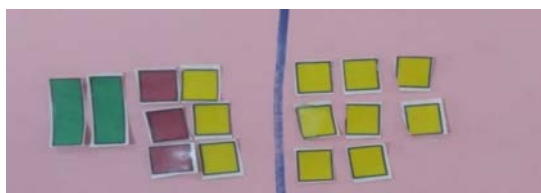


Figure 5. Modelled algebraic expression (Student sample work)

This process eliminated all zero pairs, leaving 2 positive x tiles on the left and 8 positive tiles on the right (see Figure 6).

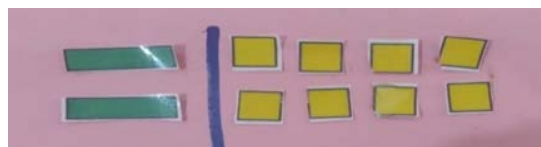


Figure 6. Modelled algebraic expression (Student sample work)

Working together, Students noticed that each positive x tile on the left corresponded to four positive unit tiles on the right. This led to the conceptual understanding that the

solution for the equation $2x - 3 = 5$ was $x = 4$ as shown in Figure 7. To give the students a chance to explore, students were tasked to work through a series of different linear equations.



Figure 7. Modelled algebraic expression (Student sample work)

3.3.3. Post intervention

Following the implementation of the intervention, the researchers administered a test to assess the students' comprehension of algebraic expressions. The evaluation of the test was carried out by the researchers through manual scoring. One point was assigned for every accurate response in both the initial and subsequent tests. The students were provided with a duration of forty-five minutes to complete the task. It was noted that a significant number of students who struggled in the initial test displayed familiarity and contentment with the subject afterward. The outcomes strongly demonstrated an overall substantial enhancement in their performance.

4. Results and Discussion

The pre and post scores on an algebra test for the grade 7 students taught using algebra tiles were analysed using a t-test. The data were coded and inputted into the statistical package SPSS to find out the relationship between the pre-test and the post-test after the use of manipulative aids (algebra tiles) to improve upon the conceptual understanding of the students in solving algebraic expressions and simple linear equations. The analysis and results are presented in Table 1.

Research Question 1: Does the use of algebra tiles as a visual and manipulative tool enhance junior high school students' understanding of basic algebraic concepts?

Table 1. Paired Samples Statistics of students' performance

Variable	Mean	N	Std. Deviation	Std. Error Mean
Pair Pre-Test	4.5666	30	3.23904	.661
Post-Test	12.0333	30	4.13918	.79820

Table 1 above displays the mean, sample size, standard deviation and standard error for the pre- test and post-test performances of students. From Table 1 above, it is seen that the mean score of students before the intervention was (4.5666) which is smaller than that of the Post-test (12.0333). This shows a massive improvement in the performance of students after engaging them with the appropriate manipulative aids (algebra tiles).

Similarly, there is a difference of (0.90) in standard deviation from (4.14) to (3.24). This shows a better performance of students in the post-test. It suggests that

the use of Visual and manipulative aids such as the algebra tiles in teaching algebra concepts improves the performance of the students.

Moving to Table 2 below, it provides insight into the correlation between the utilization of tangible educational aids (specifically, algebra tiles) and the proficiency level of grade 7 students in handling algebraic concepts.

Table 2. Paired Samples Correlations

Variable	N	Correlation	Sig
Pair Pre-Test & Post-Test	30	.746	.000

Paired Sample Correlation of pre-test and post-test of the use of manipulative aids.

From Table 2 above, there is a strong positive correlation between the post-test and pre-test. This also indicates that there is a relationship between the use of manipulative aids (algebra tiles) and achievement in students' performance.

Table 3 below contained the paired differences between the pre-test, where the students learn the use of manipulative aids (algebra tiles) and the post-test, where the students learn algebraic expression after mastering the use of algebra tiles.

Research Question 2: Is there a statistically significant mean difference between the pre-test and post-test scores after the use of algebra tiles?

The following null hypothesis was tested:

H_0 : There is no significant difference between pre-test and post-test scores after the use of algebra tiles.

H_1 : There is a significant difference between pre-test and post-test scores after the use of algebra tiles.

From the table, at a 95% confidence interval estimation of the boundaries between which the true mean difference lies in 95% of all possible interventions for the pre-test and post-test results.

For the pre-test and post-test scores, there is a lower score of 8.21211 and an upper score of 6.30678. The t static for the pre-test and post-test scores was 23.1545. The sig. (2-tailed) the column displays the probability of obtaining a t static whose absolute value is equal to or greater than the absolute t static. In Table 3 above, we see that the significant value for change in performance by the students' conceptual understanding of algebraic expression is less than 0.005.

The post-test recorded high mean performance ($M=12.0333$, $SD=4.13918$) better than the pre-test ($M=4.5666$, $SD=3.23904$). A mean difference of 7.4667 was revealed in favour of the post-test. This finding clearly shows that the performance of students in algebraic

expression in the post-test is better as compared to the pre-test. Hence, at a 5% significance level, there appears to be a significant mean difference of 7.4667 between pre-test and post-test scores after the use of algebra tiles. The researchers reject the null hypotheses.

This research aligns with the findings of previous studies conducted by [14,28,29,14], as well as [30]. These studies collectively demonstrate a noteworthy divergence in outcomes, favouring the experimental group that utilized algebra tiles, when compared to the control group.

The effect size of the intervention involving algebra tiles was computed to assess the magnitude of its impact. The calculated effect size, denoted as $d=2.38$, reflects a substantial effect size.

Based on the aforementioned discoveries and analyses, the researchers reached the conclusion that the apparent subpar performance of grade 7 students at Boxer JHS in understanding algebraic expressions is not a result of chance, but rather stems from the absence of appropriate instructional and learning resources, such as algebra tiles. The observed advancement in students' comprehension and achievement in algebra underscores the efficacy of algebra tiles as valuable teaching aids. The favourable perception of algebra tiles by JHS students underscores their ability to render intricate algebraic concepts more comprehensible. The study's findings emphasize the significance of educators in mathematics augmenting their pedagogical expertise and incorporating suitable instructional tools to facilitate productive learning encounters during mathematics lessons.

5. Conclusions

This study contributes valuable insights into the realm of algebra instruction by demonstrating the efficacy of algebra tiles in enhancing JHS students' comprehension and academic performance. The positive outcomes advocate for the integration of tangible teaching aids to facilitate better understanding of abstract mathematical concepts. Mathematics educators are encouraged to consider innovative approaches, like algebra tiles, to create more engaging and effective learning environments. Further research in this domain could explore the long-term effects of algebra tiles on students' algebraic skills and their transferability to advanced mathematical concepts.

Conceptually oriented teaching in school mathematics is best facilitated by appropriate teaching and learning materials. The students' performance had improved due to the appropriate use of teaching and learning materials, thus, the algebra tiles.

Table 3. Paired Samples Test

Variable	Paired Differences					t	Df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Pre-Test – Post-Test	7.4667	2.1104	.3224	8.2121	6.30678	19.988	29	.000

6. Recommendations

This study highlights the importance of helping grade 7 students at Boxer JHS grasp algebraic concepts and develop a genuine interest in learning math. To achieve this, the Ghana Education Service (GES) is advised to extend support to schools by providing suitable teaching and learning resources. These resources should be easily accessible to both educators and students at the foundational levels, as they lay the groundwork for comprehending mathematical concepts. This understanding is crucial for pursuing math and science disciplines at higher educational levels.

Furthermore, the study suggests that school administrators and supervisors should ensure that teachers incorporate interactive teaching materials when delivering math lessons. This approach aims to enhance students' grasp of mathematical concepts and their ability to apply mathematical principles in everyday situations.

The study also emphasizes the need for regular in-service training for mathematics teachers. This training would keep educators up to date with new teaching methods, strategies, and techniques across various aspects of the subject. Overall, the study underscores the significance of effective teaching practices, accessible learning materials, and ongoing teacher development in fostering a solid foundation in mathematics education.

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