

Geometric Deviation Representation and Variation Propagation Modeling in Multistage Machining Processes

Heping Peng*, Qianpeng Han

School of Intelligent Manufacturing, Jiangnan University, Wuhan 430056, China

*Corresponding author: hbjhun_penn@outlook.com

Received November 19, 2023; Revised December 20, 2023; Accepted December 27, 2023

Abstract The development of 3D manufacturing variation propagation model for multistage machining processes (MMPs) plays an important role for estimating the dimensional and geometric quality of machined part, monitoring the machining processes and diagnosing the variation sources, and realizing process planning evaluation and selection. In this paper, after the error sources and their propagation modes in the part machining processes are analyzed, the differential motion vectors are employed to describe locating datum-, fixture-, and machining-induced variations, a series of coordinate transformations of variation sources are used to realize the deviation accumulation and transformation, and the mathematical relationship between the key product characteristics and various machining process parameters is established, so as to realize the quantitative analysis of error propagation, error accumulation and coupling, and establish the linear explicit Stream of Variation model (SoV) of multistage machining processes. The establishment of the model reveals the dynamic evolution law of the generation, coupling and propagation of product error flow and provides effective scientific guidance for the optimal control, system design, and error source diagnosis of MMPs. The effectiveness of the proposed model is verified by a case study.

Keywords: multistage machining processes (MMPs), stream of variation (SoV), differential motion vector, variation propagation modeling

Cite This Article: Heping Peng, and Qianpeng Han, “Geometric Deviation Representation and Variation Propagation Modeling in Multistage Machining Processes.” American Journal of Mechanical Engineering, vol. 11, no. 4 (2023): 166-174. doi: 10.12691/ajme-11-4-6.

1. Introduction

The multistage machining processes (MMPs) are the most common, the most important, and the most difficult to control complex machining processes in manufacturing industry. In the MMPs, the workpiece in different machining processes may have different locating layouts, even need to be processed on different machine tools, it can be seen that the entire machining process has many error sources. The locating and processing of workpiece are two indispensable operations to complete the workpiece machining in each process. The locating operation is divided into two steps: the first step is to clamp the workpiece in the fixture; the second step is to clamp and locate the fixture on the machine tool. In the first step, when the feature surface manufactured by the previous process is used as the locating datum of the current processing, the machining errors of the previous process will accumulate to the current process to form the datum errors. In the second step of the locating operation, the fixture errors will also have an impact on the error accumulation of the workpiece. The machining errors can

be attributed to the machining operation. It can be seen that the workpiece errors in multistage machining is composed of several parts: the fixture errors and the machining errors occurred in the current machining process, and the datum errors inherited from the previous machining process [1,2]. If these deviations cannot be effectively controlled in time, with the accumulation of time and space, the size of the machined part will emerge a trend of mean shift and variance expansion. The mean shift means that product size deviates far from its design target value, while the variance expansion means that the product manufacturing size fluctuates more sharply.

The modeling of variation propagation and the diagnostic analysis of error sources in MMPs have become the critical techniques to evaluate geometric and dimensional quality of machined parts, generate robust process plans, eliminate downstream manufacturing issues, shorten production cycles and improve product quality. Currently, the state space model has been developed and extensively studied to represent the variation propagation of MMPs, in which the system control theory is applied to the variation analysis of MMPs, and the concept of output controllability in system control theory is employed to simulate and predict error propagation in multistage machining process [3].

2. Literature Review

The development of variation propagation model in MMPs is of great significance for machining variation prediction, fault diagnosis, process planning evaluation and selection, process-oriented tolerancing, etc. [4-6]. The research in this field is traced back to state space modeling method for the size control of sheet metal assemblies proposed by Jin and Shi [7]. Based on the SoV model, Ding et al. [8] further carried out the systematic diagnosability study for multi-station assembly process. Camelio et al. [9] analyzed several variation sources and proposed the evaluation method of variations propagation in a multi-stage compliant assembly system. However, these methods are used only in the multi-station assembly systems rather than the multistage machining processes. In order to expand the application scope of SoV model, Huang et al. [10] additionally proposed a systematic diagnostic method based on the SoV model for monitoring quality-related problems in the MMPs. Later, Huang et al. [11] further extended their own research to develop a state space model to describe the part dimensional errors and manufacturing variation stack-ups in the MMPs. Based on the modeling work, Huang and Shi [12] performed variation propagation analysis for multi-operational machining processes and developed a method for identifying erroneous operations. In the last two decades, the linearization of state space models has been studied extensively. Zhou et al. [13] improved the modeling work in Huang et al. [11] and developed a linear state space model by using differential motion vectors to describe the geometrical deviations of the parts. Du et al. [3] developed a linear state space model for multi-station machining system by linearizing the relationships between the part size features and machining errors. Du et al. [14] also extended the application of SoV to product lifecycle oriented quality control through an application instance. Wang et al. [15] formulated a variation propagation model using the equivalent fixture error concept, with which the datum and machine tool errors can be converted into equivalent fixture locator errors. Loose et al. [1] proposed a linear variation propagation model of MMPs through kinematic analysis. Liu and Jiang [2] derived a linear error propagation model for MMPs by using the rigid-body kinematics theories. Later, Yang et al. [16] conducted a comprehensive study on the three linearized SoV modeling methods in MMPs, namely differential motion vectors, equivalent fixture error and kinematic analysis, respectively.

In the studies of variation propagation modeling for MMPs, the most commonly used fixture scheme is the 3-2-1 locating scheme [11,13]. Along with research going deep, the fixture scheme is also extended beyond the 3-2-1 locating scheme based on punctual locators. The variation propagation model proposed by Loose et al. [1] can deal with the generic fixture layouts for workpieces other than the 3-2-1 layout situation. Abellán and Liu [5] extended the SoV model to consider the effect of fixture- and datum-induced variations for MMPs when fixtures with locating surfaces are applied. Yacob and Semere [17] proposed a dual quaternions-based method for modeling variation propagation in MMPs, which can not only mitigated the large matrix size but also achieved lower prediction errors. Based on techniques of dual quaternions

and skin model shapes, Yacob et al. [18] also further investigated the part quality prediction model in MMPs while taking into account the part with form errors and N-2-1 ($N>3$) fixture layout.

Shi [19] systematically studied the SoV methodology in his monograph, and developed the approaches for machining process monitoring, variation source diagnosis, rapid failure recovery, and defect prevention, etc. The SoV model was also employed to reveal how the roller dimensional errors are accumulated and propagated in the cold roll forming [20]. For the purpose of optimizing setup planning, the SoV model was extended to enable datum selection and setup sequence determination [21]. Abellán-Nebot et al. [22] further proposed the ways of process route optimization selection by combining shop floor quality data with the SoV model in the MMPs. Beyond that, in order to achieve more accurate variation prediction in MMPs, Abellán-Nebot et al. [4] proposed an extended SoV model by combining manufacturing-induced variation with the conventional SoV model; the model is capable of addressing more complex issues such as the product quality improvement, machining process diagnosis, and process-oriented tolerance allocation, etc. What follows was that Abellán-Nebot et al. [23] proposed a process-oriented tolerancing approach by using this extended SoV model. Abellán-Nebot et al. [6] also reviewed two 3D manufacturing variation models, namely the Stream of Variation model (SoV model) and the Model of the Manufactured Part (MoMP model), in the multi-stage manufacturing systems, and has made some analyses and comparisons on characteristics and applications of these two models. Based on the Piecewise Linear Regression Trees (PLRTs), Jin and Shi [24] proposed a modeling approach for MMP control. As an extension of the above modeling approach, Jin and Liu [25] used PLRT to identify multiple baseline models for multiple variation propagation in a Serial-Parallel MMP. Loose et al. [26] also integrated the GD&T characteristics of part features into the SoV model, so as to adapt the Geometric Dimensioning and Tolerancing (GD&T) standards in MMPs. Yacob and Semere [27] presented a multilayer shallow neural network regression approach for geometrical deviation prediction and variation source identification in MMPs. Du et al. [28] developed a three-dimensional variation propagation model for multistage turning processes of rotary workpieces. By extending the existing SoV model, Wang et al. [29] presented a unified variation propagation model for general shape workpieces in MMPs. Further more, Wang et al. [30] constructed a variation propagation framework in MMPs for the variable stiffness structure workpieces by combining the elastic deformation variations in the traditional SoV model. By considering the quality-cost equilibrium in MMPs, Wang et al. [31] also proposed a variation management framework for key control characteristics to achieve deviation control of key product characteristics. Zuo et al. [32] presented an error sensitivity analysis and precision allocation method for MMPs by combining multi-objective optimization technique with the SoV model.

The existing manufacturing variation models for the MMPs have a certain range of applications, some of which only integrate incomplete dynamic information of the manufacturing process. In this paper, a 3D digital

model of MMP is presented which is capable of integrating the dynamic variation sources during the machining process, such as locating datum-induced variations, fixture-induced variations, machining-induced variations. The remainder of the paper is organized as follows: Section 3 introduces geometric deviation representations and part vector model; Section 4 discusses the modeling methods of various variations in multistage machining process of part, and derives the variations propagation model; A case study is given in Section 5 for model validation; Finally, the conclusions and potential model applications are presented in Section 6.

3. Geometric Deviation Description

Suppose a part contains n feature surfaces related to error propagation, which include machined surfaces, design datum surfaces, manufacturing and measurement datum surfaces. Here, the vector surface model is employed to describe the part [33]. For feature surface $\mathbf{X}_i$ with direction vector $\mathbf{n}_i = [n_{ix}, n_{iy}, n_{iz}]^T$, position vector $\mathbf{p}_i = [p_{ix}, p_{iy}, p_{iz}]^T$ and size vector $\mathbf{d}_i = [d_{i1}, d_{i2}, \dots, d_{im}]^T$, it can be described by an $(6+m)$ dimensional vector as:

$$\mathbf{X}_i = [\mathbf{n}_i^T, \mathbf{p}_i^T, \mathbf{d}_i^T]^T_{(6+m) \times 1} \quad (i=1, 2, \dots, n) \quad (1)$$

where $\mathbf{n}_i = [n_{ix}, n_{iy}, n_{iz}]^T$, n_{ix} , n_{iy} and n_{iz} denote three direction cosines of feature vector along axes x , y and z in part coordinate system, respectively; $\mathbf{p}_i = [p_{ix}, p_{iy}, p_{iz}]^T$, p_{ix} , p_{iy} and p_{iz} are three position coordinate values of the feature vector in part coordinate system; $\mathbf{d}_i = [d_{i1}, d_{i2}, \dots, d_{im}]^T$, d_{i1} , d_{i2} , ..., d_{im} are dimension parameters of size vector $\mathbf{d}_i$, and m is the dimension of vector $\mathbf{d}_i$.

Part vector model is:

$$\mathbf{X} = [\mathbf{X}_1^T, \mathbf{X}_2^T, \dots, \mathbf{X}_n^T]^T_{n(6+m) \times 1} \quad (2)$$

The deviation is the amount by which surface feature $\mathbf{X}_i$ deviates from its nominal feature, then one has:

$$\Delta \mathbf{X}_i = [\Delta \mathbf{n}_i^T, \Delta \mathbf{p}_i^T, \Delta \mathbf{d}_i^T]^T_{(6+m) \times 1} \quad (3)$$

where $\Delta \mathbf{n}_i = [\Delta n_{ix}, \Delta n_{iy}, \Delta n_{iz}]^T$, $\Delta \mathbf{p}_i = [\Delta p_{ix}, \Delta p_{iy}, \Delta p_{iz}]^T$, $\Delta \mathbf{d}_i = [\Delta d_{i1}, \Delta d_{i2}, \dots, \Delta d_{im}]^T$.

The part deviation vector is denoted by $\Delta \mathbf{X}$, then one gets:

$$\Delta \mathbf{X} = [\Delta \mathbf{X}_1^T, \Delta \mathbf{X}_2^T, \dots, \Delta \mathbf{X}_n^T]^T_{n(6+m) \times 1} \quad (4)$$

4. Variation Propagation Modeling

4.1. Definition and Transformation of Coordinate Systems

As shown in Figure 1, in order to represent the manufacturing deviations and operation errors of part, three different coordinate systems, namely part coordinate system (PCS), fixture coordinate system (FCS), and machine tool coordinate system (MCS), are defined as reference systems.

- PCS is fixed to the part, the origin O_p , which is used to represent the geometric features of the part.
- FCS is fixed to the fixture, the origin O_F , in which the part is located and oriented.
- MCS is fixed at working table, the origin O_M , in which the fixture is located and oriented.

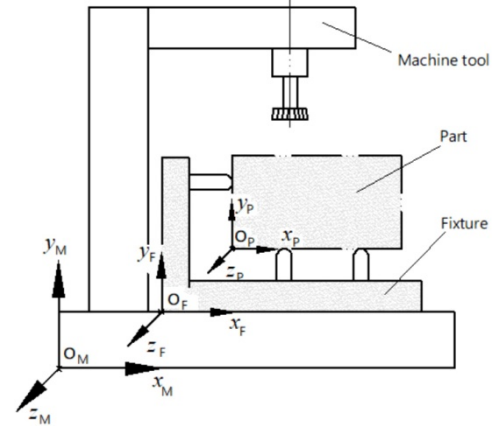


Figure 1. Coordinate systems

It is well known that any motion of a rigid body in three-dimensional coordinate system $O_0X_0Y_0Z_0$ can be achieved by three independent rotations (α , β , γ) around the original axes x_0 , y_0 and z_0 , and three independent translations (u , v , w) along these three coordinate axes, respectively. In this way, after a rotation transformation and a translation transformation being applied, the coordinates of any point (x_0, y_0, z_0) on the rigid body become (x, y, z) , which can be represented as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{R} \cdot \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + \mathbf{T} \quad (5)$$

where $\mathbf{T} = [u \ v \ w]^T$ denotes the translation matrix, u , v and w are three translation components; $\mathbf{R}$ denotes the rotation matrix and its expression is:

$$\mathbf{R} = \begin{bmatrix} C\beta C\gamma & S\alpha S\beta C\gamma - C\alpha S\gamma & C\alpha S\beta C\gamma + S\alpha S\gamma \\ C\beta S\gamma & S\alpha S\beta S\gamma + C\alpha C\gamma & C\alpha S\beta S\gamma - S\alpha C\gamma \\ -S\beta & S\alpha C\beta & C\alpha C\beta \end{bmatrix} \quad (6)$$

where α , β and γ are three rotation components; variables C and S in Eq.(6) correspond to the *Cosine* and *Sine* functions, respectively. For three differential rotations α , β , γ , the rotation matrix above can be approximated as:

$$\mathbf{R} \approx \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix} \quad (7)$$

In the same way, the transformation between any two coordinate systems also includes a rotation transformation and a translation transformation. If the i -th feature of a

part has m feature parameters, the rotation matrix of this feature from the PCS to the FCS at stage k can be expressed as:

$${}^i\mathbf{R}_P^F(k) = \begin{bmatrix} {}^0\mathbf{R}_P^F(k) & 0_{3 \times 3} & 0_{3 \times m} \\ 0_{3 \times 3} & {}^0\mathbf{R}_P^F(k) & 0_{3 \times m} \\ 0_{m \times 3} & 0_{m \times 3} & \mathbf{I}_{m \times m} \end{bmatrix}_{(6+m) \times (6+m)} \quad (8)$$

where $\mathbf{I}_{m \times m}$ is the identity matrix; the matrix ${}^0\mathbf{R}_P^F$ can be obtained according to Eq.(6).

For the part with n features, the rotation matrix of the whole part from the PCS to the FCS is:

$$\mathbf{R}_P^F(k) = \begin{bmatrix} {}^1\mathbf{R}_P^F(k) & & 0 \\ & {}^2\mathbf{R}_P^F(k) & \\ 0 & & \dots \\ & & {}^n\mathbf{R}_P^F(k) \end{bmatrix}_{n(6+m) \times n(6+m)} \quad (9)$$

Assuming that the origin coordinate of part in the FCS is $O_P [X_P^F(k), Y_P^F(k), Z_P^F(k)]$ at stage k ; then the translation matrix of the i -th feature of the part from the PCS to the FCS is:

$${}^i\mathbf{T}_P^F(k) = [0 \ 0 \ 0 \ X_P^F(k) \ Y_P^F(k) \ Z_P^F(k) \ 0 \ \dots \ 0]^T_{(6+m) \times 1} \quad (10)$$

At the k -th stage, the translation matrix of all parts features from the PCS to the FCS is:

$$\mathbf{T}_P^F(k) = [{}^1\mathbf{T}_P^F(k)]^T \ [{}^2\mathbf{T}_P^F(k)]^T \ \dots \ [{}^n\mathbf{T}_P^F(k)]^T]^T_{n(6+m) \times 1} \quad (11)$$

If the i -th feature of a part has m feature parameters, the rotation matrix of the i -th feature from the FCS to the MCS is:

$${}^i\mathbf{R}_F^M(k) = \begin{bmatrix} {}^0\mathbf{R}_F^M(k) & 0_{3 \times 3} & 0_{3 \times m} \\ 0_{3 \times 3} & {}^0\mathbf{R}_F^M(k) & 0_{3 \times m} \\ 0_{m \times 3} & 0_{m \times 3} & \mathbf{I}_{m \times m} \end{bmatrix}_{(6+m) \times (6+m)} \quad (12)$$

where $\mathbf{I}_{m \times m}$ is the identity matrix; the matrix ${}^0\mathbf{R}_F^M$ can be obtained according to Eq.(6).

For the part with n features, the rotation matrix of the whole part from the FCS to the MCS is:

$$\mathbf{R}_F^M(k) = \begin{bmatrix} {}^1\mathbf{R}_F^M(k) & & 0 \\ & {}^2\mathbf{R}_F^M(k) & \\ & & \dots \\ 0 & & {}^n\mathbf{R}_F^M(k) \end{bmatrix}_{n(6+m) \times n(6+m)} \quad (13)$$

Assuming that the origin coordinate of fixture in MCS is $O_F [X_F^M(k), Y_F^M(k), Z_F^M(k)]$ at stage k ; then the translation matrix of the i -th feature of the fixture from the FCS to the MCS is:

$${}^i\mathbf{T}_F^M(k) = [0 \ 0 \ 0 \ X_F^M(k) \ Y_F^M(k) \ Z_F^M(k) \ 0 \ \dots \ 0]^T_{(6+m) \times 1} \quad (14)$$

At the k -th stage, the translation matrix of all parts features from the FCS to the MCS is:

$$\mathbf{T}_F^M(k) = [{}^1\mathbf{T}_F^M(k)]^T \ [{}^2\mathbf{T}_F^M(k)]^T \ \dots \ [{}^n\mathbf{T}_F^M(k)]^T]^T_{n(6+m) \times 1} \quad (15)$$

4.2. Fixture-induced Variations

In practice, the 3-2-1 locating scheme is the most commonly used fixture scheme. As shown in Figure 2, the primary datum plane is determined by the locators H_1 , H_2 and H_3 ; the locators H_4 and H_5 constitute the second locating datum; and the locator H_6 is the tertiary locating datum.

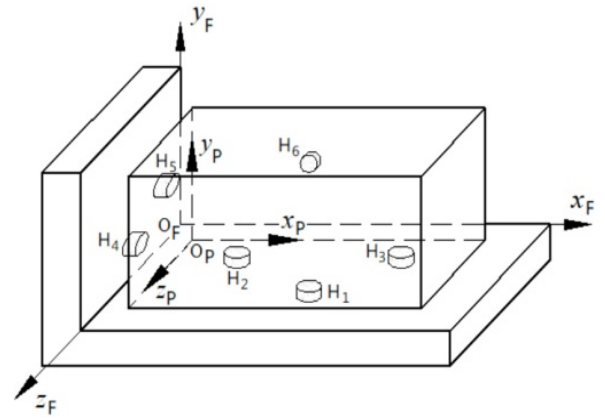


Figure 2. Fixture with 3-2-1 locating scheme

At stage k , the fixture vector in MCS can be expressed as:

$$\mathbf{H}(k) = [H_1^{yM} \ H_2^{yM} \ H_3^{yM} \ H_4^{xM} \ H_5^{xM} \ H_6^{zM}]^T \quad (16)$$

The fixture deviation caused by fixture setup elements can be expressed as:

$$\Delta\mathbf{H}(k) = [\Delta H_1^{yM} \ \Delta H_2^{yM} \ \Delta H_3^{yM} \ \Delta H_4^{xM} \ \Delta H_5^{xM} \ \Delta H_6^{zM}]^T \quad (17)$$

The deviations of the rotation and translation transformation induced by the fixture deviations are:

$$\Delta\mathbf{R}_F^M(k) = \left[\frac{\partial [\mathbf{R}_F^M(k)]_{ij}}{\partial \mathbf{H}(k)} \cdot \Delta\mathbf{H}(k) \right]_{n(6+m) \times n(6+m)} \quad (18)$$

$$\Delta\mathbf{T}_F^M(k) = \left[\frac{\partial [\mathbf{T}_F^M(k)]_i}{\partial \mathbf{H}(k)} \cdot \Delta\mathbf{H}(k) \right]_{n(6+m) \times 1} \quad (19)$$

where $[\mathbf{R}_F^M(k)]_{ij}$ represents the $\mathbf{R}_F^M(k)$ in the i -th row and the j -th column; $[\mathbf{T}_F^M(k)]_i$ represents the $\mathbf{T}_F^M(k)$ in the i -th row.

With the consideration of the impacts of fixture deviation, the transformation from FCS to MCS is:

$$\mathbf{R}_F^M(k) = {}^0\mathbf{R}_F^M(k) + \Delta\mathbf{R}_F^M(k) \quad (20)$$

$$\mathbf{T}_F^M(k) = {}^0\mathbf{T}_F^M(k) + \Delta\mathbf{T}_F^M(k) \quad (21)$$

where ${}^0\mathbf{R}_F^M(k)$ and ${}^0\mathbf{T}_F^M(k)$ are the nominal rotation and translation matrices of the part from the FCS to the MCS, respectively.

4.3. Locating Datum-induced Variations

The setup process of the machined part generally involves locating the workpiece on a fixture, and then fixing the workpiece together with the fixture on a machine tool table for processing. The variations caused by the locating datums will affect the error accumulations of part. Here, we define a datum selection matrix $\mathbf{D}(k)$ for selecting the locating datum surface of the part at stage k , which can be expressed as:

$$\mathbf{D}(k) = \begin{bmatrix} {}^1\mathbf{Q}(k) & & & 0 \\ & {}^2\mathbf{Q}(k) & & \\ & & \dots & \\ 0 & & & {}^n\mathbf{Q}(k) \end{bmatrix}_{n(6+m) \times n(6+m)} \quad (22)$$

where ${}^i\mathbf{Q}(k)$ is the $(6+m) \times (6+m)$ diagonal matrix. If surface $\mathbf{X}_i$ is chosen as the primary datum, the direction vector of the primary datum needs to be specified, and the ${}^i\mathbf{Q}(k)$ is expressed as follows:

$${}^i\mathbf{Q}(k) = \text{diag}(1, 1, 1, 0, \dots, 0)_{(6+m) \times (6+m)} \quad (23)$$

$\mathbf{D}_H(k)$ denotes locating datum, one gets:

$$\mathbf{D}_H(k) = \mathbf{D}(k) \cdot \mathbf{X}(k-1) \quad (24)$$

The datum deviation is:

$$\Delta\mathbf{D}_H(k) = \mathbf{D}(k) \cdot \Delta\mathbf{X}(k-1) \quad (25)$$

In actual parts processing, the wear, damage and loosening of fixture locating elements can cause locating datum deviation. The existence of locating datum deviation affects the rotation and translation transformation from the PCS to the FCS, and the resulting deviation is:

$$\Delta\mathbf{R}_P^F(k) = \left[\frac{\partial [\mathbf{R}_P^F(k)]_{ij}}{\partial \mathbf{D}_H(k)} \cdot \Delta\mathbf{D}_H(k) \right]_{n(6+m) \times n(6+m)} \quad (26)$$

$$\Delta\mathbf{T}_P^F(k) = \left[\frac{\partial [\mathbf{T}_P^F(k)]_i}{\partial \mathbf{D}_H(k)} \cdot \Delta\mathbf{D}_H(k) \right]_{n(6+m) \times 1} \quad (27)$$

where $[\mathbf{R}_P^F(k)]_{ij}$ represents the $[\mathbf{R}_P^F(k)]$ in the i -th row and the j -th column; $[\mathbf{T}_P^F(k)]_i$ represents the $\mathbf{T}_P^F(k)$ in the i -th row.

Then, the actual rotation and translation matrices from PCS to FCS will be written as:

$$\mathbf{R}_P^F(k) = {}^0\mathbf{R}_P^F(k) + \Delta\mathbf{R}_P^F(k) \quad (28)$$

$$\mathbf{T}_P^F(k) = {}^0\mathbf{T}_P^F(k) + \Delta\mathbf{T}_P^F(k) \quad (29)$$

where ${}^0\mathbf{R}_P^F(k)$ and ${}^0\mathbf{T}_P^F(k)$ are the nominal rotation and translation matrices of the part from the PCS to the FCS, respectively.

And according to Eqs.(20), (21), (28) and (29), the rotation and translation matrices from the PCS to the MCS can be obtained as follows:

$$\begin{aligned} \mathbf{R}_P^M(k) &= \mathbf{R}_F^M(k) \cdot \mathbf{R}_P^F(k) \\ &= [{}^0\mathbf{R}_F^M(k) + \Delta\mathbf{R}_F^M(k)] \cdot [{}^0\mathbf{R}_P^F(k) + \Delta\mathbf{R}_P^F(k)] \end{aligned} \quad (30)$$

$$\begin{aligned} \mathbf{T}_P^M(k) &= \mathbf{R}_F^M(k) \cdot \mathbf{T}_P^F(k) + \mathbf{T}_F^M(k) \\ &= [{}^0\mathbf{R}_F^M(k) + \Delta\mathbf{R}_F^M(k)] \cdot [{}^0\mathbf{T}_P^F(k) + \Delta\mathbf{T}_P^F(k)] + {}^0\mathbf{T}_F^M(k) + \Delta\mathbf{T}_F^M(k) \end{aligned} \quad (31)$$

By neglecting the higher-order terms beyond the first order of Eqs.(30) and (31), we get the deviation matrices of $\mathbf{R}_P^M(k)$ and $\mathbf{T}_P^M(k)$:

$$\Delta\mathbf{R}_P^M(k) \approx \Delta\mathbf{R}_F^M(k) \cdot {}^0\mathbf{R}_P^F(k) + {}^0\mathbf{R}_F^M(k) \cdot \Delta\mathbf{R}_P^F(k) \quad (32)$$

$$\Delta\mathbf{T}_P^M(k) \approx \Delta\mathbf{R}_F^M(k) \cdot {}^0\mathbf{T}_P^F(k) + {}^0\mathbf{R}_F^M(k) \cdot \Delta\mathbf{T}_P^F(k) + \Delta\mathbf{T}_F^M(k) \quad (33)$$

4.4. Machining-induced Variations

After the part is installed on the machine tool, its state vector $\mathbf{X}(k-1)$ corresponding to the transformation from the PCS to the MCS is rewritten as $\mathbf{X}^M(k-1)$; And a new part state vector denoted as $\mathbf{X}^M(k)$ is generated after the part has been processed at stage k . At this moment, part features can be divided into those that need to be machined and those that do not need to be machined in the current operation. Let $\mathbf{X}^{M*}(k)$ be the state vector of the feature machined in the k -th process and $\mathbf{B}(k)$ be the selection matrix of machined features, and the machined feature in the k -th process will be $\mathbf{B}(k) \cdot \mathbf{X}^{M*}(k)$, then the expression of $\mathbf{X}^M(k)$ can be obtained as follows:

$$\mathbf{X}^M(k) = \mathbf{A}(k) \cdot \mathbf{X}^M(k-1) + \mathbf{B}(k) \cdot \mathbf{X}^{M*}(k) \quad (34)$$

where $\mathbf{B}(k)$ labels the features to be processed at stage k , $\mathbf{A}(k)$ labels the non-processing features at stage k , and $\mathbf{A}(k) = \mathbf{I} - \mathbf{B}(k)$.

$$\begin{aligned} \mathbf{B}(k) &= \text{diag}[\mathbf{I}_1(k), \mathbf{I}_2(k), \dots, \mathbf{I}_i(k), \dots, \mathbf{I}_n(k)]_{n(6+m) \times n(6+m)} \end{aligned} \quad (35)$$

where $\mathbf{I}_i(k) = \begin{cases} \mathbf{I}_{(6+m) \times (6+m)} & \mathbf{X}_i \text{ is processed at } k \\ 0 & \text{Otherwise} \end{cases}$

From Eq. (34), the part features are segmented into the machined features $\mathbf{B}(k) \cdot \mathbf{X}^{M*}(k)$ and the non-machined

features $\mathbf{A}(k) \cdot \mathbf{X}^M(k-1)$ at stage k . Machining deviation of part which depends on $\mathbf{B}(k) \cdot \mathbf{X}^{M*}(k)$ is caused by the machining operation.

4.5. Variations Propagation Model

At stage k , the part state vector $\mathbf{X}^M(k-1)$ in the MCS can be obtained by performing the rotation and translation transformation on vector $\mathbf{X}(k-1)$, that is:

$$\mathbf{X}^M(k-1) = \mathbf{R}_P^M(k) \cdot \mathbf{X}(k-1) + \mathbf{T}_P^M(k) \quad (36)$$

According to Eqs.(34) and (36), one gets:

$$\mathbf{X}^M(k) = \mathbf{A}(k) \cdot [\mathbf{R}_P^M(k) \cdot \mathbf{X}(k-1) + \mathbf{T}_P^M(k)] + \mathbf{B}(k) \cdot \mathbf{X}^{M*}(k) \quad (37)$$

$$\mathbf{X}^M(k) = \mathbf{R}_P^M(k) \cdot \mathbf{X}(k) + \mathbf{T}_P^M(k) \quad (38)$$

Combining Eqs.(37) and (38), $\mathbf{X}(k)$ can be derived as follows.

$$\mathbf{X}(k) = \mathbf{R}_M^P(k) \cdot \mathbf{A}(k) \cdot \mathbf{R}_P^M(k) \cdot \mathbf{X}(k-1) + \mathbf{R}_M^P(k) \cdot \mathbf{B}(k) \cdot [\mathbf{X}^{M*}(k) - \mathbf{T}_P^M(k)] \quad (39)$$

Here,

$$\begin{aligned} \mathbf{X}^{M*}(k) &= {}^0\mathbf{X}^M(k) + \Delta\mathbf{X}^{M*}(k) \\ &= {}^0\mathbf{R}_P^M(k) \cdot {}^0\mathbf{X}(k) + {}^0\mathbf{T}_P^M(k) + \Delta\mathbf{X}^{M*}(k) \end{aligned} \quad (40)$$

Eq. (39) can be re-written as follows.

$$\begin{aligned} \mathbf{X}(k) &= \mathbf{R}_M^P(k) \cdot \mathbf{A}(k) \cdot \mathbf{R}_P^M(k) \cdot \mathbf{X}(k-1) \\ &+ \mathbf{R}_M^P(k) \cdot \mathbf{B}(k) \cdot [{}^0\mathbf{R}_P^M(k) \cdot {}^0\mathbf{X}(k) + \Delta\mathbf{X}^{M*}(k) - \Delta\mathbf{T}_P^M(k)] \end{aligned} \quad (41)$$

We also have $\mathbf{X}(k) = {}^0\mathbf{X}(k) + \Delta\mathbf{X}(k)$ and $\mathbf{X}(k-1) = {}^0\mathbf{X}(k-1) + \Delta\mathbf{X}(k-1)$, then Eq. (41) can be re-described as follows.

$$\begin{aligned} &{}^0\mathbf{X}(k) + \Delta\mathbf{X}(k) \\ &= \mathbf{R}_M^P(k) \cdot \mathbf{A}(k) \cdot \mathbf{R}_P^M(k) \cdot [{}^0\mathbf{X}(k-1) + \Delta\mathbf{X}(k-1)] \\ &+ \mathbf{R}_M^P(k) \cdot \mathbf{B}(k) \cdot [{}^0\mathbf{R}_P^M(k) \cdot {}^0\mathbf{X}(k) + \Delta\mathbf{X}^{M*}(k) - \Delta\mathbf{T}_P^M(k)] \end{aligned} \quad (42)$$

There exists the following equation.

$$\begin{aligned} \Delta\mathbf{X}(k) &= \mathbf{R}_M^P(k) \cdot \mathbf{A}(k) \cdot \Delta\mathbf{R}_P^M(k) \cdot {}^0\mathbf{X}(k) \\ &+ \mathbf{R}_M^P(k) \cdot \mathbf{A}(k) \cdot \mathbf{R}_P^M(k) \cdot \Delta\mathbf{X}(k-1) \\ &+ \mathbf{R}_M^P(k) \cdot \mathbf{B}(k) \cdot [\Delta\mathbf{X}^{M*}(k) - \Delta\mathbf{T}_P^M(k)] \end{aligned} \quad (43)$$

The rotation matrix $\mathbf{R}_M^P(k)$ of part from the MCS to the PCS, namely the inverse matrix of $\mathbf{R}_P^M(k)$, can be expressed as:

$$\begin{aligned} \mathbf{R}_M^P(k) &= \mathbf{R}_F^P(k) \cdot \mathbf{R}_M^F(k) \\ &= [{}^0\mathbf{R}_F^P(k) + \Delta\mathbf{R}_F^P(k)] \cdot [{}^0\mathbf{R}_M^F(k) + \Delta\mathbf{R}_M^F(k)] \end{aligned} \quad (44)$$

where $\mathbf{R}_M^F(k)$ and $\mathbf{R}_F^P(k)$ are the rotation matrices from the MCS to the FCS and from the FCS to the PCS, respectively.

Substituting Eqs. (30) and (44) into Eq. (43) and neglecting the higher-order terms beyond the first order, we have:

$$\begin{aligned} \Delta\mathbf{X}(k) &= {}^0\mathbf{R}_F^P(k) \cdot {}^0\mathbf{R}_M^F(k) \cdot \mathbf{A}(k) \cdot \Delta\mathbf{R}_P^M(k) \cdot {}^0\mathbf{X}(k) \\ &+ {}^0\mathbf{R}_F^P(k) \cdot {}^0\mathbf{R}_M^F(k) \cdot \mathbf{A}(k) \cdot {}^0\mathbf{R}_F^M(k) \cdot {}^0\mathbf{R}_P^F(k) \cdot \Delta\mathbf{X}(k-1) \\ &+ {}^0\mathbf{R}_F^P(k) \cdot {}^0\mathbf{R}_M^F(k) \cdot \mathbf{B}(k) \cdot [\Delta\mathbf{X}^{M*}(k) - \Delta\mathbf{T}_P^M(k)] \end{aligned} \quad (45)$$

Substituting Eqs. (32) and (33) into the above equation, we further get:

At the right side of Eq. (46), the first term represents the machining deviations accumulated from the previous process to the current process, the remaining items are the deviations caused by machining errors, fixture errors and datum errors of current process, respectively.

$$\begin{aligned} \Delta\mathbf{X}(k) &= {}^0\mathbf{R}_F^P(k) \cdot {}^0\mathbf{R}_M^F(k) \cdot \mathbf{A}(k) \cdot {}^0\mathbf{R}_F^M(k) \cdot {}^0\mathbf{R}_P^F(k) \cdot \Delta\mathbf{X}(k-1) \\ &+ {}^0\mathbf{R}_F^P(k) \cdot {}^0\mathbf{R}_M^F(k) \cdot \mathbf{A}(k) \\ &\cdot \{\Delta\mathbf{R}_F^M(k) \cdot {}^0\mathbf{R}_P^F(k) + {}^0\mathbf{R}_F^M(k) \cdot \Delta\mathbf{R}_P^F(k)\} \cdot {}^0\mathbf{X}(k) \\ &+ {}^0\mathbf{R}_F^P(k) \cdot {}^0\mathbf{R}_M^F(k) \cdot \mathbf{B}(k) \\ &\cdot [\Delta\mathbf{X}^{M*}(k) - \Delta\mathbf{R}_F^M(k) \cdot {}^0\mathbf{T}_P^F(k) - {}^0\mathbf{R}_F^M(k) \cdot \Delta\mathbf{T}_P^F(k) - \Delta\mathbf{T}_P^M(k)] \end{aligned} \quad (46)$$

5. Model Validation

The proposed model is validated on a workpiece with multistage machining processes whose design specifications are shown in Figure 3. According to the design specifications, the key product characteristics are determined as follows: the distance between surface X_2 and datum surface A and the parallelism requirement of surface X_2 with respect to datum surface A ; The distance between surface X_6 and datum surface A and the parallelism requirement of surface X_6 with respect to datum surface A ; The distance between surface X_7 and datum surface B , the parallelism requirement of surface X_7 with respect to datum surface B and the positional requirement of surface X_8 relative to datum surface B .

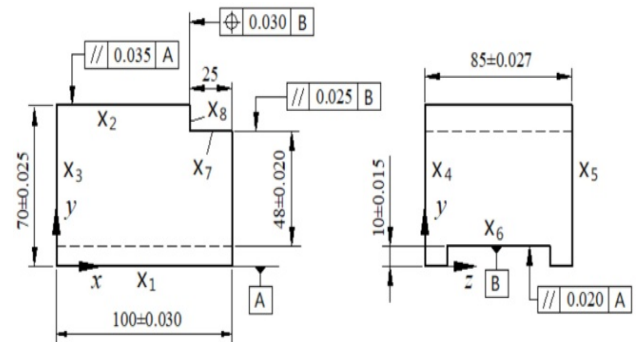
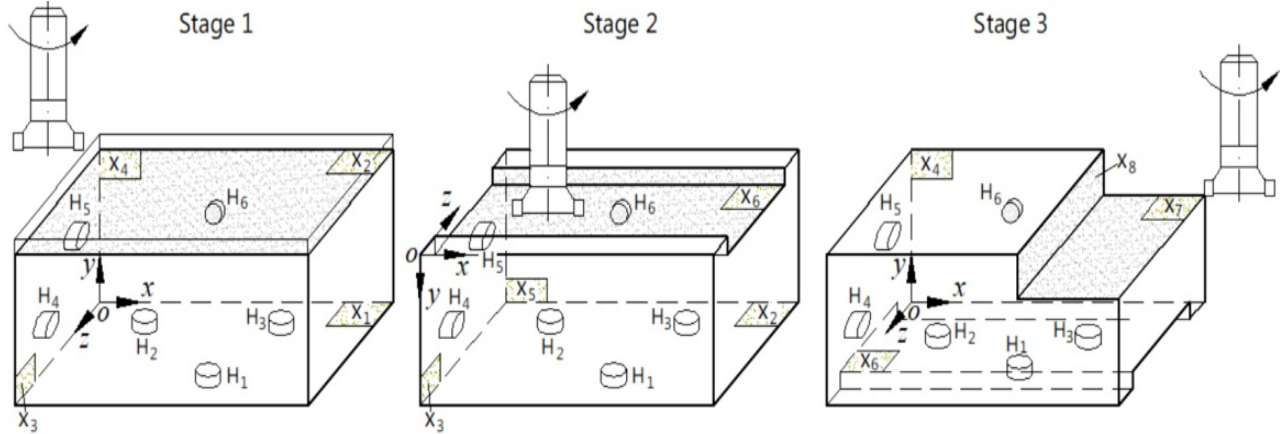


Figure 3. Part design specifications

Table 1. Part locating datum and machining sequence

Stage	Datum surfaces	Machined surfaces	Specifications
1	X_1, X_3, X_4	X_2	$70(\pm 0.025)$ Parallelism 0.035
2	X_2, X_3, X_5	X_6	$10(\pm 0.015)$ Parallelism 0.020
3	X_6, X_3, X_4	X_7, X_8	$48(\pm 0.020)$ Parallelism 0.025 Position 0.030

**Figure 4.** The multistage machining processes of part 5.2. Model Parameter

Three machining operations are required to satisfy part design specifications. The first operation is to mill surface X_2 by choosing surfaces X_1, X_3 and X_4 as the locating datums; followed by the milling of surface X_6 by taking the machined surface X_2 in the first step, surfaces X_3 and X_5 as the locating datums; Finally, the machined surfaces X_6 , surfaces X_3 and X_4 are employed as locating datums to mill the surfaces X_7 and X_8 . The operation sequence of the workpiece is graphically shown in Figure 4, where the locating datums are achieved by six locators, namely $H_1, H_2, \dots, H_6$, and the primary locating datum consists of H_1, H_2 and H_3 . Each operation, the corresponding locating datum and machining sequence are list in Table 1.

5.1. Part Vector Model

According to the previous section, the part vector model can be constructed by using eight feature planes $X_1, X_2, X_3, X_4, X_5, X_6, X_7$ and X_8 .

$$\mathbf{X}(k) = \left[\mathbf{X}_1(k)^T, \mathbf{X}_2(k)^T, \dots, \mathbf{X}_8(k)^T \right]^T$$

And the deviation vector $\Delta \mathbf{X}$ is:

$$\Delta \mathbf{X}(k) = \left[\Delta \mathbf{X}_1(k)^T, \Delta \mathbf{X}_2(k)^T, \dots, \Delta \mathbf{X}_8(k)^T \right]^T$$

For the representation of manufacturing deviation of the part, we assume that coordinate plane xoz of PCS coincides with the feature plane X_1 ; and the origin of PCS is the intersection point of three mutually perpendicular planes X_1, X_3 and X_4 , as shown in Figure 4. The nominal orientations, positions and feature parameters of these surface features in the final stage are listed in Table 2.

Note that these data in Table 2 are relative to the PCS.

Table 2. Part feature vectors

X_i	$\mathbf{n}_i = [n_{ix}, n_{iy}, n_{iz}]^T$	$\mathbf{p}_i = [p_{ix}, p_{iy}, p_{iz}]^T$	$\mathbf{d}_i$
Plane X_1	[0, -1, 0]	[50, 0, 42.5]	0
Plane X_2	[0, 1, 0]	[37.5, 70, 42.5]	0
Plane X_3	[-1, 0, 0]	[0, 35, 42.5]	0
Plane X_4	[0, 0, -1]	[50, 35, 0]	0
Plane X_5	[0, 0, 1]	[50, 35, 85]	0
Plane X_6	[0, -1, 0]	[50, 10, 42.5]	0
Plane X_7	[0, 1, 0]	[87.5, 58, 42.5]	0
Plane X_8	[1, 0, 0]	[75, 64, 42.5]	0

Table 3 lists the parameters required to construct the variation propagation model, in which the Euler angles α, β , and γ are used to construct the rotation matrices ${}^0\mathbf{R}_P^F(k)$ and ${}^0\mathbf{R}_F^M(k)$; and p_x, p_y , and p_z are used for translation matrices ${}^0\mathbf{T}_P^F(k)$ and ${}^0\mathbf{T}_F^M(k)$. The fixture errors are used to build matrices $\Delta \mathbf{R}_F^M(k)$ and $\Delta \mathbf{T}_F^M(k)$; and the machining errors are used to construct matrix $\Delta \mathbf{X}^{M*}(k)$.

Table 3. Setup Operations

3	PCS to FCS (nominal)	FCS to MCS (nominal)	Fixture errors	Machining errors
α (rad)	0	0	0.0002	0.0009
β (rad)	0	0	0.0003	0.0008
γ (rad)	0	0	0.0005	0.0009
p_x (mm)	60	0	0.0036	0.0058
p_y (mm)	45	0	0.0055	0.0089
p_z (mm)	50	0	0.0028	0.0090

5.3. Analysis of Machining Operations

The part processing operations are implemented on a processing station using 3-2-1 fixtures involve three stage operations to meet the design specifications. The setting of the two nominal coordinate systems PCS and FCS is shown in Figure 2. And without loss of generality, we assume that the nominal MCS coincides with the nominal FCS exactly, in this case, ${}^0\mathbf{R}_F^M(k)$ and ${}^0\mathbf{R}_M^F(k)$ are identity matrices. We assume that there is no datum error in the first stage. In contrast, the datum errors in the second and third stages will contribute to the total error of the part, because the primary positioning datums used in these stages are the surface features processed in the previous processes.

5.4. Simulation Results

In order to predict the variations of key product characteristics, a program based on the proposed model has been developed in MATLAB R2014b for simulation and calculation. After entering the fixture errors and machining errors (Table 3) into the program, the prediction variations at each stage can be obtained. The program prediction results and the actual CMM measurements for key product characteristics are listed in Table 4.

Table 4. Comparison the model prediction values with the CMM measurements (unit: mm)

Char.	Spec.	Pred.	CMM
D _{X2}	70(±0.025)	69.9776	69.9860
Pa _{X2}	Parallelism 0.035	0.0328	0.0305
D _{X6}	10 (±0.015)	9.9980	10.0085
Pa _{X6}	Parallelism 0.020	0.0153	0.0195
D _{X7}	48 (±0.020)	48.0142	47.9895
Pa _{X7}	Parallelism 0.025	0.0175	0.0118
Po _{X8}	Position 0.030	0.0229	0.0126

By comparing the experiment measurements and the model prediction values in Table 4, we can draw the following conclusions:

- It can be seen from Table 4 that there are tiny deviations between the model prediction results and the actual measurement values, and this deviation range is acceptable, thus indicating the effectiveness of the proposed model.
- The differences between the model prediction results and the actual measurement values of feature surface X₂ are smaller than those of feature surface X₆. This is because the milling operation of surface X₆ in stage 2 takes the surface X₂ machined in stage 1 as the primary locating datum, so the machining errors of surface X₂ are accumulated in the machining of surface X₆. Similarly, the feature surface X₆ machined in stage 2 needs to be taken as the primary locating datum when processing feature surfaces X₇ and X₈ in stage 3, the machining errors of surface X₆ form the locating errors of surfaces X₇ and X₈ in the machining process, which results in the differences between the model prediction results

and the actual measurement values of feature surfaces X₇ and X₈ are generally greater than those of feature surface X₆. All of these indicate the propagation and accumulation of machining errors in multistage machining processes.

6. Conclusions

The multistage machining process is a typical discrete dynamic process. In this paper, after analyzing the product machining processes, the key product characteristics and key control characteristics that affect the product processing quality are extracted, and a part vector model for product feature deviation control is obtained. On the basis of this model, the differential motion vectors are employed to represent various variation sources on the workpiece. Through a series of coordinate transformations of these variation sources, the mathematical relationships between key product characteristics and these error sources are captured, and the linear explicit state space model of error propagation in MMPs is constructed.

For future studies, the proposed model needs to be extended to take into account the affects of various other variations. The model presented in this paper mainly considers the variations caused by the locating datum, fixture and machining, but there are still many variation errors that are not included, such as vibration errors of machine tools and cutting tools, thermal deformation errors, etc. In order to improve the prediction accuracy of the model, comprehensively considering these errors in the model is one of the future research directions. In addition, the multi-scale variations should also be considered in the model. The current study is limited to dimensional errors; In fact, geometrical errors, waviness and roughness also participate in the propagation and accumulation of errors, which will result in the variations of machining features. Therefore, considering the effect of multi-scale variation in the model is also a research direction in the future. Besides, the variation propagation model can be applied to many fields, such as: optimal allocation of process tolerances, optimal selection of process parameters, optimization design of detection strategies, quality monitoring and process diagnostics of products, etc. The proposed model can ultimately achieve the purpose of controlling product quality, improving the stability of manufacturing system, and enhancing the competitiveness of products in the market.

ACKNOWLEDGMENTS

The authors would like to acknowledge the financial support of the National Natural Science Foundation of China (Grant No. 52075222).

References

- [1] Loose J P, Zhou S, Ceglarek D. Kinematic analysis of dimensional variation propagation for multistage machining processes with general fixture layouts. *IEEE T Autom Sci Eng*, 2007, 4(2): 141-152.
- [2] Liu D and Jiang P. Modelling of machining error flow based on form features for multistage processes. *Int J Comput Integ Manuf*,

- 2009, 22(9): 857-876.
- [3] Du S, Xi L, Pan E, et al. Modeling and control of dimensional quality of a serial multi-station machining system. *Int J Reliab Qual Saf Eng*, 2006, 13(5): 399-419.
 - [4] Abellán-Nebot J V, Liu J, Subirón F R, et al. State space modeling of variation propagation in multistation machining processes considering machining-induced variations. *J Manuf Sci Eng*, 2012, 134(2): 1-13.
 - [5] Abellán J V and Liu J. Variation propagation modelling for multi-station machining processes with fixtures based on locating surfaces. *Int J Prod Res*, 2013, 51, (15): 4667-4681.
 - [6] Abellán-Nebot J V, Subirón F R, Mira J S. Manufacturing variation models in multi-station machining systems. *Int J Adv Manuf Technol*, 2013, 64 (1-4): 63-83.
 - [7] Jin J and Shi J. State space modeling of sheet metal assembly for dimensional control. *ASME J Manuf Sci Eng*, 1999, 121(4): 756-762.
 - [8] Ding Y, Shi J, Ceglarek D. Diagnosability analysis of multi-station manufacturing processes. *J Dyn Syst Meas Control*, 2002, 124(1): 1-13.
 - [9] Camelio J A, Hu S J, Ceglarek D J. Modeling variation propagation of multi-station assembly systems with compliant parts. *J Mech Design*. 2003, 125: 673-681.
 - [10] Huang Q, Zhou S, Shi J. Diagnosis of multi-operational machining processes through variation propagation analysis. *Robot Cim-Int Manuf*, 2002, 18(3-4): 233-239.
 - [11] Huang Q, Shi J, Yuan J. Part dimensional error and its propagation modeling in multi-operational machining processes. *J Manuf Sci Eng*, 2003, 125(2): 255-262.
 - [12] Huang Q and Shi J. Variation transmission analysis and diagnosis of multi-operational machining processes. *IIE Trans*, 2004, 36(9): 807-815.
 - [13] Zhou S, Huang Q, Shi J. State space modeling of dimensional variation propagation in multistage machining process using differential motion vectors. *IEEE Trans Robot Autom*, 2003, 19(2): 296-309.
 - [14] Du S, Xi L, Ni J, et al. Product lifecycle-oriented quality and productivity improvement based on stream of variation methodology. *Comput Ind*, 2008, 59(2-3): 180-192.
 - [15] Wang H, Huang Q, Katz R. Multi-operational machining processes modeling for sequential root cause identification and measurement reduction. *J Manuf Sci Eng*, 2005, 127: 512-521.
 - [16] Yang F, Jin S, Li Z. A comprehensive study of linear variation propagation modeling methods for multistage machining processes. *Int J Adv Manuf Technol*, 2017, 90: 2139-2151.
 - [17] Yacob F and Semere D. Variation propagation modelling in multistage machining processes using dual quaternions. *Int J Adv Manuf Technol*, 2020, 111(9-10): 2987-2998.
 - [18] Yacob F, Semere D, Anwer N. Variation propagation modeling in multistage machining processes considering form errors and N-2-1 fixture layouts. *Int J Adv Manuf Technol*, 2021, 116(1): 507-522.
 - [19] Shi J. Stream of variation modeling and analysis for multistage manufacturing processes. CRC Press, Taylor & Francis Group: Boca Raton, FL, 2007.
 - [20] Zhang L, Ni J, Lai X. Dimensional errors of rollers in the stream of variation modeling in cold roll forming process of quadrate steel tube. *Int J Adv Manuf Technol*, 2008, 37: 1082-1092.
 - [21] Liu J, Shi J, Hu S J. Quality-assured setup planning based on the stream-of-variation model for multi-stage machining processes. *IIE Trans*, 2009, 41(4): 323-334.
 - [22] Abellán-Nebot J V, Liu J, Subirón F R. Design of multi-station manufacturing processes by integrating the stream-of-variation model and shop-floor data. *J Manuf Syst*, 2011, 30(2): 70-82.
 - [23] Abellán-Nebot J V, Liu J, Subirón F R. Process-oriented tolerancing using the extended stream of variation model. *Comput Ind*, 2013, 64(5): 485-498.
 - [24] Jin R and Shi J. Reconfigured piecewise linear regression tree for multistage manufacturing process control. *IIE Trans*, 2012, 44: 249-261.
 - [25] Jin R and Liu K. Multimode variation modeling and process monitoring for serial-parallel multistage manufacturing processes. *IIE Trans*, 2013, 45(6): 617-629.
 - [26] Loose J P, Zhou Q, Zhou S, et al. Integrating GD&T into dimensional variation models for multistage machining processes. *Int J Prod Res*, 2010, 48, (11): 3129-3149.
 - [27] Yacob F and Semere D. A multilayer shallow learning approach to variation prediction and variation source identification in multistage machining processes. *J Intell Manuf*, 2021, 32(4): 1173-1187.
 - [28] Du S, Yao X, Huang D, et al. Three-dimensional variation propagation modeling for multistage turning process of rotary workpieces. *Comput Ind Eng*, 2015, 82: 41-53.
 - [29] Wang K, Du S, Xi L. Three-dimensional tolerance analysis modelling of variation propagation in multi-stage machining processes for general shape workpieces. *Int J Precis Eng Manuf*, 2020, 21: 31-44.
 - [30] Wang K, Li G, Du S, et al. State space modelling of variation propagation in multistage machining processes for variable stiffness structure workpieces. *Int J Prod Res*, 2021, 59 (13): 4033-4052.
 - [31] Wang K, Yin Y, Du S, et al. Variation management of key control characteristics in multistage machining processes considering quality-cost equilibrium. *J Manuf Syst*, 2021, 59(6): 441-452.
 - [32] Zuo X, Li B, Yang J. Error sensitivity analysis and precision distribution for multi-operation machining processes based on error propagation model. *Int J Adv Manuf Technol*, 2016, 86: 269-280.
 - [33] Martinsen K. Vectorial tolerancing for all types of surfaces. *ASME Adv Des Automat*, 1993, 2: 187-198.

