

Integrated Optimization Model for Concurrent Tolerancing Allocation and Alternative Manufacturing Process Selection

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Abstract Most of the existing concurrent tolerancing methods can only be used to solve optimal tolerance allocation problems under the conditions of given manufacturing processes and economical process tolerance ranges, which are difficult to get the global optimization solutions of process and design tolerances, as well as the most economical processing routes. With the aim to address these shortcomings, an optimization mathematical model was developed in this paper, in which we consider the selection of alternative manufacturing processes by introducing a manufacturing process selection coefficient into the existing concurrent tolerancing model. The recommended model simultaneously allocates the design and process tolerances with alternative machining process selection by minimizing the total cost including the production cost and the expected quality loss, and taking the product function requirements, the economic machining tolerance range for each alternative process operation, and alternative manufacturing process selection as constraints. In order to accurately solve this discrete integrated optimization model, the MATLAB R2016b is adopted as a nonlinear programming technique to achieve the optimization solution of concurrent tolerancing allocation and manufacturing process selection. The numerical example of a gear assembly proves that the proposed model can not only select manufacturing processes but also realize concurrent optimization allocation of tolerances.

Keywords: Concurrent Tolerancing, Quality loss function, Alternative manufacturing processes, Optimization, Process tolerance

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1. Introduction

Tolerance is very important geometric information transmitted in various stages of product design, production and verification, which has a great impact on performance quality and production cost of a product. A product feature (dimensional or geometrical feature) with loose tolerances means low production cost, but the variation of product quality characteristics is large, which will bring about greater quality loss. On the other hand, the feature with tight tolerances indicates little variation in product quality characteristics, but will lead to higher production cost. Tolerances have become the link to achieve the best trade-off between product quality and production cost [1]. Traditionally, tolerance allocation consists of two serial independent phases: product tolerance allocation and process tolerance allocation. Instead of this serial tolerance allocation mode, concurrent tolerancing allocation joints these two relatively independent phases together, thus shortening product development cycles,

improving design efficiency, and reducing production costs. In the early days, Ye and Salustri [2] developed a non-linear optimization model with the objective function of minimizing the summation of production cost and quality loss for simultaneously allocating the process tolerances and design tolerances. For the sake of quality improvement and cost reduction, Jeang [3] combined tolerance design and parameter design into one equation to simultaneously obtain the optimized values of process tolerance, process mean, and design tolerance; Jeang et al. [4] further extended this work for simultaneously determining the optimal process mean and process tolerance based on manufacturing cost and asymmetric mass loss function. After deriving the multivariate quality loss function for interrelated critical characteristics, Peng et al. [5] proposed a new concurrent tolerancing allocation model for mechanical assemblies with interrelated dimensional chains based on minimizing the sum of expected quality loss and production cost. But this study mainly focuses on the concurrent optimal allocation of dimensional tolerances, without considering geometric tolerances. Then, Peng HP and Peng ZQ [6] conducted an

extended study on the model to realize concurrent tolerancing allocation including geometrical tolerances and dimensional tolerances. Janakiraman and Saravanan [7] adopted the design of experiments approach to systematically investigate the machining process parameters that affect product quality, and established a mathematical model to simultaneously achieve the optimal allocation of tolerances and the determination of procedure parameters with the aim of minimizing the total machining cost and quality loss function. In order to get better product assemblability and lower production cost, Lu et al. [8] investigated the key technologies for concurrent tolerance design with non-cooperative game theory, taking the perspective of the actual processing and assembling processes of product. Peng [9] introduced the present worth of expected quality loss to capture the quality loss caused by product degradation over time, and developed a new tolerance optimization model based on the minimization of the sum of the present worth of expected quality loss and production cost to achieve the parallel tolerancing allocation of products with correlated characteristics. Then, Balamurugan et al. [10] extended Peng's model to implement the concurrent optimization allocation of geometrical tolerances and dimensional tolerances based on maximizing the summation of weight process geometrical and dimensional tolerances and minimizing the present worth of expected quality loss using evolutionary optimization techniques.

For manufactured parts with multiple machining processes (MMPs), various machines, machining tools, process plans, scheduling plans, etc. are employed to transform the raw materials or semi-finished products into final products, in which process planning and scheduling are two crucial functions that are usually executed sequentially. The integration of process planning and scheduling is crucial for optimal utilization of manufacturing resources, and various optimization algorithms are also adopted to solve this kind of integrated optimization problems [11,12,13,14]. And machining operation sequencing has always been an important research direction in computer-aided process planning; an optimal process sequence could greatly improve production efficiency and reduce production cost [15]. Utilizing the extended spline model, Yeo et al. [16] proposed an approach for determining optimal process tolerances and generating optimal process sequence based on the minimum production cost criterion. Robles and Roy [17] investigated a model for optimal tolerance allocation and process sequence selection by taking the minimization of manufacturing cost and the quality loss as objective function, and taking design tolerance constraint, process machining capacity constraint, and process sequence selection constraint as constraint conditions. Geetha et al. [18] suggested a method for concurrent tolerance allocation and scheduling that simultaneously considered the total tolerance cost and total machine idle time cost, and the genetic algorithm (GA) was introduced to allocate the part tolerances and determine the optimal production sequence. Mohapatra et al. [19] constructed a multi-objective optimization model for integrating process planning and scheduling. In order to obtain the optimal and robust solution, an improved controlled elitist non-dominated sorting genetic algorithm was employed to

solve this integration problem. Deja and Siemiatkowski [20] presented an extended feature classification for the needs of the rational process plan selection, and developed a feature-based reasoning method for generating machining sequences and allocating machine alternatives.

The machining processes selection and the optimal tolerance allocation are two important factors affecting product cost in MMPs; In this connection, developing more efficient approaches to solve the machining processes selection problem and tolerance allocation problem has become another important research direction in CAPP. Ming and Mak [21] constructed a model for optimal tolerances allocation and manufacturing operations selection, in which Hopfield neural network and genetic algorithm are used to solve the manufacturing operations selection problem and the tolerance optimization problem of each manufacturing operation, respectively. Singh et al. [22] formulated the optimal tolerance allocation problem involving alternative machining processes selection and utilized genetic algorithm to solve the complex tolerancing problem. Compared with Singh et al.'s approach mentioned above, the new algorithm developed by Kumar et al. [23] is more cost-effective in solving the optimal tolerance synthesis problem involving alternative machining processes selection. Sivakumar et al. [24] proposed a multi-objective tolerance optimization method with the optimization objectives of minimizing tolerance stack-up, manufacturing cost and quality loss, and the Multi-Objective Particle Swarm Optimization and the Elitist Non-dominated Sorting Genetic Algorithm are adopted to solve this optimal problem to achieve alternative manufacturing process selection and concurrent tolerance allocation. Geetha et al. [25] also constructed a multi-objective optimization model by concurrently optimizing machining parameters: total machining time, the machine idle time cost and manufacturing cost; and the genetic algorithm was employed solve this model to achieve optimal tolerance synthesis with process and machine selection.

In the process of product concurrent tolerancing allocation, there often exist various feasible process plans, machining sequences, and specific machining methods for the workpieces, how to select the economical and reasonable process plans, machining sequences, and specific machining methods to obtain the optimization process and design tolerances is of great significance for ensuring the product functional requirements and reducing the product production costs. Nevertheless, most of the existing concurrent tolerancing allocation methods are: the designer determines the manufacturing processes and process routes for each process of the part in advance, and then solves the established concurrent tolerance optimization model, so as to achieve concurrent allocation of process tolerances and design tolerances. But these methods based on pre-determined manufacturing processes and process routes do not take into account the impact of the diversity of manufacturing processes and process routes in product design and manufacturing on the results of concurrent tolerance optimization allocation, which can easily lead to the problems such as suboptimal tolerance results and insufficient economical process routes for parts. To overcome these shortcomings, the focus of this research is to extend traditional concurrent

tolerancing allocation model and develop an integrated optimization model for concurrent tolerancing allocation and alternative machining process selection, aiming to overcome the shortcomings of previous concurrent tolerancing allocation model that can only perform tolerance optimization allocation under pre-determined manufacturing processes and process routes.

2. Mathematical Model

2.1. Manufacturing Process

The selection of manufacturing processes is to choose a reasonable machining process for each feature surface of the parts with quality requirements. The commonly used processing methods for mechanical parts include turning, milling, planing, drilling, grinding, etc. In general, the turning operation can process part feature surfaces including internal and external cylindrical surfaces, conical surfaces, internal and external threads, forming rotary surfaces and end surfaces, etc.; the milling can process various planes (including horizontal, vertical and inclined planes), forming surfaces, spiral surfaces, various grooves, etc.; while the grinding can process internal and external cylindrical surfaces, conical surfaces, planes, forming surfaces, etc.

In general, different workpiece materials, workpiece shapes and sizes, machining precision and surface quality need to adopt different manufacturing processes. In concurrent tolerancing allocation, we consider the selection of manufacturing processes by introducing an alternative manufacturing process selection coefficient into concurrent tolerancing allocation model.

2.2. Production Cost

In general, a tighter tolerance usually incurs higher production cost; while a looser tolerance will reduce production cost of the machined part. The relationship between cost and tolerance is generally formulated as the cost-tolerance model, and the acquisition of high quality empirical cost-tolerance data is a necessary prerequisite for establishing a reliable and practical cost-tolerance model. Many studies focused on mathematical modeling of production cost-tolerance relations based on empirical cost-tolerance data from all frequently used production processes. Several commonly-used cost-tolerance models include: the Reciprocal Model, the Reciprocal Squared Model, the Reciprocal Powers Model, the Exponential Model, the Combined Reciprocal Powers and Exponential Model, the Reciprocal Powers and Exponential Hybrid Model, etc.

The total production cost is the sum of the machining costs of the various machining processes of all relevant parts; it can be expressed as follows:

$$MC(T_{ij}) = \sum_{i=1}^p \sum_{j=1}^q C(T_{ij}) \quad (1)$$

where $C(T_{ij})$ denotes the production cost of machining the i^{th} part feature dimension to tolerance T_{ij} in the j^{th} machining process; p is the number of part feature

dimensions; q is the number of machining processes of the i^{th} part dimension.

Each machining operation of a part may have multiple different manufacturing processes, and different manufacturing processes correspond to different production equipment, machining time, etc., which will cause changes in production costs. Here, we recommend a manufacturing process selection coefficient μ_{ijk} to consider the impact of different manufacturing processes, Eq. (1) can be written as:

$$MC(T_{ijk}) = \sum_{i=1}^p \sum_{j=1}^q \sum_{k=1}^r \mu_{ijk} C(T_{ijk}) \quad (2)$$

where $C(T_{ijk})$ represents the production cost of obtaining required process tolerance T_{ijk} by selecting the k^{th} manufacturing process in the j^{th} process of the i^{th} part dimension; r is the number of alternative machining processes in the j^{th} machining process of the i^{th} part dimension; μ_{ijk} is the manufacturing process selection coefficient, if the k^{th} manufacturing process is selected in the j^{th} machining process of the i^{th} part dimension, then μ_{ijk} is 1, otherwise it is 0.

In this study, we adopted the Combined Reciprocal Powers and Exponential Function proposed by Dong et al. [26] as the cost-tolerance model, i.e.

$$C(T_{ijk}) = a_0 + a_1 T_{ijk}^{-a_2} + a_3 e^{-a_4 T_{ijk}} \quad (3)$$

where the model parameters a_0 , a_1 , a_2 , a_3 , and a_4 can be determined according to the empirical data.

Dong et al. also introduced several specific cost-tolerance models on the basis of in-depth research on the empirical cost-tolerance data of typical machining processes such as surface milling, turning, rotary surface grinding, hole machining and casting. According to the research of Dong et al. [26], several typical cost-tolerance models are listed as follows:

For surface milling:

$$C(T_{ijk}) = 0.00001 + 0.00001 T_{ijk}^{-1.836} + 455.3 e^{-18.24 T_{ijk}} \quad (4)$$

For turning:

$$C(T_{ijk}) = 0.005228 + 0.36 T_{ijk}^{-0.9044} + 63.39 e^{-2.531 T_{ijk}} \quad (5)$$

For rotary surface grinding

$$C(T_{ijk}) = 39.03 + 17.90 T_{ijk}^{-0.00001} + 30.84 e^{-63.47 T_{ijk}} \quad (6)$$

2.3. Expected Quality Loss

The functional requirements of a product are guaranteed by its critical quality characteristics. Assuming that the u^{th} product characteristic parameter z_u ($u=1, 2, \dots, n$) is the function of p part design dimensions, we have:

$$z_u = g_u(y_1, y_2, \dots, y_p) \quad (7)$$

where y_i ($i=1, 2, \dots, p$) is the i^{th} part dimension.

By extending Taguchi's quality loss function, Lee and Tang [27] proposed a general format for evaluating the quality loss of the product with multiple quality characteristics as follows:

$$L(z_u) = \sum_{u=1}^n \sum_{v=1}^u A_{uv} (z_u - m_u)(z_v - m_v) \quad (8)$$

where A_{uv} is the quality loss coefficient, the calculation method of which has been mentioned in many studies [9,10,27]; z_u is the u^{th} product quality characteristic with its target value m_u ; z_v is the v^{th} quality characteristic with its target value m_v . And the expected quality loss for batch products can be written as:

$$EL(z_u) = \sum_{u=1}^n A_{uu} [\sigma_{z_u}^2 + \delta_{z_u}^2] + \sum_{u=2}^n \sum_{l=1}^{u-1} A_{ul} [\sigma_{z_u z_l}^2 + \delta_{z_u} \delta_{z_l}] \quad (9)$$

where $\sigma_{z_u}^2$ is the variance of product characteristic parameter z_u ; $\sigma_{z_u z_v}^2$ is the covariance between characteristic parameters z_u and z_v , for $u, v=1, 2, \dots, n, u \neq v$. δ_{z_u} and δ_{z_v} represent the biases between the mean values of characteristic parameters z_u and z_v and their target values, respectively.

In the condition of mass production, it can be assumed that the variations of part design dimensions are in accord with normal distribution centered on its mean value. According to statistical theory, the variance of the variations in assembly characteristic parameter z_u can be expressed as:

$$\sigma_{z_u}^2 = \sum_{i=1}^p \left(\frac{\partial g_u}{\partial y_i} \right)^2 \sigma_{y_i}^2 \quad (10)$$

$$\sigma_{z_u z_v}^2 = \sum_{i=1}^p \left(\frac{\partial g_u}{\partial y_i} \right) \left(\frac{\partial g_v}{\partial y_i} \right) \sigma_{y_i}^2 \quad (11)$$

where $\sigma_{y_i}^2$ is the variance of the variations in the part design dimension y_i .

According to the 6 σ tolerance analysis method, approximately 99.73% of the variations in part design dimension are distributed within interval $[-3\sigma, +3\sigma]$ centered on its mean value [6], and we have:

$$\sigma_{y_i}^2 = \left(\frac{T_i}{3} \right)^2 \quad (12)$$

where T_i is the bilateral tolerance of part design dimension y_i .

In the machining process, the part dimension y_i is expressed as the equation of relevant process dimensions, that is:

$$y_i = f_i(x_{i1}, x_{i2}, \dots, x_{iq}) \quad (13)$$

where x_{ij} ($j = 1, 2, \dots, q$) is the j^{th} process dimension of y_i , and q is the number of machining processes of y_i .

Suppose the variations of process dimensions are normally distributed, according to statistical theory, the variance of the design dimension y_i variations is similarly expressed as:

$$\sigma_{y_i}^2 = \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \sigma_{x_{ij}}^2 \quad (14)$$

where $\sigma_{x_{ij}}^2$ is the variance of the variations in the process dimension x_{ij} .

Similar to Eq.(12), we can also have:

$$\sigma_{x_{ij}}^2 = \left(\frac{T_{ij}}{3} \right)^2 \quad (15)$$

where T_{ij} is the bilateral tolerance of process dimension x_{ij} .

By substituting Eqs. (12) and (15) into Eq.(14), the function relation formula between design tolerance and process tolerances can be obtained as follows:

$$T_i^2 = \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 T_{ij}^2 \quad (16)$$

Substituting Eq. (15) into Eq. (14), we have:

$$\sigma_{y_i}^2 = \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \left(\frac{T_{ij}}{3} \right)^2 \quad (17)$$

Substituting Eq. (17) into Eqs. (10) and (11), one has:

$$\sigma_{z_u}^2 = \sum_{i=1}^p \left(\frac{\partial g_u}{\partial y_i} \right)^2 \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \left(\frac{T_{ij}}{3} \right)^2 \quad (18)$$

$$\sigma_{z_u z_v}^2 = \sum_{i=1}^p \left(\frac{\partial g_u}{\partial y_i} \right) \left(\frac{\partial g_v}{\partial y_i} \right) \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \left(\frac{T_{ij}}{3} \right)^2 \quad (19)$$

And substituting Eqs. (18) and (19) into Eq. (9), the expected quality loss expressed as the function of process tolerances T_{ij} can be represented as:

$$EL(T_{ij}) = \sum_{u=1}^n A_{uu} \left[\sum_{i=1}^p \left(\frac{\partial g_u}{\partial y_i} \right)^2 \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \left(\frac{T_{ij}}{3} \right)^2 \right] + \sum_{u=2}^n \sum_{l=1}^{u-1} A_{ul} \left[\sum_{i=1}^p \left(\frac{\partial g_u}{\partial y_i} \right) \left(\frac{\partial g_l}{\partial y_i} \right) \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \left(\frac{T_{ij}}{3} \right)^2 \right] \quad (20)$$

If there exist r alternative manufacturing processes in the j^{th} machining process of the i^{th} part dimension, we introduce a binary variable μ_{ijk} for selecting the manufacturing process. Thus, Eq. (20) will be written as:

$$EL(T_{ijk}) = \sum_{u=1}^n A_{uu} \left[\sum_{i=1}^p \left(\frac{\partial g_u}{\partial y_i} \right)^2 \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \sum_{k=1}^r \mu_{ijk} \left(\frac{T_{ijk}}{3} \right)^2 \right] + \sum_{u=2}^n \sum_{l=1}^{u-1} A_{ul} \left[\sum_{i=1}^p \left(\frac{\partial g_u}{\partial y_i} \right) \left(\frac{\partial g_l}{\partial y_i} \right) \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \sum_{k=1}^r \mu_{ijk} \left(\frac{T_{ijk}}{3} \right)^2 \right] \quad (21)$$

2.4. Model Formulation

The concurrent allocation of process and design tolerances involved in the selection of manufacturing processes usually affects both production cost and product quality. Based on the idea of combining the concurrent tolerancing allocation with alternative manufacturing process selection, an integrated optimization mathematical model is established in this section with the objective function of minimizing the sum of the production cost and the expected quality loss, and the constraints of assembly function requirement, economic machining accuracy range, and manufacturing process selection.

$$\min \{ MC(T_{ijk}) + EL(T_{ijk}) \} \quad (22)$$

Substituting Eqs. (2) and (21) into Eq.(22), we get:

$$\min \left\{ \sum_{i=1}^p \sum_{j=1}^q \sum_{k=1}^r \mu_{ijk} C(T_{ijk}) + \sum_{u=1}^n A_{uu} \left[\sum_{i=1}^p \left(\frac{\partial g_u}{\partial y_i} \right)^2 \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \sum_{k=1}^r \mu_{ijk} \left(\frac{T_{ijk}}{3} \right)^2 \right] + \sum_{u=1}^n A_{uu} \left[\sum_{i=1}^p \left(\frac{\partial g_u}{\partial y_i} \right)^2 \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \sum_{k=1}^r \mu_{ijk} \left(\frac{T_{ijk}}{3} \right)^2 \right] \right\} \quad (23)$$

Subject to the following constraints:

(1) The assembly function requirement constraint with the RSS criterion:

$$\sum_{i=1}^p \sum_{j=1}^q \left(\frac{\partial g}{\partial y_i} \right)^2 \left(\frac{\partial f_j}{\partial x_{ij}} \right)^2 \sum_{k=1}^r \mu_{ijk} \cdot T_{ijk}^2 \leq T_{zu}^2 \quad (24)$$

where T_{zu} is the assembly functional tolerance of the product.

(2) The economical process tolerance range for each manufacturing process:

$$T_{ijk}^- \leq T_{ijk} \leq T_{ijk}^+ \quad \forall i=1, 2, \dots, p; j=1, 2, \dots, q \quad (25)$$

where T_{ijk}^- and T_{ijk}^+ are respectively the lower and upper bounds of economical process tolerance T_{ijk} when the k^{th} manufacturing process is selected in the j^{th} machining process of the i^{th} part dimension.

(3) Manufacturing process selection constraint

The purpose of this constraint is to ensure that only one of the most economical and reasonable manufacturing

processes is ultimately selected for each process.

$$\sum_{k=1}^r \mu_{ijk} = 1 \quad (26)$$

where $\forall i=1, 2, \dots, p; j=1, 2, \dots, q;$

$$\mu_{ijk} = \begin{cases} 1 & \text{if the } k^{\text{th}} \text{ manufacturing process is selected} \\ 0 & \text{else} \end{cases}$$

3. Numerical Example

In this section, an example of gear shaft assembly is used to verify the recommended model. The assembly shown in Figure 1 is composed of gear shaft 1, right bushing 2, gearbox 3, cover 4, left bushing 5, etc. The variation of horizontal gap z (assembly characteristic parameter) between the left shoulder of the gear shaft and the right end face of the left bushing needs to be controlled to guarantee the performance of this assembly. Suppose the nominal design dimensions have already been assigned according to the functional requirements of the product, they are: $y_1=140\text{mm}$, $y_2=15\text{mm}$, $y_3=125\text{mm}$, $y_4=45\text{mm}$, $y_5=15\text{mm}$, respectively. The assembly characteristic parameter is $z = 0 \pm T_z$, (unit: mm). Assume that when the assembly gap z deviates from its design target will lead to product failure and cause a loss of \$100.

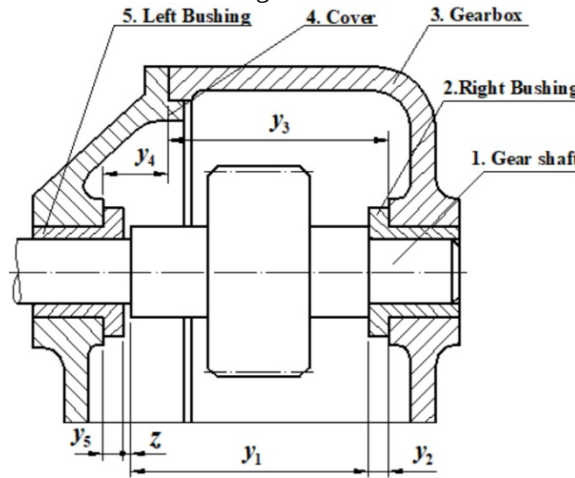


Figure 1. A gear shaft assembly

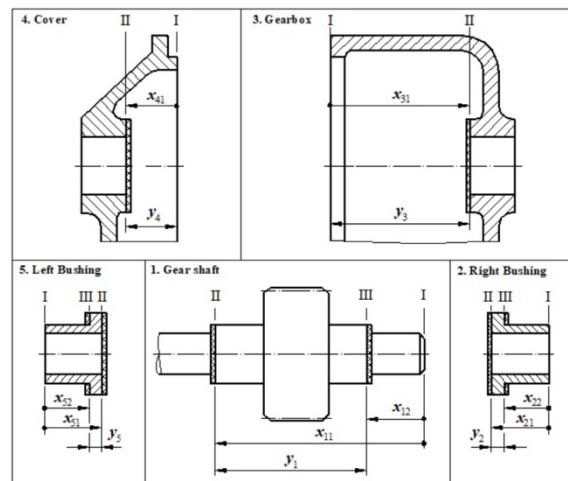


Figure 2. Manufacturing processes of parts

Table 1. Manufacturing processes of parts (mm)

Parts	Process no.	Reference surface	Machined surface	Manufacturing operation	Process dimension	Process tolerance	Tolerance range
1.Gear shaft	11	I	II	Rough turning	X_{111}	T_{111}	0.290-0.460
				Semi-extractive turning	X_{112}	T_{112}	0.046-0.185
				Finish turning	X_{113}	T_{113}	0.020-0.046
				Rough grinding	X_{114}	T_{114}	0.046-0.072
				Semi-finished grinding	X_{115}	T_{115}	0.029-0.046
				Fine grinding	X_{116}	T_{116}	0.014-0.029
	12	I	III	Rough turning	X_{121}	T_{121}	0.160-0.250
				Semi-extractive turning	X_{122}	T_{122}	0.025-0.100
				Finish turning	X_{123}	T_{123}	0.011-0.025
				Rough grinding	X_{124}	T_{124}	0.025-0.039
				Semi-finished grinding	X_{125}	T_{125}	0.016-0.025
				Fine grinding	X_{126}	T_{126}	0.007-0.016
2/5.Right/Left Bushing	21/51	I	II	Rough milling	X_{211}/X_{511}	T_{211}/T_{511}	0.190-0.460
				Half finish milling	X_{212}/X_{512}	T_{212}/T_{512}	0.046-0.190
				Finish milling	X_{213}/X_{513}	T_{213}/T_{513}	0.019-0.046
				Rough turning	X_{214}/X_{514}	T_{214}/T_{514}	0.190-0.300
				Semi-extractive turning	X_{215}/X_{515}	T_{215}/T_{515}	0.030-0.120
				Finish turning	X_{216}/X_{516}	T_{216}/T_{516}	0.013-0.030
				Rough grinding	X_{217}/X_{517}	T_{217}/T_{517}	0.030-0.046
				Semi-finished grinding	X_{218}/X_{518}	T_{218}/T_{518}	0.019-0.030
	22/52	I	III	Fine grinding	X_{219}/X_{519}	T_{219}/T_{519}	0.008-0.019
				Rough turning	X_{221}/X_{521}	T_{221}/T_{521}	0.160-0.250
				Semi-extractive turning	X_{222}/X_{522}	T_{222}/T_{522}	0.025-0.100
				Finish turning	X_{223}/X_{523}	T_{223}/T_{523}	0.011-0.025
				Rough grinding	X_{224}/X_{524}	T_{224}/T_{524}	0.025-0.039
				Semi-finished grinding	X_{225}/X_{525}	T_{225}/T_{525}	0.016-0.025
				Fine grinding	X_{226}/X_{526}	T_{226}/T_{526}	0.007-0.016
				Rough milling	X_{311}	T_{311}	0.250-0.630
3.Gearbox	31	I	II	Half finish milling	X_{312}	T_{312}	0.063-0.250
				Finish milling	X_{313}	T_{313}	0.025-0.063
				Rough turning	X_{314}	T_{314}	0.250-0.400
				Semi-extractive turning	X_{315}	T_{315}	0.040-0.160
				Finish turning	X_{316}	T_{316}	0.018-0.040
				Rough grinding	X_{317}	T_{317}	0.040-0.063
				Semi-finished grinding	X_{318}	T_{318}	0.025-0.040
				Fine grinding	X_{319}	T_{319}	0.012-0.025
4.Cover	41	I	II	Rough milling	X_{411}	T_{411}	0.160-0.390
				Half finish milling	X_{412}	T_{412}	0.039-0.160
				Finish milling	X_{413}	T_{413}	0.016-0.039
				Rough turning	X_{414}	T_{414}	0.160-0.250
				Semi-extractive turning	X_{415}	T_{415}	0.025-0.100
				Finish turning	X_{416}	T_{416}	0.011-0.025
				Rough grinding	X_{417}	T_{417}	0.025-0.039
				Semi-finished grinding	X_{418}	T_{418}	0.016-0.025
				Fine grinding	X_{419}	T_{419}	0.007-0.016

3.1. Problem Analysis

According to the manufacturing processes of parts as shown in Figure 2, the surface features to be machined are all planes, and there are 6-9 alternative manufacturing processes (rough milling, half finish milling, finish milling, rough turning, semi-extractive turning, finish turning, rough grinding, semi-finished grinding, fine grinding) for each process. The two processes of gear shaft 1 are involved in the machining of end surfaces, and the manufacturing processes of which can be turning and grinding; The left end face of right bushing 2 (left bushing 5) is a round plane, and the manufacturing processes used will be milling, turning, and grinding, while its left end

face is a toroidal plane, and the manufacturing processes commonly used are turning and grinding; the manufacturing processes for gearbox 3 and cover 4 include milling, turning and grinding. The manufacturing processes of relevant parts and the economical process tolerance ranges are listed in Table 1.

3.2. Objective Function

As mentioned above, the objective of this optimization issue is to minimize the total assembly cost. The total assembly cost consists of the production cost and the expected quality loss, in which the production cost is the sum of processing cost of each machining operation for all

process dimensions; the expected quality loss can be calculated by using Eq. (21). In this example, the functional requirement of this assembly is guaranteed by a single critical quality characteristic z , the Eq. (21) can be simplified as:

$$EL(T_{ijk}) = A_{11} \cdot \left[\sum_{i=1}^p \left(\frac{\partial g}{\partial y_i} \right)^2 \sum_{j=1}^q \left(\frac{\partial f_i}{\partial x_{ij}} \right)^2 \sum_{k=1}^r \mu_{ijk} \left(\frac{T_{ijk}}{3} \right)^2 \right] \quad (27)$$

Assuming that the tolerance interval of assembly characteristic parameter z is located in $[-T_z, T_z]$, and the quality loss due to product failure is Q_0 , then the quality loss coefficient A_{11} can be calculated as:

$$A_{11} = \frac{Q_0}{T_z^2} \quad (28)$$

The two rotational surfaces specified by process dimensions X_{11} and X_{12} (the corresponding process tolerances T_{11} and T_{12} , respectively) can be machined by using turning and surface grinding, the production cost associated with the process dimensions X_{11} and X_{12} can be modeled by turning and rotary surface grinding model, which can be respectively represented as follows.

$$\begin{aligned} \sum_{k=1}^6 \mu_{11k} C(T_{11k}) = & \sum_{k=1}^3 \mu_{11k} \cdot (0.005228 + 0.36T_{11k}^{-0.9044} + 63.39e^{-2.531 T_{11k}}) \\ & + \sum_{k=4}^6 \mu_{11k} \cdot (39.03 + 17.90T_{11k}^{-0.00001} + 30.84e^{-63.47 T_{11k}}) \\ \sum_{k=1}^6 \mu_{12k} C(T_{12k}) = & \sum_{k=1}^3 \mu_{12k} \cdot (0.005228 + 0.36T_{12k}^{-0.9044} + 63.39e^{-2.531 T_{12k}}) \\ & + \sum_{k=4}^6 \mu_{12k} \cdot (39.03 + 17.90T_{12k}^{-0.00001} + 30.84e^{-63.47 T_{12k}}) \end{aligned}$$

The two rotational surfaces specified by X_{21} and X_{22} (the corresponding process tolerances T_{21} and T_{22} , respectively) can be machined by using the face milling, turning and surface grinding, so the production cost related to process dimensions X_{21} and X_{22} can be modeled by using the face milling, turning, and rotary surface grinding model, which will be respectively expressed as follows.

$$\begin{aligned} \sum_{k=1}^9 \mu_{21k} C(T_{21k}) = & \sum_{k=1}^3 \mu_{21k} \cdot (0.00001 + 0.00001T_{21k}^{-1.836} + 455.3e^{-18.24 T_{21k}}) \\ & + \sum_{k=4}^6 \mu_{21k} \cdot (0.005228 + 0.36T_{21k}^{-0.9044} + 63.39e^{-2.531 T_{21k}}) \\ & + \sum_{k=7}^9 \mu_{21k} \cdot (39.03 + 17.90T_{21k}^{-0.00001} + 30.84e^{-63.47 T_{21k}}) \end{aligned}$$

$$\begin{aligned} \sum_{k=1}^6 \mu_{22k} C(T_{22k}) = & \sum_{k=1}^3 \mu_{22k} \cdot (0.005228 + 0.36T_{22k}^{-0.9044} + 63.39e^{-2.531 T_{22k}}) \\ & + \sum_{k=4}^6 \mu_{22k} \cdot (39.03 + 17.90T_{22k}^{-0.00001} + 30.84e^{-63.47 T_{22k}}) \end{aligned}$$

Both flat surfaces specified by process dimensions X_{31} and X_{41} (the corresponding process tolerances T_{31} and T_{41} , respectively) can be machined by using the face milling, turning, and surface grinding, so the production cost associated with process dimensions X_{31} and X_{41} can be represented as:

$$\begin{aligned} \sum_{k=1}^9 \mu_{31k} C(T_{31k}) = & \sum_{k=1}^3 \mu_{31k} \cdot (0.00001 + 0.00001T_{31k}^{-1.836} + 455.3e^{-18.24 T_{31k}}) \\ & + \sum_{k=4}^6 \mu_{31k} \cdot (0.005228 + 0.36T_{31k}^{-0.9044} + 63.39e^{-2.531 T_{31k}}) \\ & + \sum_{k=7}^9 \mu_{31k} \cdot (39.03 + 17.90T_{31k}^{-0.00001} + 30.84e^{-63.47 T_{31k}}) \\ \sum_{k=1}^9 \mu_{41k} C(T_{41k}) = & \sum_{k=1}^3 \mu_{41k} \cdot (0.00001 + 0.00001T_{41k}^{-1.836} + 455.3e^{-18.24 T_{41k}}) \\ & + \sum_{k=4}^6 \mu_{41k} \cdot (0.005228 + 0.36T_{41k}^{-0.9044} + 63.39e^{-2.531 T_{41k}}) \\ & + \sum_{k=7}^9 \mu_{41k} \cdot (39.03 + 17.90T_{41k}^{-0.00001} + 30.84e^{-63.47 T_{41k}}) \end{aligned}$$

The dimensions of part 2 and part 5 are exactly the same, similar to the process dimensions X_{21} and X_{22} ; we get the production cost associated with process dimensions X_{51} and X_{52} .

$$\begin{aligned} \sum_{k=1}^9 \mu_{51k} C(T_{51k}) = & \sum_{k=1}^3 \mu_{51k} \cdot (0.00001 + 0.00001T_{51k}^{-1.836} + 455.3e^{-18.24 T_{51k}}) \\ & + \sum_{k=4}^6 \mu_{51k} \cdot (0.005228 + 0.36T_{51k}^{-0.9044} + 63.39e^{-2.531 T_{51k}}) \\ & + \sum_{k=7}^9 \mu_{51k} \cdot (39.03 + 17.90T_{51k}^{-0.00001} + 30.84e^{-63.47 T_{51k}}) \\ \sum_{k=1}^9 \mu_{52k} C(T_{52k}) = & \sum_{k=1}^3 \mu_{52k} \cdot (0.005228 + 0.36T_{52k}^{-0.9044} + 63.39e^{-2.531 T_{52k}}) \\ & + \sum_{k=4}^6 \mu_{52k} \cdot (39.03 + 17.90T_{52k}^{-0.00001} + 30.84e^{-63.47 T_{52k}}) \end{aligned}$$

The sum of processing costs of all machining operations constitutes the production cost of the assembly, which can be expressed as:

$MC(T_{ijk})$

$$= \sum_{k=1}^6 \mu_{11k} C(T_{11k}) + \sum_{k=1}^6 \mu_{12k} C(T_{12k}) + \sum_{k=1}^9 \mu_{21k} C(T_{21k}) + \sum_{k=1}^6 \mu_{22k} C(T_{22k}) \\ + \sum_{k=1}^9 \mu_{31k} C(T_{31k}) + \sum_{k=1}^9 \mu_{41k} C(T_{41k}) + \sum_{k=1}^9 \mu_{51k} C(T_{51k}) + \sum_{k=1}^6 \mu_{52k} C(T_{52k})$$

As shown in Figure 1, the assembly characteristic function of the gear shaft assembly is defined by:

$$z = -y_1 - y_2 + y_3 + y_4 - y_5$$

The variance of the variations in assembly characteristic parameter can be obtained from Eq. (10) as follows.

$$\sigma_z^2 = \sigma_{y_1}^2 + \sigma_{y_2}^2 + \sigma_{y_3}^2 + \sigma_{y_4}^2 + \sigma_{y_5}^2$$

From Eq. (12), we have:

$$\sigma_{y_1}^2 = \frac{1}{9} T_1^2 \quad \sigma_{y_2}^2 = \frac{1}{9} T_2^2$$

$$\sigma_{y_3}^2 = \frac{1}{9} T_3^2 \quad \sigma_{y_4}^2 = \frac{1}{9} T_4^2$$

$$\sigma_{y_5}^2 = \frac{1}{9} T_5^2$$

According to the given machining processes of parts, five machining equations can be obtained as follows:

$$y_1 = x_{11} - x_{12}$$

$$y_2 = x_{21} - x_{22}$$

$$y_3 = x_{31}$$

$$EL(T_{ijk}) = \frac{A_{11}}{9} \cdot \left[\sum_{k=1}^r (\mu_{11k} T_{11k}^2 + \mu_{12k} T_{12k}^2 + \mu_{21k} T_{21k}^2 + \mu_{22k} T_{22k}^2 + \mu_{31k} T_{31k}^2 + \mu_{41k} T_{41k}^2 + \mu_{51k} T_{51k}^2 + \mu_{52k} T_{52k}^2) \right] \\ = 177.8 \left[\sum_{k=1}^6 \mu_{11k} T_{11k}^2 + \sum_{k=1}^6 \mu_{12k} T_{12k}^2 + \sum_{k=1}^9 \mu_{21k} T_{21k}^2 + \sum_{k=1}^6 \mu_{22k} T_{22k}^2 + \sum_{k=1}^9 \mu_{31k} T_{31k}^2 + \sum_{k=1}^9 \mu_{41k} T_{41k}^2 + \sum_{k=1}^9 \mu_{51k} T_{51k}^2 + \sum_{k=1}^6 \mu_{52k} T_{52k}^2 \right]$$

And then, the objective function of this optimization problem is formulated as:

$$\min \{ MC(T_{ijk}) + EL(T_{ijk}) \}$$

It should be noted that in this example we only consider the production cost of the process dimensions involved in assembly functional requirements; and ignore the production cost of other process dimensions, as we adopted the most economical process tolerances for these process dimensions. The same considerations also apply to the expected quality loss.

3.3. Constraint Conditions

Subject to the following constraints:

(1) Assembly function requirement constraint

$$T_{11}^2 + T_{12}^2 + T_{21}^2 + T_{22}^2 + T_{31}^2 + T_{41}^2 + T_{51}^2 + T_{52}^2 \leq T_z^2$$

where T_z is the bilateral tolerance of assembly characteristic size z , $T_{11} = \sum_{k=1}^6 \mu_{11k} T_{11k}$, $T_{12} = \sum_{k=1}^6 \mu_{12k} T_{12k}$,

$$y_4 = x_{41}$$

$$y_5 = x_{51} - x_{52}$$

Under stable processing condition, combining Eqs. (14) and (15), we get:

$$\sigma_{y_1}^2 = \sigma_{x_{11}}^2 + \sigma_{x_{12}}^2 = \frac{1}{9} (T_{11}^2 + T_{12}^2)$$

$$\sigma_{y_2}^2 = \sigma_{x_{21}}^2 + \sigma_{x_{22}}^2 = \frac{1}{9} (T_{21}^2 + T_{22}^2)$$

$$\sigma_{y_3}^2 = \sigma_{x_{31}}^2 = \frac{1}{9} T_{31}^2$$

$$\sigma_{y_4}^2 = \sigma_{x_{41}}^2 = \frac{1}{9} T_{41}^2$$

$$\sigma_{y_5}^2 = \sigma_{x_{51}}^2 + \sigma_{x_{52}}^2 = \frac{1}{9} (T_{51}^2 + T_{52}^2)$$

Then we obtain the following relationships:

$$T_1^2 = T_{11}^2 + T_{12}^2$$

$$T_2^2 = T_{21}^2 + T_{22}^2$$

$$T_3^2 = T_{31}^2$$

$$T_4^2 = T_{41}^2$$

$$T_5^2 = T_{51}^2 + T_{52}^2$$

From Eqs. (27) and (28), the expected loss is expressed as:

$$T_{21} = \sum_{k=1}^9 \mu_{21k} T_{21k}, \quad T_{22} = \sum_{k=1}^6 \mu_{22k} T_{22k}, \quad T_{31} = \sum_{k=1}^9 \mu_{31k} T_{31k},$$

$$T_{41} = \sum_{k=1}^9 \mu_{41k} T_{41k}, \quad T_{51} = \sum_{k=1}^9 \mu_{51k} T_{51k}, \quad T_{52} = \sum_{k=1}^6 \mu_{52k} T_{52k}.$$

(2) Economical process tolerance range constraints

The economical process tolerance ranges are listed in the last column of Table 1.

(3) Manufacturing process selection constraints

$$\sum_{k=1}^6 \mu_{11k} = 1, \quad \sum_{k=1}^6 \mu_{12k} = 1,$$

$$\sum_{k=1}^9 \mu_{21k} = 1, \quad \sum_{k=1}^6 \mu_{22k} = 1,$$

$$\sum_{k=1}^9 \mu_{31k} = 1, \quad \sum_{k=1}^9 \mu_{41k} = 1,$$

$$\sum_{k=1}^9 \mu_{51k} = 1, \quad \sum_{k=1}^6 \mu_{52k} = 1,$$

where $\mu_{ijk} = 1$ or $0, \forall i = 1, 2, \dots, p; j = 1, 2, \dots, q$

3.4. Optimization Results

The optimization model developed in this example is a complicated discrete nonlinear optimization model. The programming software MATLAB R2016b was adopted to solve this model.

When T_z is assumed to be 0.08, 0.18, 0.25, and 0.32, respectively, the optimized process tolerances and selected machining processes are listed in Table 2.

The optimized total cost and part design tolerances are

listed in Table 3.

By analyzing and comparing the optimization results when T_z is 0.08, 0.18, 0.25, and 0.32, respectively, it can be seen that along with the increase of product assembly function tolerance, the total product cost decreases, and most of the process tolerances and design tolerances increase; In the meanwhile, the manufacturing processes have also changed, showing a trend towards decreasing the economical machining precision.

Table 2. The optimized process tolerances and selected machining processes

T_z (mm)	Process Tolerances T_{ijk} (mm)/Machining processes							
	T_{11}	T_{12}	T_{21}	T_{22}	T_{31}	T_{41}	T_{51}	T_{51}
0.08	0.02025/ T_{113}	0.02641/ T_{122}	0.02445/ T_{216}	0.01618/ T_{223}	0.03352/ T_{316}	0.04270/ T_{415}	0.03066/ T_{515}	0.02174/ T_{523}
	Finish turning	Semi-extractive turning	Finish turning	Finish turning	Finish turning	Semi-extractive turning	Semi-extractive turning	Finish turning
0.18	0.03238/ T_{113}	0.04173/ T_{122}	0.04697/ T_{215}	0.01430/ T_{223}	0.04358/ T_{315}	0.15495/ T_{412}	0.02330/ T_{516}	0.02115/ T_{523}
	Finish turning	Semi-extractive turning	Semi-extractive turning	Finish turning	Semi-extractive turning	Half finish milling	Finish turning	Finish turning
0.25	0.03755/ T_{113}	0.03891/ T_{124}	0.16210/ T_{212}	0.06118/ T_{222}	0.03289/ T_{316}	0.02420/ T_{416}	0.16106/ T_{512}	0.03005/ T_{522}
	Finish turning	Rough grinding	Half finish milling	Semi-extractive turning	Finish turning	Finish turning	Half finish milling	Semi-extractive turning
0.32	0.06862/ T_{114}	0.06561/ T_{122}	0.07311/ T_{215}	0.05479/ T_{222}	0.17280/ T_{312}	0.16142/ T_{411}	0.15889/ T_{512}	0.05685/ T_{522}
	Rough grinding	Semi-extractive turning	Semi-extractive turning	Semi-extractive turning	Half finish milling	Rough milling	Half finish milling	Semi-extractive turning

Table 3. The optimized design tolerances and total costs

T_z	Design tolerances (mm)					Total cost (\$)
	T_1	T_2	T_3	T_4	T_5	
0.08	0.03328	0.02932	0.03352	0.04270	0.03758	1223.8
0.18	0.05282	0.04910	0.04358	0.15495	0.03147	469.2
0.25	0.05407	0.17326	0.03289	0.02420	0.16384	433.6
0.32	0.09494	0.09136	0.17280	0.16142	0.16875	418.6

4. Conclusions

Most previous studies on concurrent tolerancing allocation have focused on minimizing production cost, quality loss or a combination of both, and paid little attention to the selection of manufacturing processes. In this study, the manufacturing process selection coefficient was introduced into the concurrent tolerance optimization model, and the manufacturing process selection was taken as one of the constraints of the model, so as to achieve the optimization of design and process tolerances while determine the machining processes for parts. The suggested model is validated by an example of gear shaft assembly.

In order to cover more possible applications, further research will involve the integrated optimization of concurrent tolerancing allocation and manufacturing process selection for products with multiple interrelated dimension chains and non-normally distributed process dimensions. Another direction of future research will be directed to the integrated optimization simultaneously including dimensional and geometrical tolerances, even involving material conditions related to both.

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