

Comprehensive Dynamic Modeling of a Rotary Servo Base Unit Using Frequency Response and Bump Test Techniques

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Abstract This experimental study presents a comprehensive investigation into the dynamic modeling of a Rotary Servo Base Unit, focusing on deriving its dynamics equation and transfer function using first-principles. The study begins with an analytical approach, applying fundamental physics principles to model the system's rotary motion. To validate the theoretical model, two experimental methodologies are implemented to obtain the system's transfer function. The first method involves a frequency response experiment, where the system is subjected to sinusoidal inputs at varying frequencies. By analyzing the amplitude and phase responses, the transfer function is extracted, providing insight into the system's frequency-dependent behavior. The second method employs a bump test, a dynamic excitation approach that perturbs the system to observe its transient response. Through this method, the transfer function is derived based on the system's impulse response, offering additional validation and a broader understanding of its dynamic characteristics. By integrating these analytical and experimental approaches, this study establishes a robust framework for modeling the Rotary Servo Base Unit. The findings contribute to enhanced control system design and improved performance analysis in servo-based applications.

Keywords: Rotary Servo Base Unit, Dynamic Modeling, Transfer Function, Frequency Response, Bump Test, Impulse Response, Step Response, System Identification, Control System Design, Servo Dynamics

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1. Introduction

Servo systems play a critical role in modern automation, robotics, and control applications, where precise motion control is required. Among these systems, the **Rotary Servo Base Unit** is a fundamental electromechanical system used for studying and implementing various control techniques. Understanding its dynamic behavior is crucial for optimizing performance, improving stability, and developing accurate control strategies.

This study focuses on characterizing the rotary motion of the **Rotary Servo Base Unit load shaft** by developing an accurate **transfer function** that represents its dynamic behavior. A transfer function mathematically describes how the system's output (load shaft motion) responds to an input (motor voltage), enabling engineers to predict system behavior under various conditions.

To achieve this, both **analytical modeling** and **experimental validation** are employed. The analytical approach is based on **classical mechanics principles**, where fundamental physics laws governing rotary motion—such as Newton's laws, torque equations, and

system inertia—are used to derive the governing dynamics equation and an initial theoretical transfer function. However, real-world systems often include non-idealities such as friction, electrical resistance, and mechanical backlash, which may cause deviations from theoretical predictions.

To refine and validate the analytical model, experimental methods are conducted. The **Rotary Servo Base Unit** is subjected to controlled input signals, and the resulting **load shaft motion** is measured and analyzed. Two primary experimental techniques are implemented:

- 1. Frequency Response Experiment:** The system is subjected to sinusoidal inputs at varying frequencies, and the corresponding output is recorded. The resulting amplitude and phase response data allow for the extraction of the system's **transfer function**, providing insights into its frequency-dependent behavior.
- 2. Bump Test (Impulse Response Test):** The system is excited with a sudden, short-duration force or displacement, and its transient response is recorded. The impulse response data is then used to derive the transfer function in the time domain, offering an additional perspective on system dynamics.

By integrating both analytical and experimental

approaches, this study aims to establish a **comprehensive dynamic model** of the Rotary Servo Base Unit. The resulting transfer function serves as a fundamental tool for **control system design**, enabling engineers to optimize servo performance in various industrial and research applications.

2. Methods/Approach

In this experiment we will use two approaches to observe how does the system react to different inputs, the inputs can be chosen in many different ways, we will discuss two of them here:

2.1. Frequency Response

The **frequency response experiment** examines how the **Rotary Servo Base Unit** reacts to sinusoidal input signals at different frequencies. This approach helps us understand how the system behaves in response to varying frequency inputs, which is crucial for designing effective control systems.

The **transfer function** describes the relationship between the system's input and output in the frequency domain. Mathematically, it is represented as:

$$G(s) = Y(s) \backslash U(s)$$

where **U(s)** is the input signal, **Y(s)** is the output, and **s** is the complex frequency variable. This function provides information about both the **amplitude** and **phase shift** of the system's response.

Experimental Procedure:

1. A **sinusoidal input voltage** is applied to the **Rotary Servo Base Unit's DC motor**, keeping the amplitude constant while changing the frequency.
2. The **load shaft speed** is recorded as the system's output.
3. The amplitude and phase shift of the output signal are measured for each frequency.
4. A **set of frequency response data points** is collected, and a mathematical model is fitted to derive the system's **transfer function**.
5. A **Bode plot** is created to visualize the system's gain and phase across different frequencies.

Importance:

It helps identify how the system responds to different frequency inputs.

It provides key insights into system stability, resonance, and control design.

The Bode plot helps engineers fine-tune the system for better performance

2.2. The Bump Test

The bump test, also known as the impulse response test, is a method used to evaluate the dynamic characteristics of a system, such as the Rotary Servo Base Unit, by applying a sudden and short-duration force or displacement. The fundamental concept behind this test is to observe how the system responds to a rapid change in input, which provides valuable information about its transient behavior, damping characteristics, and overall dynamic response.

In the time domain, the response of a linear time-

invariant system to an impulse input can be represented by the system's impulse response function, denoted as $h(t)$. The Laplace transform of this impulse response function, $H(s)$, is used to obtain the transfer function of the system, which is a mathematical representation of the system's behavior in the frequency domain.

During the bump test, a controlled disturbance is applied to the system, and the resulting motion or force response is carefully measured and recorded. This response is then analyzed to extract the transfer function of the system. The transfer function reveals important system parameters, such as the natural frequency, damping ratio, and other dynamic characteristics that define how the system will behave under various conditions.

The transfer function obtained through the bump test serves as a complementary tool to frequency response analysis. While frequency response tests focus on steady-state behavior and frequency-dependent characteristics, the bump test provides insight into the system's transient behavior, capturing its immediate reaction to a sudden disturbance. The results of the bump test help validate and refine the accuracy of the transfer function obtained from frequency domain analysis, ensuring a more complete understanding of the system's overall dynamic performance.

Procedure:

The q servo modeling Simulink diagram, as shown in [Figure 1.1](#), will be used to carry out experiments. The Rotary Servo Base Unit subsystem incorporates QUARC blocks that interface with the DC motor and sensors of the Rotary Servo Base Unit system. The Rotary Servo Base Unit model utilizes a Transfer Fcn block from the Simulink library to simulate the dynamics of the Rotary Servo Base Unit system. This setup allows for the simultaneous monitoring of both the measured and simulated load shaft speed in response to a given input voltage. The system provides a comprehensive view of the performance by comparing real-time measurements with simulated behavior.

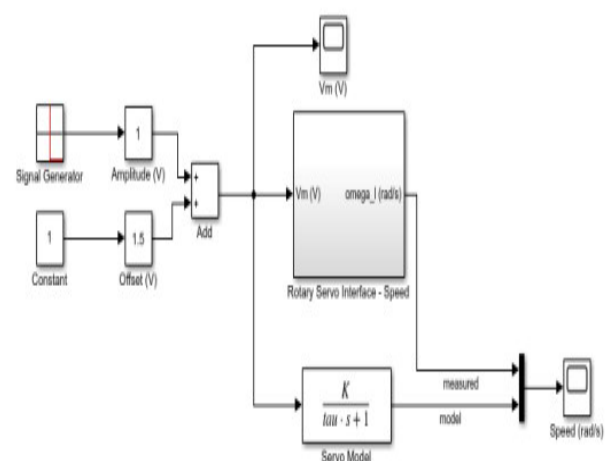


Figure 1. q_servo_modeling Simulink Diagram

q_servo_modeling Simulink diagram used to model Rotary Servo Base Unit.

At the beginning of the experiment, the amplitude of the input sine wave will be kept constant while the frequency is varied. For each frequency change, the amplitude of the corresponding output sine wave will be recorded. Once all the input and output values have been collected, these data

points will be used to create a chart that visually represents the relationship between the input frequency and the output amplitude. This chart will help analyze the system's response across different frequencies and assess its dynamic behavior.

2.3. Steady State Gain

To determine the steady-state gain of the system, the experiment will be conducted with a constant input voltage. Follow these steps to set the system for a constant 2V input voltage:

1. In the Simulink diagram, double-click the Signal Generator block and confirm the following settings:
 - Waveform: Sine
 - Amplitude: 1.0
 - Frequency: 1.0 Hz
 - Units: Hertz
2. Adjust the Amplitude (V) slider gain to 0.
3. Set the Offset (V) block to 2.0 V.
4. Open the load shaft speed scope (Speed in rad/s) and the motor input voltage scope (Vm in V).
5. Click on QUARC | Build to compile the Simulink diagram.
6. Select QUARC | Start to run the controller. You can then observe the input and output signals on the scopes. The response of the system can be seen in the load shaft speed **scope** (Figure 3), which shows the system's reaction to the constant input voltage.
7. Measure the speed of the load shaft and enter the measurement in the Table below under the $f = 0$ Hz row.
8. Calculate the steady-state gain both in linear and decibel (dB) units.

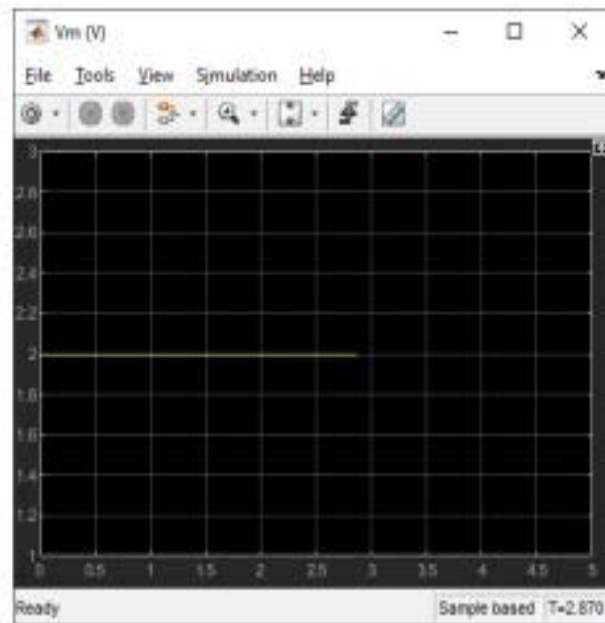


Figure 2. Constant input motor voltage

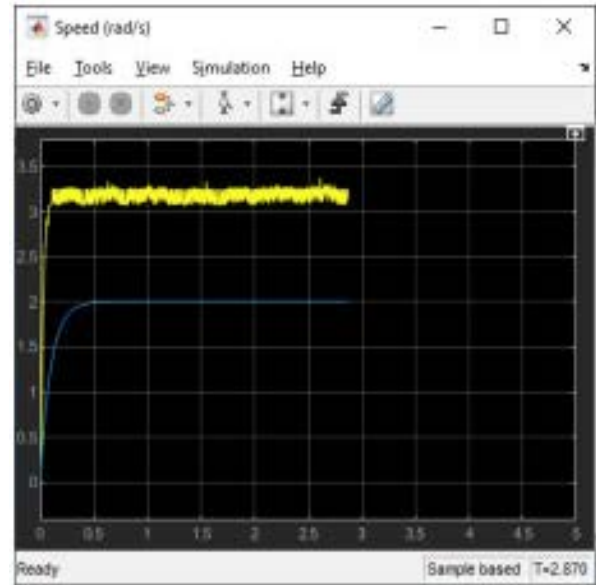


Figure 3. Load shaft speed response

2.4. Gain at Varying Frequencies:

In this part of the experiment, we will send an input sine wave at a specific frequency to the system and record the amplitude of the output signal. The frequency will be increased in each step, and the output amplitude will be observed accordingly.

To create the input sine wave:

1. In the Simulink diagram, double-click on the Signal Generator block and ensure the following settings:
 - Waveform: Sine
 - Amplitude: 1.0
 - Frequency: 1.0 Hz
 - Units: Hertz
2. Set the Amplitude (V) slider gain to 2.0 V.
3. Set the Offset (V) block to 0 V.
4. Run the q_servo_modeling QUARC controller to apply the 2V, 1 Hz sine wave to the Rotary Servo Base Unit.
5. The Rotary Servo Base Unit should begin rotating smoothly back and forth. In the following figures, you will observe the input and output signals corresponding to the applied sine wave. The output data will be recorded for analysis as the frequency is incremented.
6. Measure the maximum positive speed of the load shaft at $f = 1.0$ Hz input and enter it in the Table. As before, this measurement can be done directly from the scope using the Cursor Measurements tool.
7. Calculate the gain of the system (in both linear and dB units) and enter the results in the Table.
8. Increase the frequency to $f = 2.0$ Hz by adjusting the frequency parameter in the Signal Generator block. Measure the maximum load speed and calculate the gain. Repeat this step.

- Using the Matlab plot command and the data collected in the Table, generate a Bode magnitude plot. Make sure the amplitude and frequency scales are in decibels. When making the Bode plot, ignore the $f = 0$ Hz entry as the logarithm of 0 is not defined.

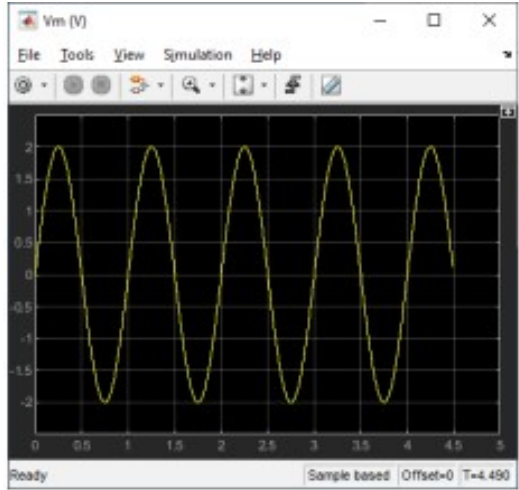


Figure 4. Input motor voltage

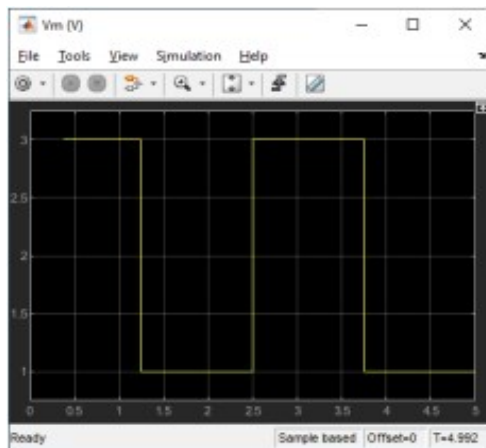


Figure 5. Square input motor voltage

rotating in the same direction and alternating between low and high speeds. The response can be observed in scope in Figure 5 and Figure 6.

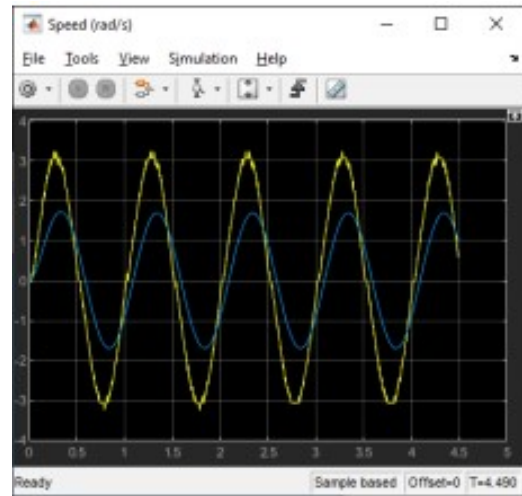


Figure 6. Load shaft speed

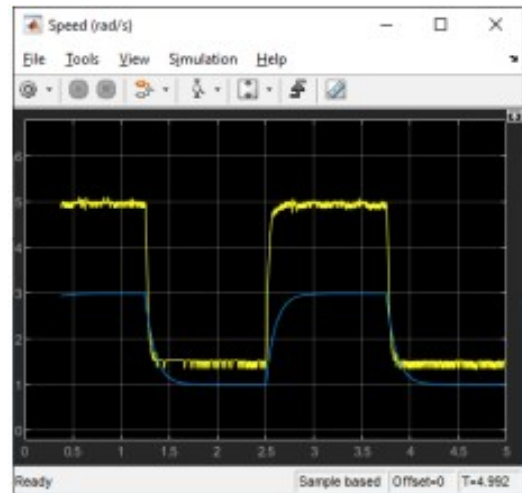


Figure 7. Load shaft speed step response

2.5. Step Response Experiment

In this part we will send step input to the rotary servo base unit, after that we will record the response of the load shaft; to achieve this part we can follow the following steps:

- Double-click on the Signal Generator block and ensure the following parameters are set:
 - Wave form: square
 - Amplitude: 1.0
 - Frequency: 0.4
 - Units: Hertz
- Set the Amplitude (V) gain block to 1.0 V.
- Set the Offset (V) gain block to 2.0 V.
- Open the load shaft speed scope, Speed (rad/s), and the motor input voltage cope, Vm (V).
- Click on QUARC | Build to compile the Simulink diagram.
- Select QUARC | Start to run the controller. The gears on the Rotary Servo Base Unit should be

3. Results

Table 1. Total results chart

f (Hz)	Amplitude (v)	Max Load Speed (rad/s)	Gain: G(w) (rad/s/v)	Gain: G(w) (rad/s/v, dB)
0	2	3.38	1.69	4.557734092
1	2	3.329	1.6645	4.42567599
2	2	3.069	1.5345	3.719337855
3	2	2.896	1.448	3.215371237
4	2	2.818	1.409	2.978219862
5	2	2.57	1.285	2.178062553
6	2	2.48	1.24	1.868433703
7	2	2.307	1.1535	1.240351977
8	2	2.17	1.085	0.708594764

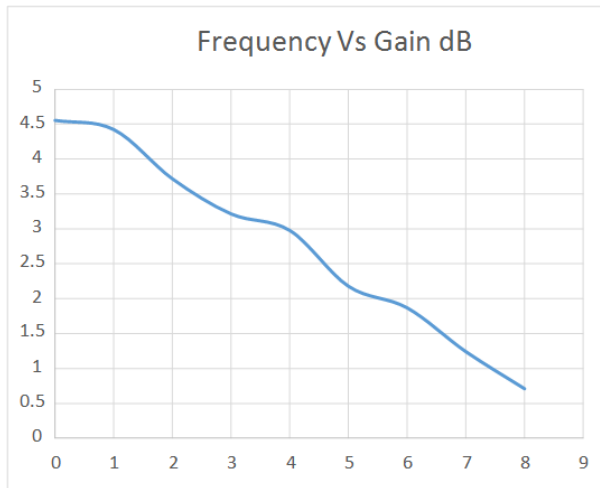


Figure 8. F vs Gain db

4. Conclusions

This experiment systematically explored the impact of varying the gain and time constant model parameters on the Rotary Servo Base Unit system's response. Both the gain, which amplifies the system's output in response to input variations, and the time constant, which determines the system's response speed and steady-state attainment, play crucial roles in shaping the overall behavior. Increasing the gain made the system more sensitive to input changes, while adjusting the time constant influenced the rate at which the system reached its steady state.

Despite the theoretical appeal of the nominal model, it failed to accurately represent the actual behavior of the Rotary Servo Base Unit. Two primary reasons for this discrepancy were identified: the nominal model assumes idealized conditions, neglecting real-world factors such as friction, backlash, and other non-linearities; and manufacturing tolerances and environmental variations introduced uncertainties, leading to deviations from the nominal model.

The comparison of three models—nominal, frequency response, and bump test models—revealed significant differences in accuracy. The nominal model, while providing a basic understanding, lacked precision due to idealized assumptions. The frequency response model, derived from systematic frequency variation experiments, offered valuable insights into the system's behavior across frequencies, improving accuracy. The bump test model, which captures transient responses to sudden changes, complemented the frequency response model by providing insights into dynamic characteristics, enhancing the overall system representation.

In conclusion, integrating different modeling approaches—nominal, frequency response, and bump test models—allowed for a more comprehensive understanding of the Rotary Servo Base Unit. This approach formed a holistic framework, balancing the strengths and limitations of each model. It offers a more accurate representation of the system's behavior, aiding informed decision-making in applications ranging from control system design to real-world implementations of the Rotary Servo Base Unit. This comprehensive modeling framework lays the groundwork for optimizing performance in various engineering applications.

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