An Ecological Model for Predicting Behaviour of Mediterranean Shrublands

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Abstract In order to build dynamic models for prediction and management of degraded Mediterranean forest areas was necessary to build MARIOLA model, which is a calculation computer program. This model includes the following subprograms. 1) BIOSHRUB program, which calculates total, green and woody shrubs biomass and it establishes the time differences to calculate the growth. 2) SELEGO program, which builds the flow equations from the experimental data. It is based on advanced procedures of statistical multiple regression. 3) VEGETATION program, which solves the state equations with Euler or Runge-Kutta integration methods. Each one of these subprograms can act as independent or as linked programs.

Keywords: allometric equations, biomass, flow equations, multiple regression, state equations, transformed functions

Cite This Article: José Luis Usó-Doménech, Josué-Antonio Nescolarde-Selva, and Miguel Lloret-Climent, Lucía González-Franco, "An Ecological Model for Predicting Behaviour of Mediterranean Shrublands." *American Journal of Systems and Software*, vol. 4, no. 1 (2016): 1-13. doi: 10.12691/ajss-4-1-1.

1. Introduction

Vast areas of primitive Mediterranean forest have become areas of shrublands, considered the final phase of the degradation process, mostly caused by the anthropic action. However, the Mediterranean shrublands play an important role in protecting the soil from the hydride erosion processes, especially in orographic areas characterized by their steep slopes, greatly harmed by irregular precipitation that quite often are local and very intense. Being able to predict the behaviour of shrub ecosystem caused by climatic variations has become a powerful weapon in adopting policies of territorial organization and planning.

The MARIOLA model [16,30,33] is a dynamic model based on the Dynamic of Systems [8], in which differential equations (state equations) are set up hypothetically, and flow variables are obtained from field observations using multiple regression equations [31,33]. The development of the MARIOLA model can predict the effect that the climatic changes of several variables produce on others. This model requires the following conditions:

1) A causal structure based on climatological, ecological and edaphological theories.

2) Functions determining the behaviour of the variables under study.

3) A validation procedure, which compares results obtained by the model and those obtained by experimentation.

The main characteristics of MARIOLA model [12] are.

1) It is deductive.
2) It is compartmental.
3) It is stochastic-deterministic.
4) It is holistic.

5) It is dynamic.
6) It is causal.
7) It is biodemographic and biogeochemical.
8) It is a non-linear model.
9) The model disaggregation is high, in order to be able to study behaviours in morphological and ecophysiological levels.

The MARIOLA model presents two options of calculation:

a) **BMARIOLA** model based on the study of the behavior of plant biomass.

b) **PMARIOLA**, population model that studies and analyzes the behavior of the plant population or the population of its constituent parts (leaves, flowers, fruits, seeds, etc.).

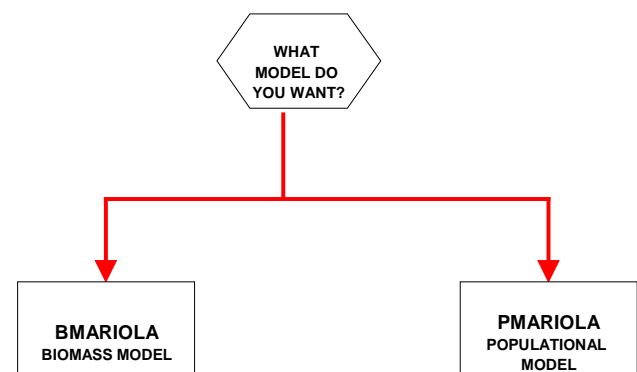The flow chart of the MARIOLA model can be seen in Figure 1.

Figure 1. Flow chart of MARIOLA model

The calculation program was built taking into account the properties sated above. The following are its main subprograms:

1) **BIOSHRUB** program, which builds the formulas for the Mediterranean bush biomass.

2) **SELEGO** program, which builds the flow equations. The growth equations of the total, green and woody biomass can be found as a particular case of flow equations, which forces the construction of a subroutine of growth, in which this is calculated from the simple difference between the existing biomass in period I and in period i+1.

3) **VEGETATION** program, which is the numeric resolution of the flow equations and the resolution of the system of first order ordinary differential equations using the Euler method. An option is to integrate the Runge-Kutta method if necessary for more precision, most of all when the model disaggregates to levels of usage of ecophysiological variables. The starting variables are obtained by Montecarlo simulation.

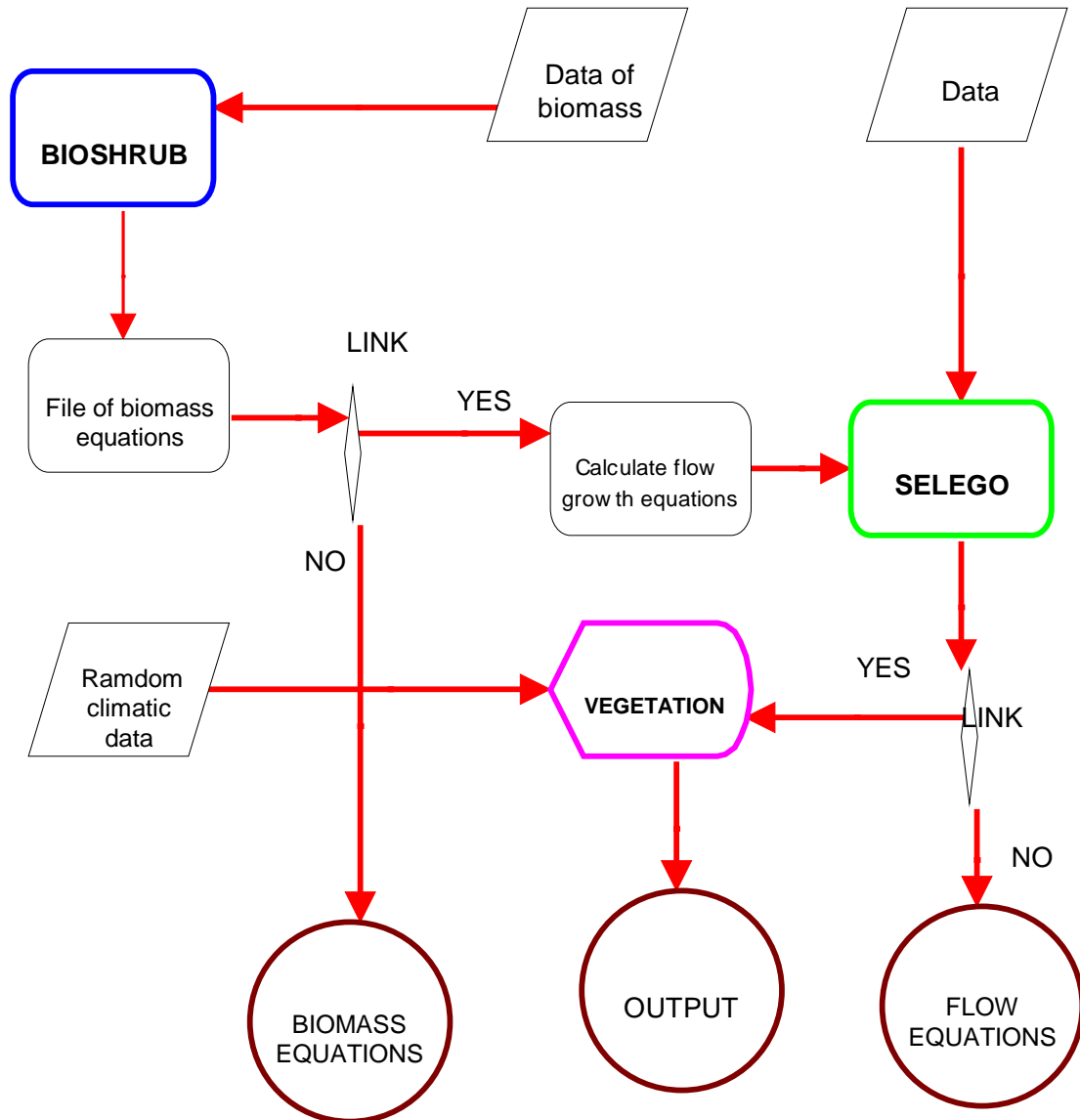The flow chart of the BMARIOLA model can be seen in Figure 2.

Figure 2. Flow chart of BMARIOLA model

The flow chart of the PMARIOLA model can be seen in Figure 3.

The aim of this work is to present, analyze and describe each one of the three subprograms defining MARIOLA model, the main computer program. Results of the application of these subprograms to real data are briefly presented throughout the paper.

2. Study Area

To carry out the experimentation and validation we had to choose two different zones with similar characteristics, which are considered to be representative of bush ecosystems in the Mediterranean mountain zone. The parameterization plot is located in the proximity of the village of Agres, in the Serra de Mariola (Province of Alicante, Spain) UTM = 30SYH19, height 850 m, with the following climatic characteristics based on 16 years of observation, mean annual temperature 14.1 ℃, temperature of the coldest month (January) 6.5 ℃, temperature of the warmest month (August) 23.1 ℃, pluviometer annual mean 600 mm, mean of potential evapotranspiration 757 mm. The geological substratum is of the cretaceous limestone, over which calcareous platforms are situated of the regololithic type with a reforestation of *Pinus halepensis.*
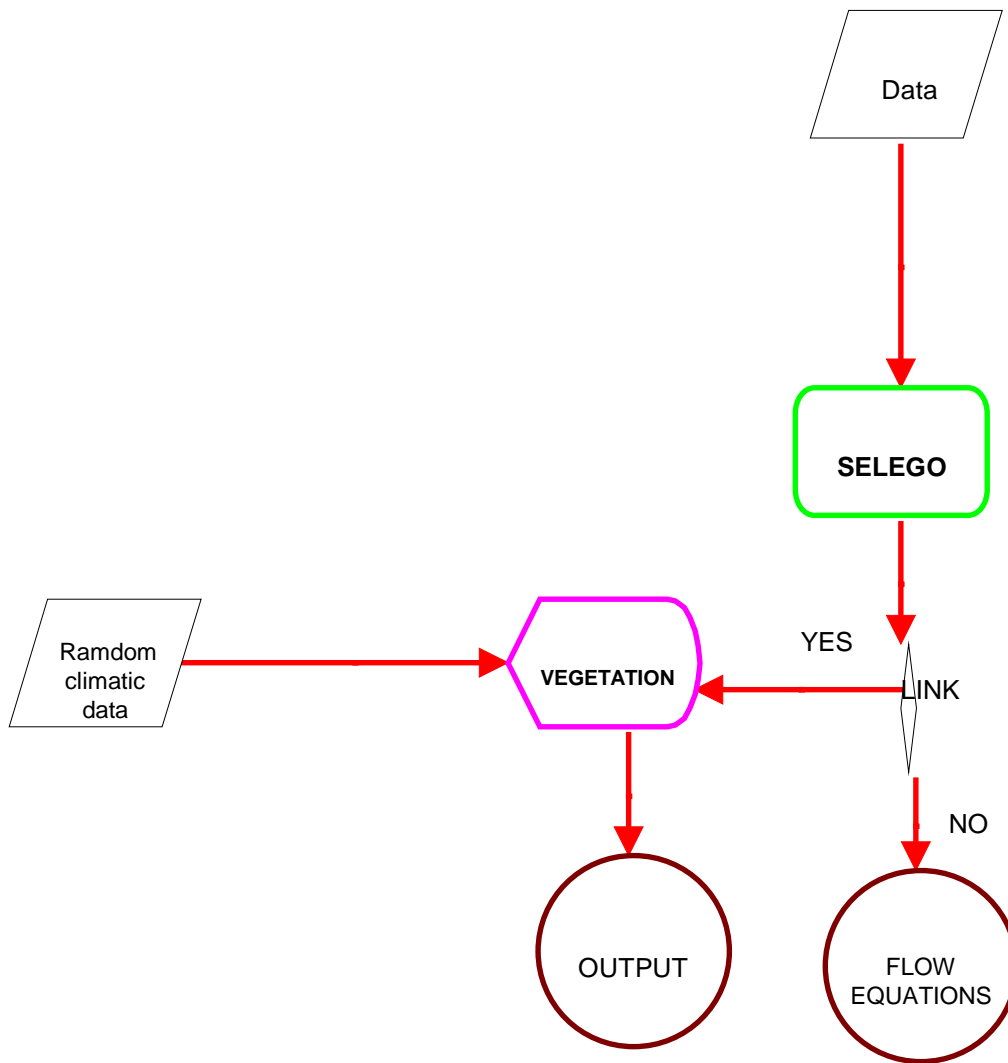
Figure 3. Flow chart of PMARIOLA model

Figure 4. Sierra de Mariola

The Serra de Mariola (Figure 4) is located within the community of Valencia, north of Alicante and south of Valencia, last Betic reliefs. It includes the municipalities of Banyeres de Mariola, Bocairent, Alfafara, Agres, Muro del Comtat, Cocentaina, Alcoi, Onil and Biar. The mountain is one of the last foothills of the Baetic Mountains [9]. It has a quadrangular form and an alignment from southwest to northeast. The predominant material is limestone, although the intense tectonic activity has caused the existence of very varied landscapes. The maximum height of the mountain is the Montcabrer, with 1390 m, the third in the province of Alicante. Besides this,

there are numerous mountains over a thousand meters, as the Alto de Mariola (1.158 m), el Contador (1,232 m) and El Portín (1,081 m). This mountain has a Mediterranean climate with a slight continental shade due to the relative existing medium altitude. It has cold winters where temperatures can drop to -15 °C in the highest areas (1000 meters or more). Summers are hot, with temperatures that can exceed 35 and even 40. Rainfall varies between 350 mm and 900 mm per year respectively (very irregular). This mountain is remarkably rich in vegetable varieties. Among them is the presence of yew. In microreserve of *Teixeira d'Agres* we can see the forest of Europe's southernmost yew, but is currently not as abundant as in the past. Most of the mountain is covered by a forest formation where the pine is predominant along with various shrubs such as gorse, rosemary and rockrose. However, it is also possible to find Mediterranean mixed forest, where species such as oak intermingle with much less common deciduous trees in Valencia such as ash, maple or gall as tree species and the presence of butcher's broom and honeysuckle shrubs. But undoubtedly the true wealth of the mountain is located in the floristic diversity. They have come to identify 1,200 species, many of them endemic to Valencia or saw it. Among them they could note Mariola sage, cat's tail, the edge chamomile, piperela, lavender, santonica, hypericum tea or rock. Due to the variety of environments the mountains has a rich fauna. Among the reptiles are highlighting the Iberian lizard or ashen lizard, the eyed lizard, snake or a viper vipers. Among the birds can be found Granivorous as partridge, or Serin Finch; insectivorous as the robin, tit or woodpecker or prey such as the golden eagle, goshawk, sparrow hawks, kestrels, owls, long-eared owl, the tawny owl, the griffon vulture and the red kite. Mammals are also abundant highlighting the rabbit, marten, weasel, genet, badger, fox and wild boar. It also highlights some insects like water fleas and mosquitoes.

The validation zone corresponds to the Font Roja Natural Park, in Alcoi, situated in the Serra del Menejador (Province of Alicante, Spain), UTM = 30 SYH 18, height 800 m. The climate characteristics obtained from 18n years of observations are the following: mean annual temperature 14.7°C, mean temperature of the coldest month (January) 7.6°C, mean temperature of the warmest month (August) 27.8°C, pluviometer annual mean 449 mm, mean of potential evapotranspiration 771 mm. Geologically the material of the sub-soil are marls of the Keuper type, over which calcareous platforms of the regolithic type are situated with an artificial repopulation of *Pinus halepensis* mixed with fragments of potential *Quercus rotundifolia* forest.

10 shrubs differing in size, shape and structure were selected as representative of Mediterranean ecosystem: *Bupleurum frucicescens L., Ulex parviflorus Pourret, Helychrysum stoechas (L.) Moench, Rosmarinus officinalis L., Lavandula latifolia Miller, Sedum sediforme (Jacq.) Pau, Genista scorpius (L.) DC, in Lam.&DC, Marrubium vulgaris L., Thymus vulgaris L, and Cistus albidus L.*

They are common plants [3,28] which play an important role in the shrub communities of the Western Mediterranean region, especially during the first ten years after a forest fire.

The experimental and validations zones were of 100 square meters.

3. The BIOSHRUB Program

The BIOSHRUB computer program is a software system which generates allometric equations for Mediterranean shrubs.

3.1. Theoretical Foundations

In order to build the dynamics of Mediterranean bushes, the calculation of biomass growth was definitely necessary. If bg is the biomass (B) growth for period (t_{i+1}-t_i), then:

$$BG = B(t_{i+1}) - B(t_i), i = 1, 2, ..., n. \qquad (1)$$

For this reason it was necessary to know the biomass (B) for each time period (monthly). Nondestructive estimation of biomass is a difficult problem in ecological studies. They have been proposed some methods of estimation [1,13,20,21,23,29], where biomass is calculated based on indirect measures, such as basal diameter. This procedure is used in measuring the biomass of trees. Other measurement methods such as those based on the length of the sheets may be more desirable in shrubs and herbaceous species. So specify Hussey and Long [10] Mark [15]; Scott [26] Measurement of biomass in alpine plants has been studied by Bliss [4]. More sophisticated non-destructive methods have been developed by Boutoon Tieszen [5], Jensen [11], Kumar Monteith [14], Mayhew et al. [17], Steven et al. [27], which they have been based on the reflectance ratio. Chapman [6] in electrical capacitance. Westman Paris [42] using radar techniques.

The most common method to determine the biomass and the production, it is through the use of regression equations allometric. Huxley was the first to introduce the concept of allometry. It is usually expressed as the ratio between the relative rate of growth of an organ, and other organ of same organism. These allometric regressions develop measuring biomass production or whole plants or their components, and correlating these data with other components more easily measured as the diameter, at the height of human breast [22]. Many of the allometric formulas are referred to trees, usually due to economic reasons. Regarding the Mediterranean species, Canadell et al. give Allometric equations for *Quercus ilex L.* However, in the literaturewe do not know Allometric equations for Mediterranean shrub species except Usó, Mateu, Karjalainen and Salvador [32]. The method proposed by the authors is based on the estimate of the biomass (B) as a function of the apparent volume of the plant (V_{ap}). To measure the apparent volume is based on the hypothesis that the plant is a hypothetical geometric figure. Chapman [6] assumes that the volume of a tree trunk, has the shape of a paraboloid of rotation. Woodwell and Whittaker (1968) observed that the volume of the parabola was quite suitable for determining the volume of the bushes, being the radius measured at the base. Azmi, Razali, Mohd and Ahmad, 1991, propose the relationship between biomass and cylinder for Malaysian bamboos.

BIOSHRUB offer a simple method, suitable for shrubs, which is based on the estimation of the aerial biomass (B) as a function of the apparent volume (V_{ap}), that is the volume of a cylinder, bush apparent form. The cylindrical shape is very appropriate for shrubs and yields average deviations of less than 9% between estimated biomass and measured biomass. Both measurement were obtained directly in the field during months of October, November and December, when the shrubs al all stated species had no flowers and fruits, using a 1.5 m-long ruler with 0.01m divisions. After taking measurements, the plant was cut at soil level and dried to constant weight. The real biomass was obtained by simple weighing the dried plant on a monoplate balance (0.1% precision). To obtain the parameters, 30 samples of every taxon were studied and 30 different samples for the validation process.

We are two hypotheses:

a) First hypothesis

The apparent volume is the volume of the circular cylinder defined by its height (h) and its diameter (Figure 5)
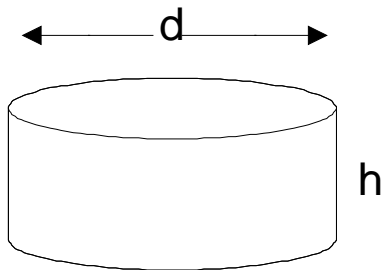

Figure 5. Apparent volume. First hypothesis

and given by

$$V_{ap} = \frac{\pi d^2 h}{4} \qquad (2)$$

b) Second hypothesis

The apparent volume has been considered as the volume of the elliptical cylinder defined by its height z and its axes x,y (Figure 6)
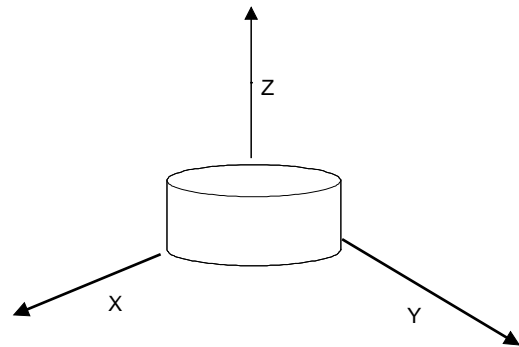

Figure 6. Apparent volume. Second hypothesis

$$V_{ap} = \pi xyz \qquad (3)$$

Both measurement were obtained directly in the field during months of October, November and December, when the shrubs al all stated species had no flowers and fruits, using a 1.5 m-long ruler with 0.01m divisions. After taking measurements, the plant was cut at soil level and dried to constant weight. The real biomass was obtained by simple weighing the dried plant on a monoplate balance (0.1% precision). To obtain the parameters, 30 samples of every taxon were studied and 30 different samples for the validation process.

For all taxes studied the highest correlation was obtained for the exponential equation

$$B = ae^{bV_{ap}} + c \qquad (4)$$

Where a, b and c are coefficients. Coefficient b is calculated by an iterative process and it varies from -50 to 50 in intervals of 0.1. Correlation coefficient is higher than or equal to 0.9. Table 1 shows the validation of the results, with a fairly insignificant deviation with respect to the real results (Table 1).

Table 1.

Taxon	d	d int. 95%	PM	D	D int. 95%	s
Bupleurum frucicescens	5.10	[-5.09, 15.28]	38.9	13.39	[-13.34, 40.12]	3.67
Ulex parviflorus	-4.58	[-17.30, 8.14]	63.75	-7.18	[-27.14, 12.77]	4.58
Helychrysum stoechas	1.92	[-11.88, 15.51]	128.75	1.47	[-9.59, 12.53]	4.93
Rosmarinus officinalis	0.08	[-16.26, 16.42]	111.42	0.07	[-14.60, 17.74]	5.89
Lavandula latifolia	-6.90	[-18.68, 4.88]	101.07	-6.83	[-18.48, 4.83]	4.24
Sedum sediforme	0.96	[-0.62, 2.54]	5.24	18.32	[-11.75, 48.39]	0.57
Genista scorpius	-14.40	[-40.07, 11.27]	165.96	-5.62	[-24.14, 6.79]	9.25
Marrubium vulgaris	-2.96	[-25.09, 20.12]	135.81	-2.19	[-19.21, 14.83]	8.33
Thymus vulgaris	6.44	[-4.87, 17.75]	49.4	13.04	[-9.86, 35.93]	4.07
Cistus albidus	-1.46	[-14.83, 11.91]	100.51	-1.45	[-14.76, 11.85]	4.83

d = average difference between biomasses.
PM = average biomass.
D = 100d/PM
s = standard deviation of the differences.
d int. 95% and D int. 95% 0 intervals of d and D estimated considering a grade of significance of 95% if t-Student of 2.776.

Table 2 presents the results for ten species.

Table 2.

Taxon	c	c int.95%	a	a int.95%	b	r
Bupleurum frucicescens	-37150.3	[-40159.8,-34140.5]	37164.0	[34156.4, 40171.6]	0.01	0.98
Ulex parviflorus	-825.5	[-974.2, -678.7]	846.4	[711.2, 981.5]	0.50	0.95
Helychrysum stoechas	-401.7	[-447.7, -355.6]	422.7	[383.3, 462.1]	1.00	0.93
Rosmarinus officinalis	-367.0	[-418.1, -317.6]	381.8	[338.9, 424.7]	2.00	0.98
Lavandula latifolia	9.6	[0.9, 18.9]	19.6	[17.9, 22.0]	10.00	0.96
Sedum sediforme	144.2	[105.0, 183.5]	-141.9	[-181.7, -101.9]	-7.00	0.85
Genista scorpius	504.7	[434.8, 574.6]	-485.7	[-567.2, -404.3]	-1.00	0.95
Marrubium vulgaris	591.5	[475.5, 707.4]	-581.4	[-718.5, -444.4]	-1.00	0.93
Thymus vulgaris	46249.4	[34218.4, 58268.4]	-46247.3	[-58282.3, -34212.5]	-0.05	0.89
Cistus albidus	54752.7	[46703.9, 69901.6]	-54736.0	[-63797.2, -46674.8]	-0.01	0.98

Values of the parameters of the equation (4) giving the highest correlation factor. Intervals of c and a are obtained with an estimation of 95%.
r = correlation coefficient.

Table 3 contains Allometric formulas of some Mediterranean shrubs

Table 3. Allometric formulas of the BIOSHRUB subprogram

Taxa	Allometric equation	r
Ulex parviflorus	$B = 846.4e^{0.50V_{ap}} - 825.5$	0.95
Helychrisum stoechas	$B = 422.7e^{1.0V_{ap}} - 401.7$	0.93
Rosmarinus officinalis	$B = 381.8e^{2.0V_{ap}} - 367.0$	0.98
Lavandula latifolia	$B = 19.6e^{10.0V_{ap}} - 9.6$	0.96
Sedum sediforme	$B = -141.9e^{-7.0V_{ap}} + 144.2$	0.85
Genista scorpius	$B = -485.7e^{-1.0V_{ap}} + 504.7$	0.95
Marrubium vulgaris	$B = -581.4e^{-1.0V_{ap}} + 591.5$	0.93
Thymus vulgaris	$B = -4627.3e^{-0.05V_{ap}} + 46249.4$	0.89
Cistus albidus	$B = -54736.0e^{-0.01V_{ap}} + 54752.7$	0.98
Salvia mariolensis	$B = 334.7e^{3V_{ap}} - 305.6$	0.95
Phlonis lygnitis	$B = -5112.8e^{-1.0V_{ap}} + 5113.8$	0.95
Brachipodium retrisum	$B = 60.4e^{10V_{ap}} - 59.2$	0.90
Scabiosa turolensis	$B = -558.9e^{-2V_{ap}} + 559.0$	0.97
Sanguisorba rupicola	$B = 1732.3e^{0.5V_{ap}} - 1731.9$	0.95
Ononis minutissima	$B = -26.7e^{-10V_{ap}} + 27.5$	0.88
Erica multiflora	$B = 6.9e^{3V_{ap}} - 7.1$	0.91
Rubia peregrina	$B = 20.4e^{8V_{ap}} - 19.6$	0.93
Erinacea antyllis	$B = -145106.1e^{-0.05V_{ap}} + 145095.3$	0.99
Daphne gnidium	$B = 3486.8e^{0.1V_{ap}} - 3495.6$	0.99
Bupléurum fruticescens	$B = 37164.0e^{0.01V_{ap}} - 37150.3$	0.98

r = correlation coefficient.

Function of global biomass (4) is the result of the integration of the differential equation

$$\frac{dB}{dV_{ap}} = b\left(V_{ap} - c\right) \qquad (5)$$

This approach seems convenient for some purposes, especially under field conditions, as it provides a way of preventing the irregular characteristics of environmental factors. This equation suggests that the increase in the biomass of a plant is proportional to the difference between the present biomass and its maximum or minimum value. Therefore "a + c" represents the initial biomass, i.e. the biomass corresponding to an apparent volume equal to zero. It is evident that "a + c" must be

zero in all cases, but the results obtained (Table 2) show that because of experimental errors and deviations in the fitting of the function this is not true. It is also important to note that the sign of "a" must be the same as the sign of "b", because if not $\dfrac{dB}{dV_{ap}}$ is negative, and this does not make sense biologically. It is noted, if we make the derivative of (4), that

$$B' = abe^{bV_{ap}} \qquad (6)$$

and

$$B'' = ab^2 e^{bV_{ap}}. \qquad (7)$$

Consequently the curvature radius (R) will be:

$$R = \frac{\left[1 + a^2 b^2 e^{2bV_{ap}}\right]^{\frac{3}{2}}}{a^2 b e^{bV_{ap}}} > ab^2 e^{2bV_{ap}}. \qquad (8)$$

For positive value of "a" and "b" the defined curve will be concave, and the higher positive values are, the higher the curvature radius is for a given V_{ap} and consequently the closer to a straight line the corresponding function is. On the other hand, if "a" and "b" are negative, the curve will be convex, and the more negative these values are, the higher the curvature radius and consequently the closer to a straight line the corresponding function will be.

The biological interpretation of this is the following [32]:

a) For positive values of "a", the higher the apparent volume is, the higher is the apparent density $\dfrac{B}{V_{ap}}$.

b) For the negative values of "a" and "b", the higher the apparent volume, the lower the apparent density.

Therefore "a" and "b" have similar biological interpretation, but "b" has a greater influence on the equation because it is in exponential form. So, the parameter "b" can be considered as a plant which shows the development of the apparent density $\dfrac{B}{V_{ap}}$ with respect to the apparent volume. Parameter "c" has a different interpretation depending on whether the defined curves are concave (a > 0, b > 0). In the form case "c" represents the asymptotic values of an exponential curve and does not have biological sense. In the latter "c" is also the horizontal asymptotic value, but in the case of a concave curve it can be interpreted biologically as the maximal biomass which the plant can accumulate.

In the case of second hypothesis we derive biomass B respect to three directions of space (x, y, z):

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial V_{ap}} \frac{\partial V_{ap}}{\partial x} = b(B - c)\pi yz$$

$$\frac{\partial B}{\partial y} = \frac{\partial B}{\partial V_{ap}} \frac{\partial V_{ap}}{\partial y} = b(B - c)\pi xz \qquad (9)$$

$$\frac{\partial B}{\partial z} = \frac{\partial B}{\partial V_{ap}} \frac{\partial V_{ap}}{\partial z} = b(B - c)\pi xy$$

The solution of the system of differential equations is.

$$B = \pi b(B - c)xyz \qquad (10)$$

or

$$B = b(B - c)V_{ap} \qquad (11)$$

being

$$d_{ap} = \frac{B}{V_{ap}} = b(B - c) \qquad (12)$$

the apparent density. It seems possible to consider the value of "b" as the main indicator of the type of curve. We denominate it as *coefficient of intrusivity [32]*. Observing the results obtained for the same period of time (Table 2), the extreme value of "b" are 10 and -7, and for the most of these cases the values lies between 1 and -1. Value of b = 10 correspond to Lavandula *latifolia Miller*, which possesses a high proportion of lignified tissue. At other extreme, the value of b = -7 corresponds to *Sedum sediforme (Jacq.) Pau*, a succulent plant, with a high proportion of water stored in their tissues. It is possible to establish two strategies in the aerial biomass accumulation process.

1) An *intrusive strategy* with high positive values of b.

2) An *extrusive strategy* with high negative values of b.

The two strategies would correspond to response of plant to climatic factors.

3.2. Program Characteristics

BIOSHRUB program has the following characteristics:

1) Each variable, height (h) and diameter (d) represents an attribute of the object under study.

2) Variable biomass (B) has been selected as a dependent variable with the other one (apparent volume V_{ap}) considered as independent.

3) The program carries out successive runs (trials) with a set of possible ways of constructing the allometric function [7,32]

4) If a correlation coefficient R_1 is obtained with an allometric formula, another allometric formula can be found to vary the b coefficients, in which correlation coefficient R_2 will be higher than R_1.

5) When the program finds the higher coefficient R_1, it prints results in a specific file.

The flow chart of the BIOSHRUB is the following (Figure 7).

Once we get the allometric equations, BIOSHRUB program allows for the necessary interpolations taking as input variables only the maximum height (h) and width (d) of the plant.

The program has two main parts: one is the calculation of the allometric formula if this still unknown and the other one is to obtain the estimated biomass under each of the different hypothesis explained. The program has an additional file in which the name of each plant and its allometric equation are recorded. Figure 8 shows the flow chart of the process.

Biometric file saves the allometric shrub equations for further use, either independently or linked to the rest of MARIOLA through the growth subroutine.
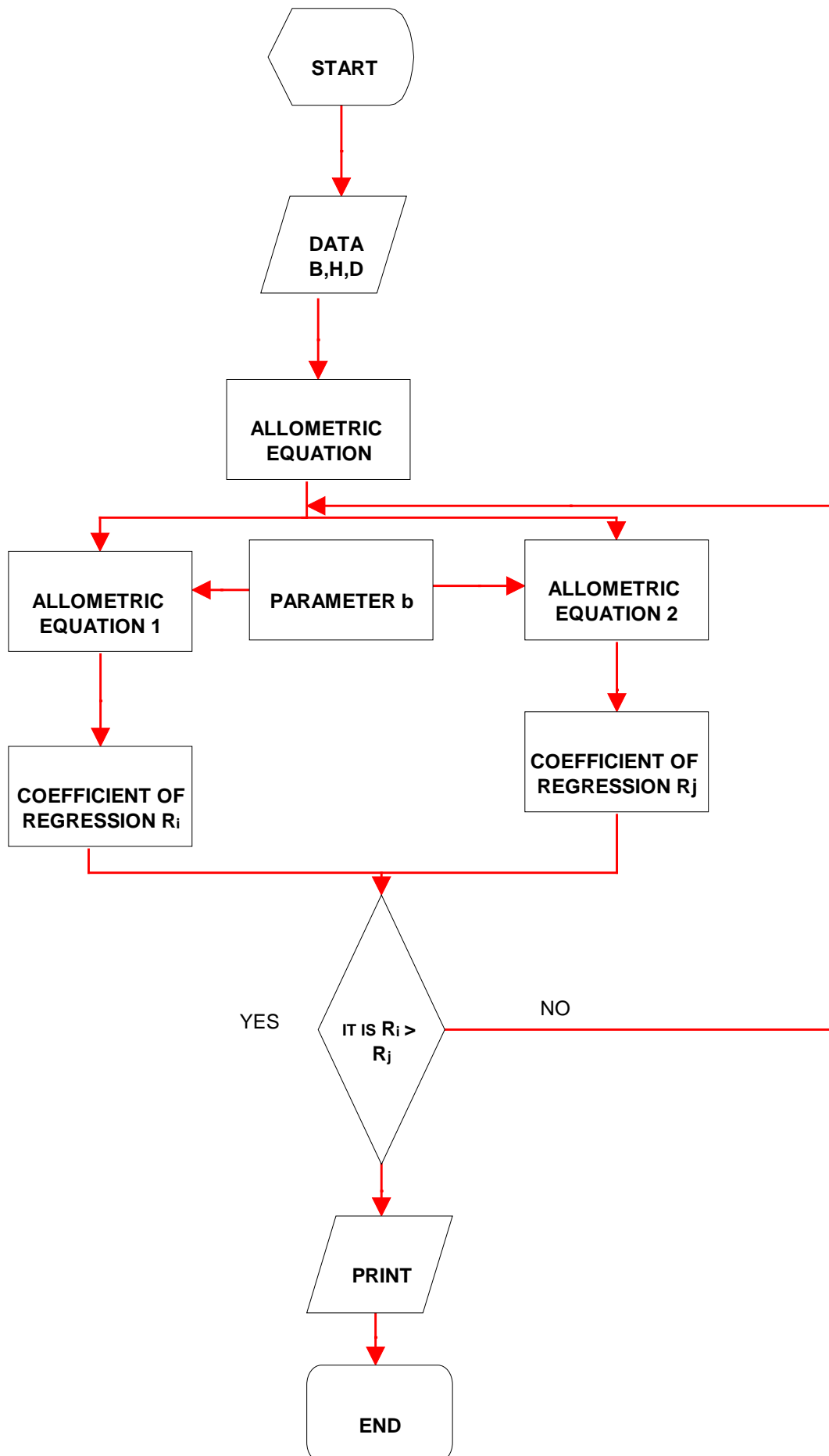
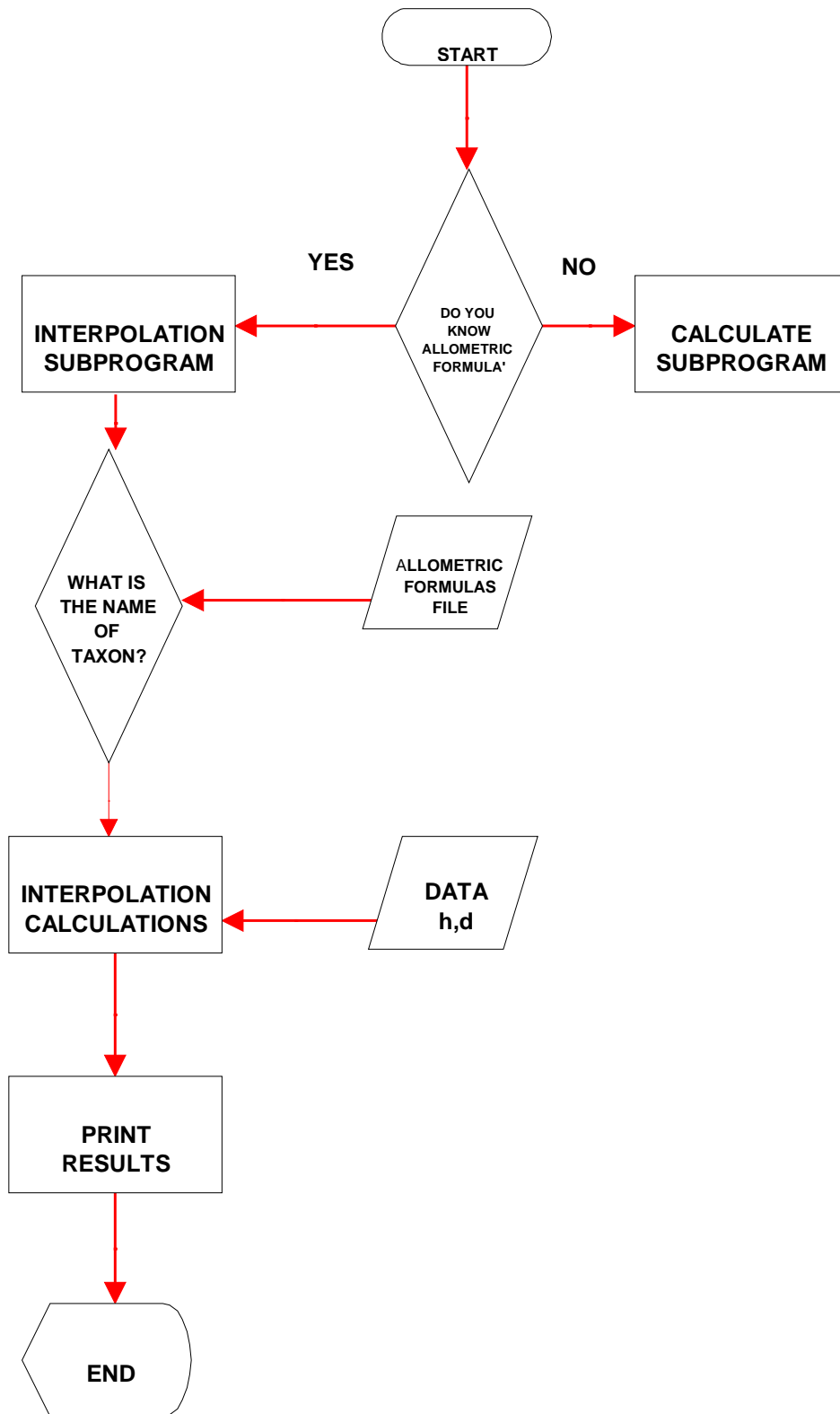**Figure 7.** Flow chart of BIOSHRUB program

Figure 8. Flow chart of the process of interpolation

4. The Growth Subroutine

If BG is the biomass growth for the period (t_{i+1}-t_i), then we get the next equation [33]:

$$BG = B(t_{i+1}) - B(t_i), i = 1, 2, ..., n \qquad (13)$$

Expressions of growth equations for the Mediterranean bush taxons are [18,19,30,31,32,33]:

$$GBG = B(a_1 T + b_1 H + c_1) + d_1 PLU + f_1 \qquad (14)$$

$$WBG = a_2 B + b_2 B \cdot H + c_2 B \cdot PLU + d_2 \qquad (15)$$

$$B = GB + WB \qquad (16)$$

where a_i, b_i, c_i, d_i,…(i = 1,2,…,n) are parameters, GBG is the rate of production by growth of the green biomass (in grams, g), WBG the rate of the woody biomass (g), B the total biomass (g), GB the green biomass (g), T the

environmental temperature (ºC), H the environmental humidity (%) and PLU the precipitation (in liters, l).

Results of recalling (4), (13), (14) and (15).

$$GBG = GB(t_{i+1}) - GB(t_i)$$
$$= \left(ae^{bVap(t_{i+1})} + c \right) - \left(ae^{bVap(t_i)} + c \right) \quad (17a)$$
$$= a \left(e^{bVap(t_{i+1})} - e^{bVap}(t_i) \right)$$

$$WBG = WB(t_{i+1}) - WB(t_i)$$
$$= \left(\alpha e^{\beta Vap(t_{i+1})} + \gamma \right) - \left(\alpha e^{\beta Vap(t_i)} + \gamma \right) \quad (17b)$$
$$= \alpha \left(e^{\beta Vap(t_{i+1})} - e^{\beta Vap(t_i)} \right)$$

where

$$GBG = a \left(e^{bVap(t_{i+1})} - e^{bVap}(t_i) \right)$$
$$= B(a_1 T + b_1 H + c_1) + d_1 PLU + f_1 \quad (18a)$$

$$WBG = \alpha \left(e^{\beta Vap(t_{i+1})} - e^{\beta Vap(t_i)} \right)$$
$$= a_2 B + b_2 B \cdot H + c_2 B \cdot PLU + d_2 \quad (18b)$$

Equations 18a and 18b are considered flow equations and they are the VEGETATION program input.

5. The Selego Subprogram

In the classic method of Dynamic of Systems [8], non linear process are built through tables (graphics). The process is carried out by lineal or polynomial interpolations. The authors replace this method with the construction of equations based on the lineal combination of transformed functions [7,18,19,24,25,34,35,36,37,39,40,41].

We consider a particular case based on transformed functions of order 0 and 1 of the SELEGO program which generates well adjusted functions (flow variables) from field data. A generator and adjuster of non-linear functions within the numerical range of the variables have been used. By means of an iterative process, the program looks for the best fit with data among a set of possible functions. SELEGO computer program has the following characteristics:

1) Each variable considered represents an attribute of the object under study.

2) One variable has been selected as a dependent variable with a set of others being considered as independent.

3) The program carries out successive runs (trials) with a set of possible ways of building the function. These functions will always have the following generic form:

$$\Psi = \sum_k \sum_L T_k^i T_L^i D_{kL} \quad (19)$$

where D_{kL} are the coefficient that have to be determined and T_k^i, T_k^i are the transformed functions.

4) If, with a lineal combination of n-transformed functions, a correlation coefficient R_1 is obtained, one can discover another lineal combination of n+1 transformed functions [37,39], in which the correlation coefficient R_2, will probably be $R_2 > R_1$.

5) When it is a question of discovering one function with n transformed functions from m variables with n < m, the variables will always be the most significant. The flow diagram of the SELEGO program can be observed in Figure 9.
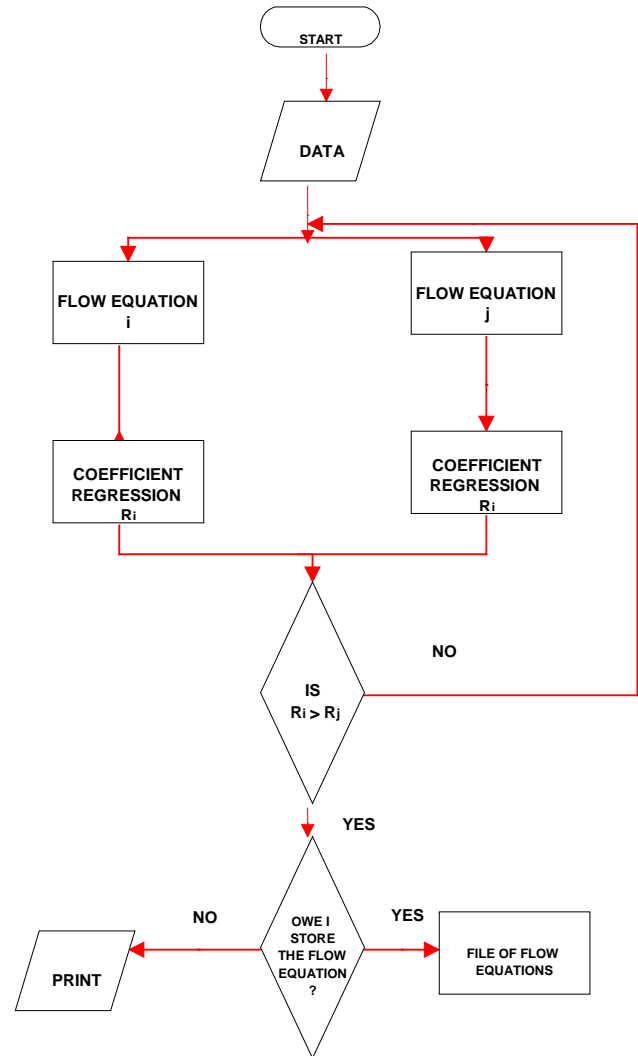

Figure 9. Flow chart of SELEGO program

6. The Vegetation Subprogram

The VEGETATION program is based on the theory of transparent boxes from the General Theory of Systems. This expert system presents the following characteristics:

1) The program asks for information about the name of the model (MARIOLA) and the full name of the already defined variables. The codification process is completely automatic.

2) The program asks for information about the quality dependencies that exist between variables.

3) The simple and multiple dependencies are introduced, and if they not table like, they can be expressed as an equation like a sentence.

4) The program asks for information about the refeeding connections.

5) Last of all, the instructions are given, to operate with the built model.

6) The type and range of each variable is not specified because all of them are real and vary from $+\infty$ to 0.

The VEGETATION program carries out a calculation sequence that allows the obtainment of the evolution of the behaviour of studied attributes at the moment that the system is modeled. The VEGETATION model builds three information programs for each model: the input program, the calculation program and the output program. We can summarize the mechanism of the VEGETATION program in Figure 10.
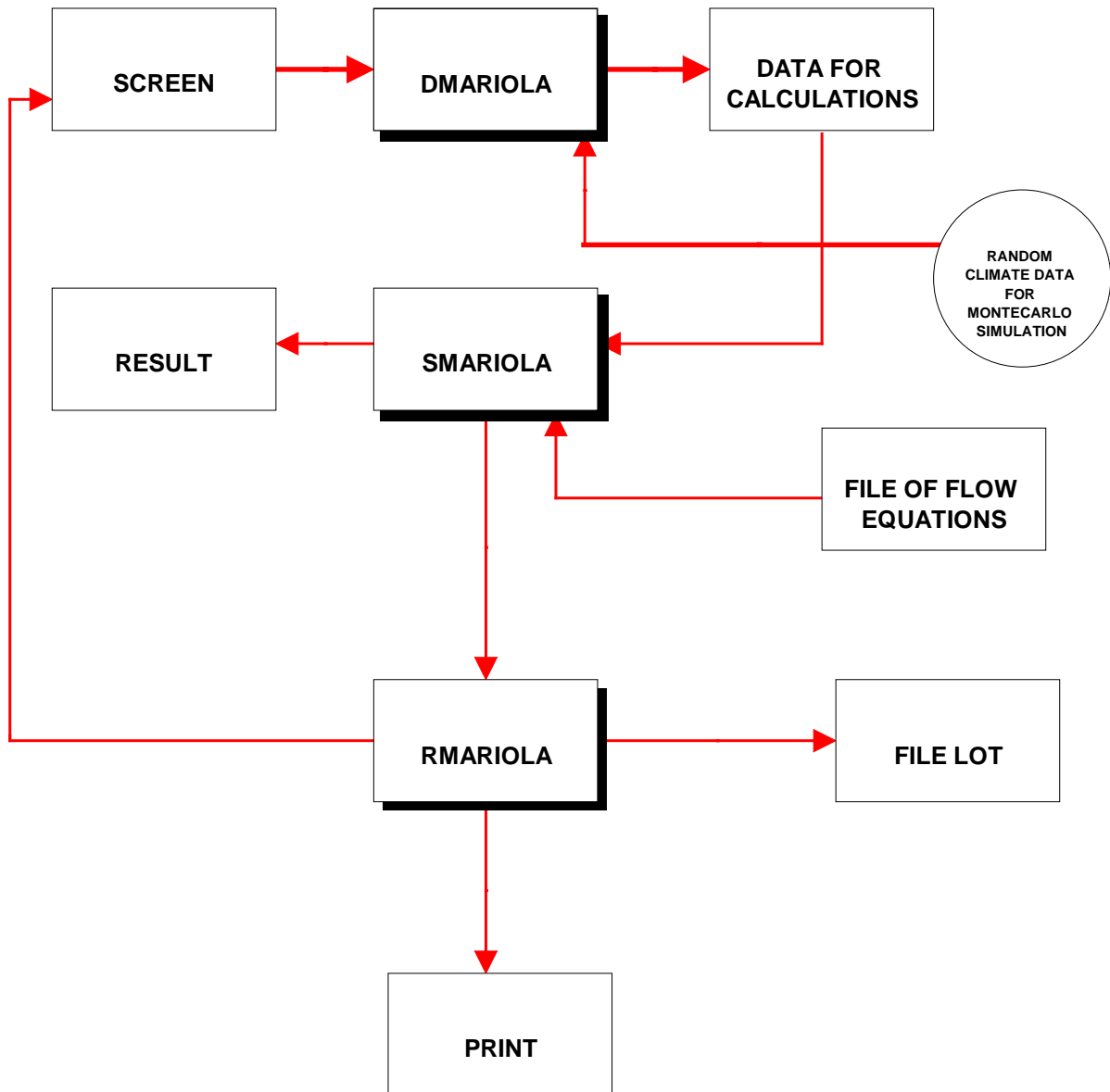

Figure 10. Flow chart of the VEGETATION program

1) The input program DMARIOLA asks for the values of the input variables and it prepares the input file for the operative program.

2) The operative program SMARIOLA obtains data from the input file and it creates an output file.

3) The output program RMARIOLA asks for some data about the experimental connection, the election of the mechanism on the form of the desired relationship, i.e., the number of desired periods, the number of related variables, the format of the graphics or tables, exit through the screen or printer, etc.

6.1. Theorical Foundations

We assume that the dynamics of the system can be modeled starting off with a set of ordinary differential equations as follows:

$$\frac{dy_i}{dt} = F(x), \overline{x} = x(t), t \geq 0; \overline{x}(0) = x_0, \forall j \qquad (20)$$

$$x.[0,+\infty] \rightarrow R^n; y(t) = F\big(x(t)\big); F : R^n \rightarrow R$$

where R^n is the phase space, t the time and y is the state variable.

Hence we start off with a system of non-linear differential equations

$$\frac{dy_i}{dt} = \sum_{i=1}^{n} x_{ij}, \quad \forall j = 1, 2, ..., n \qquad (21)$$

where x_{ij} are the flow variables which produce the state variable y_j. The flow variables determine the variations of the system states and characterize the actions that are

taken in it, which are accumulated in the corresponding levels or states. The flow variables determine how the available information is converted into an action. Equations that define the behaviour of the system are associated with a flow variable. These equations associated with a flow variable receive the name of *flow equations* or *decision equations*. These equations represent the biological, chemical and physical processes in the ecosystems. They are the relationship between the external variables (forcing functions) and state variables.

Each one of the flow variables can depend either on the input variables or on state variables. We will call z the set formed by the state and input variables and we will identify it as an open subset of R^n. it is possible to write it as follows:

$$\forall x_{ij}, x_{ij} = f_{ij}\left(z_1(t), z_2(t), ..., z_n(t)\right) \qquad (22)$$

$$z_i : [0, +\infty] \rightarrow z \in R^n, \left(f_{ij} : R^{n^2} \rightarrow R^n\right).$$

Our goal is to express every x_{ij} as a linear combination of transformed functions, so that they adjust to the model studied through linear regression.

For convenience, the transformed functions of order 0 are expressed by $T^1(z_r)$. The transformed function of order 1 by T^2 where $T^2(z_r, z_s) = z_r \circ z_s$. In general, a transformed function of order k-1, which is a composition of $z_{i1}, z_{i2}, ..., z_{ik}$, is expressed as

$$T^k\left(z_{i1}, z_{i2}, ..., z_{ik}\right) = z_{i1} \circ z_{i2} \circ ... \circ z_{ik} \qquad (23)$$

And

$$x_{ij}(t) = \sum_{r=1}^{n} c_r^1 T^1(z_r) + \sum_{r=1}^{n}\sum_{s=1}^{n} c_{rs}^2 T^2(z_r, z_s)$$
$$+ \sum_{r=1}^{n}\sum_{s=1}^{n}\sum_{h=1}^{n} c_{rsh}^3 T^3(z_r, z_s, z_h) + ... \qquad (24)$$
$$+ \sum_{r=1}^{n}\sum_{s=1}^{n}...\sum_{u=1}^{n} c_{rs...u}^p T^p(z_r, z_s, ..., z_u) + ...$$

Using the statistical regression methodology

$$x_{ij}(t) = \sum_{r=1}^{n} c_r^1 T^1(z_r) + \sum_{r=1}^{n}\sum_{s=1}^{n} c_{rs}^2 T^2(z_r, z_s)$$
$$+ \sum_{r=1}^{n}\sum_{s=1}^{n}\sum_{h=1}^{n} c_{rsh}^3 T^3(z_r, z_s, z_h) + ... \qquad (25)$$
$$+ \sum_{r=1}^{n}\sum_{s=1}^{n}...\sum_{u=1}^{n} c_{rs...u}^p T^p(z_r, z_s, ..., z_u) + ... + b$$

Therefore, the equation of the state variable becomes

$$\frac{dy_j}{dt} = \sum_{i=1}^{n}\sum_{r=1}^{n} c_r^1 T^1(z_r) + \sum_{i=1}^{n}\sum_{r=1}^{n}\sum_{s=1}^{n} c_{rs}^2 T^2(z_r, z_s)$$
$$+ \sum_{i=1}^{n}\sum_{r=1}^{n}\sum_{s=1}^{n}\sum_{h=1}^{n} c_{rsh}^3 T^3(z_r, z_s, z_h) + ... \qquad (26)$$
$$+ \sum_{i=1}^{n}\sum_{r=1}^{n}\sum_{s=1}^{n}...\sum_{u=1}^{n} c_{rs...u}^p T^p(z_r, z_s, ..., z_u) + ...$$

Integrating using Euler's method

$$y_j(t+1)$$
$$= y_j(t) + \delta_t \left[\begin{array}{l} \sum\limits_{i=1}^{n}\sum\limits_{r=1}^{n} c_r^1 T^1(z_r) \\ + \sum\limits_{i=1}^{n}\sum\limits_{r=1}^{n}\sum\limits_{s=1}^{n} c_{rs}^2 T^2(z_r, z_s) + ... \\ + \sum\limits_{i=1}^{n}\sum\limits_{r=1}^{n}\sum\limits_{s=1}^{n}...\sum\limits_{u=1}^{n} c_{rs...u}^p T^p\left(\begin{array}{l} z_r, z_s, \\ ..., z_u \end{array}\right) + ... \end{array} \right] \qquad (27)$$

and in condensed form

$$y_j(t+1) = y_j(t) + \delta_t \left[\sum_{i=1}^{n} \Phi_{ij}(t) \right]. \qquad (28)$$

7. Conclusions

MARIOLA model, being has been operating since 1995, and modeling different shrubs in many places along the Valencian Community. This model, with all its limitations, has replied to the expectations put on it. It has allowed us to calculate with enough credibility a series of variables determining the development and behaviour of bushes in the Mediterranean ecosystems. It not only allows us to follow the mentioned development in the ecosystem level but it can also be used for the pursuit of one sample.

1) BIOSHRUB program can be considered a closed program. BIOMASS files can be extraordinary enlarged while experimental work increases the information of the taxons. However, parameter b needs a more adequate interpretation of its meaning from a vegetable edaphophysiological point of view. It is possible that the step (now of 0.1) has to be enlarged to 0.01 to be able to reduce the mistake margins of the biological interpretation of such a parameter.

2) Mathematically, all flow equations (SELEGO program) are a combination of transformed functions of variables by multiple regressions. They are the input and outputs of ordinary differential equations (and partial derivatives differential equations also). It is not a normal way of building a model. Stability and semantic problems are opened. The differential equations are integrated with a "noise" and uncertainty that derives from the same nature of the data of the real and of the real methods for building the flow equations.

3) MARIOLA model is flexible. Its structure allows the addition of new goals and relationships. Its enlargement depend on the aggregation level desired, which is at the same time a function of the objectives of the model itself.

MARIOLA can open perspectives of research upon the effects of erosion and desertification in the Mediterranean ecosystems.

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