

Characterization and Modeling of Rigid Pile Behavior

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Abstract The aim of this study is to gain a better understanding of the behavior of rigid piles subjected to lateral loading and head moment in clay soil, taking into account soil-structure interaction. In order to establish the behavioral model of the assembly, soil-pile interaction models according to different authors were presented. According to several authors, the soil-pile interface is characterized by the soil reaction modulus E_s . This parameter, which characterizes the interaction between the soil and the structure, depends not only on the mechanical and geometric properties of the concrete, but also on the foundation soil. Within the framework of our work, the model of [1] is used to determine the modulus of reaction E_s . This is a function of the rheological parameters of the soil (α and E_M) and the geometric characteristics of the pile. After solving the behavior model of the rigid soil-pile assembly, the Python programming tool was used to perform the parametric analysis. The results showed the significant impact of the interaction model adopted and the forces applied at the pile head. The study shows that soil parameters have a greater influence on foundation behavior than pile parameters. The results also show that the forces applied at the pile head have a non-negligible influence on the behavior model.

Keywords: Rigid pile, lateral loads, soil-pile interaction, analytical calculation, numerical modelling

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1. Introduction

At present, vertical piles are designed to withstand axial loads, but they can sometimes be subjected to lateral loads. The latter may be quasi-static (impact of a ship docking, sudden braking of a convoy on a bridge, etc.) or dynamic (swell, effect of wind on structures, earthquake, etc.). The calculation of foundation structures is an important and complex task, since it involves soil-structure interaction. In this context, a complete characterization of the soil-pile interaction is a prerequisite for good control of the structure's behavior, and this has led to the development of several research works [1,2,3,4,5,6,7,8]. It is in this context that we are interested in studying the behavior of rigid piles under lateral loads. In this work, the pile is considered as a for and the soil is modeled as a set of horizontal springs with a reaction modulus E_s . The aim of this work is to characterize the soil-pile interaction and establish a behavior model for the assembly. This model will then be solved in order to carry out a parametric study.

2. Modeling the Behavior of Rigid Piles Under Lateral Loads and Characterizing Soil-pile Interaction

Several studies have been carried out to characterize the behavior of piles under lateral loads. These studies have led to the development of several approaches that can be classified into four categories:

- Reaction modulus method [9],
- p - y curve method [10] & [11],
- Elastic continuum method [12],
- Finite element or finite difference numerical methods.

Although somewhat complex, finite element and finite difference methods are often used.

2.1. Behavior Model

Winkler's analytical method is the oldest, and can be used to predict the lateral reaction of the soil. It involves modeling the interaction between the soil and the pile using a series of independent springs of varying stiffness. The stiffness provides a direct link between the lateral reaction of the soil (p) and the lateral displacement of the pile (y) under lateral loading (Figure 1). This method is the basis of the p - y curves, where the "springs" have a non-linear behavior.

Winkler's model defines soil as a stack of independent slices. Each slice of soil is modelled by a lateral spring (Figure 2) on which the pile rests.

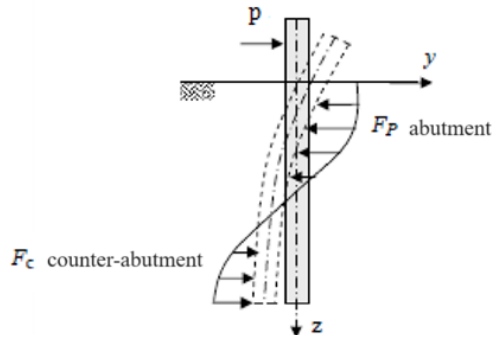


Figure 1. Pile mobilizing lateral soil reaction [9]

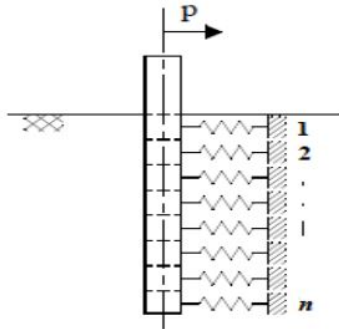


Figure 2. Representation of the Winkler Model [9]

The pressure p on a soil "slice" depends only on its lateral displacement and on a soil reaction coefficient, called k_h (MN / m^3) in the case of lateral loading.

$$p = k_h(z) \cdot D \cdot y \quad (1)$$

This equation is also expressed in the following form:

$$p = E_S \cdot y \quad (2)$$

Where

P : Soil reaction per unit length of pile

E_S : Soil reaction modulus, ($E_S = k_h \cdot D$)

D : Pile diameter or width

The pile is idealized as a laterally loaded elastic beam. The soil is modelled by horizontal springs, independent of each other, and of stiffness E_S . Thus, the pile's behavior is governed by the equation of a beam on elastic supports:

$$E_p I_p \frac{d^4 y}{dz^4} = -p \cdot D \quad (3)$$

This leads to the following equation, which governs the behavior of the pile.

$$E_p I_p \frac{d^4 y}{dz^4} + k_h(z) \cdot D \cdot y = 0 \quad (4)$$

The solutions to this equation can be obtained either analytically or numerically. The main advantage of this method is that at any point along the pile, the soil-pile interaction can be defined. But this definition is restricted by the assumption that the pressure at a point is a linear function of the displacement at that point, and by its dependence on the soil-pile interaction model characterizing the entire structure.

2.2. Characterization of Soil-structure Interaction

Defining the reaction modulus profile is the main difficulty in studying pile behavior. It depends on numerous parameters such as pile stiffness, loading level, soil type, etc. Pressiometric test results are commonly used for foundation design. [13] lists most of the models used to predict soil-structure interaction. He concludes that the reaction modulus E_S can be determined either from Young's modulus E or from the pressiometric modulus E_M . Several authors have worked on the characterization of soil-structure interaction, namely (Table 1 to 3):

Table 1. Reaction modulus according to various authors

Authors	Relationship
[1]	$\frac{E_s}{E_M} = \begin{cases} \frac{3}{\frac{2}{3} \left(\frac{D_0}{D} \right) \left(2,65 \frac{D}{D_0} \right)^\alpha + \frac{\alpha}{2}} & \text{pour } D > D_0 \\ \frac{18}{4,2,65^\alpha + 3\alpha} & \text{pour } D < D_0 \end{cases}$
[2]	$\frac{E_s}{E} = \frac{1}{1,35} = 0,74$ $E = A \cdot \gamma \cdot z$
[3]	$\frac{E_s}{E} = 0,82$
[4]	$E_s = K \left(0,308 + 1,584 \frac{D}{L} \right) \frac{z}{r \cdot L}$ $r = D + D \cdot \tan \beta$
[10] & [11]	$E_s = 1,3^{12} \sqrt{\frac{E \cdot D^4}{E_p I_p (1 - \nu^2)}}$

The different parameters involved in the relationships of Table 1 are defined below:

γ : Soil density

A : Dimensionless coefficient as a function of sand density given in Table 2

D_0 : Reference diameter equal to 0.6

α : Rheological coefficient depending on soil type

E_M : Pressuremeter modulus

E : Soil modulus of elasticity

D : Pile diameter

$E_p I_p$: Flexural rigidity of the pile

ν : Poisson's ratio

L : Pile length

Z : Depth

D : Pile diameter

β : Angle dispersion between $\phi/4$ and ϕ

ϕ : Angle of ground friction

L : Pile length

K : Soil parameter (Table 3)

One of the disadvantages of this Poulos method [3] is that it cannot be extended to a stratified soil medium, nor can the influence factors be calculated using the equation of Mindlin [16]. Indeed, Mindlin's equation is not applicable to a non-homogeneous stratified medium. Furthermore, the assumption that the pile is a rectangular strip embedded in the soil is only approximately valid if the pile has a square or **I-shaped** cross-section. In the case of piles with a circular cross-section, this idealization

needs to be brought closer, but seems reasonable. This method has been used in practice by several engineers.

Another approach is the **P-y method**, a generalization of the Winkler model. It is a semi-empirical method, because the prediction and construction of curves for the study of an isolated pile is based on laboratory or in situ tests. Each soil is represented by a series of **P-y** curves. In effect, the soil is assimilated to linear or non-linear elastic supports (*commonly referred to as springs*). This is translated into (**P**, **y**) diagrams (Figure 3), i.e. relationships between lateral reaction, **P**, and lateral displacement, **y**.

Table 2. A coefficient values from [2]

Sand density	Let go	Medium	Dense
A value	100-300	300-1000	1000-2000

Table 3. K parameter values [4]

Soil type	K
Dense sand	200-400
Medium-density gravel	150-300
Medium-dense sand	100-250
Fine sand	80-200
Stiff clay	60-180
Saturated stiff clay	30-100
Plastic clay	10-80
Clay	2-30

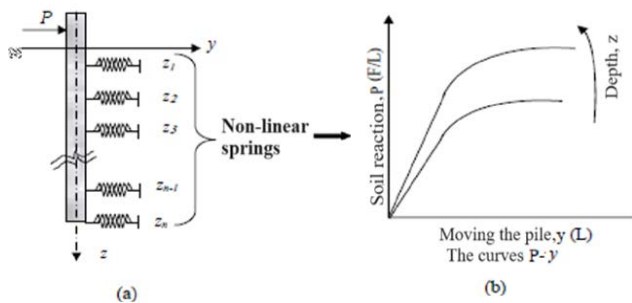


Figure 3. (a) Winkler model for lateral loading and (b) reaction curves (p-y) [9]

For a soil-pile system subjected to lateral loading, let's consider what happens at a section (or slice of the pile)

located at depth z . At rest, after installation, the section is subjected to lateral earth pressure, the resultant of which is zero.

When the pile is subjected to lateral loading, the section under consideration is displaced laterally by y_i and the stress state is modified in such a way that the lateral resultant on the section under consideration has a direction opposite to the displacement y_i . Over the entire height of the pile, for a given depth, similar behavior with varying intensities can be observed. This makes it possible to study the entire pile for any loading and any soil type. Non-linear **P-y** curves that vary with depth and soil type are obtained along the entire length of the pile (Figure 4).

Since a **p-y** curve represents the behavior of a pile at a given cross-section, and therefore for different slabs for the whole pile, assuming that the cross-sections are independent, several researchers have proposed methods for determining them in order to dimension piles. For the shape of the pile cross-section, tests carried out by [15] show that the shape has very little influence on lateral pressure distribution and ultimate pile strength. The methods devised and developed by numerous researchers use a variety of approaches: in situ tests, laboratory tests, physical modelling or numerical modelling. The diversity of these approaches leads to as many **P-y** reaction curves. Among the various methods of calculating the reaction modulus E_s , two are applicable to clays, namely the formulas of [1] and [3].

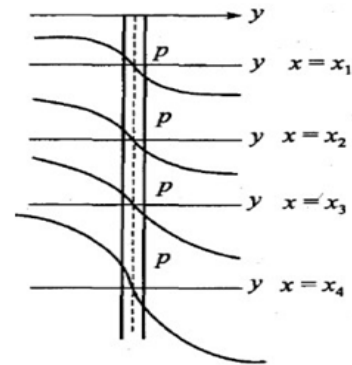


Figure 4. Series of p-y curves for a pile [14]

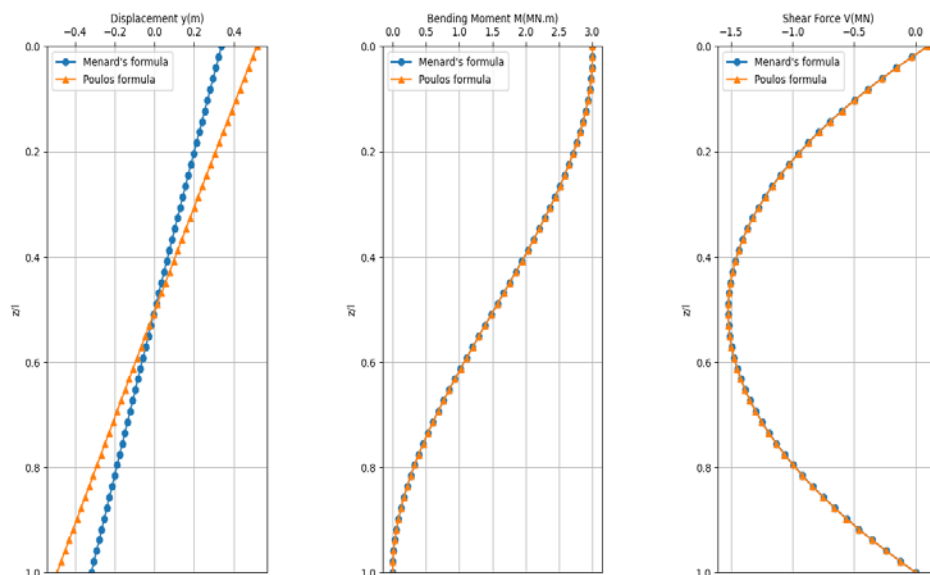


Figure 5. Comparative study of the Ménard and Poulos formulas

First, we carry out a comparative analysis of these two formulations to determine which is closer to the exact results. Code programs in Python have enabled us to plot the displacement, moment and force curves of flexible and rigid piles subjected to a moment M_0 and a lateral force V_0 at the head, as a function of depth. To make our results more applicable and accessible, we have employed adimensional coefficients.

In this way, we have thus represented the displacements $y(z)$, the bending moments $M(z)$, and the shear forces $V(z)$ as a function of the ratio between the depth and the length of the pile (z/l) for a value of the moment at the head $M_0 = 3 \text{ MN.m}$ and $V_0 = 0.1 \text{ MN}$ (Figure 5).

3. Resolution of the Behavioural Model of a Rigid Pile Subjected at the Head to a Moment M_0 and a Lateral Force V_0

The behavior of the foundation depends on both its own bending stiffness (E_p, I_p) and that of the soil E_s , i.e. the relative stiffness of the pile-soil. This is expressed hereafter as the transfer length l_0 . Let's consider a rigid pile subjected at the head to a moment M_0 and a force V_0 in linearly elastic clay (Figure 6).

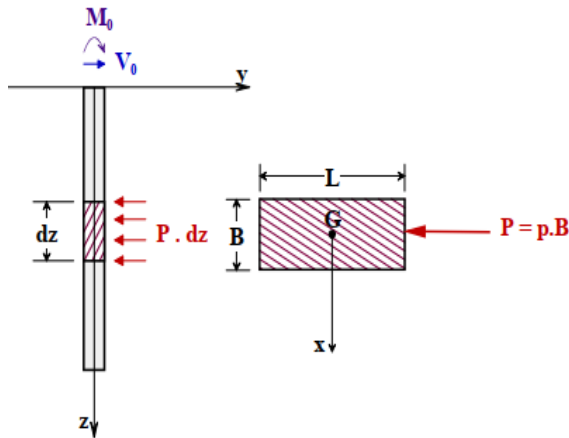


Figure 6. Diagram of a pile under M_0 and V_0

Let's consider a beam section loaded by a distributed load P and bounded by two infinitely adjacent cross-sections dz apart (Figure 7).

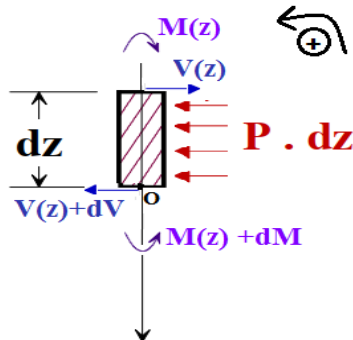


Figure 7. Diagram of an infinitesimal pile section under V and M

We obtain the equilibrium equations:

$$\begin{cases} \Sigma F = 0 \\ \Sigma M_{/O} = 0 \end{cases} \Rightarrow$$

$$\begin{cases} -V(z) + V(z) + dV + P \cdot dz = 0 \\ -M(z) - V(z) \cdot dz + P \frac{dz^2}{2} + M(z) + dM = 0 \end{cases} \quad (5)$$

By assuming $dz^2 = 0$

$$P = -\frac{dV}{dz} \quad (6)$$

$$V = \frac{dM}{dz} \quad (7)$$

We find the equations for straight beams subjected to a uniformly distributed force P (kN/m):

$$P = -\frac{dV}{dz} = -\frac{d^2M}{dz^2} = -E_p I_p \frac{d^4y}{dz^4} \quad (8)$$

$$V = \frac{dM}{dz} = E_p I_p \frac{d^3y}{dz^3} \quad (9)$$

$$M = E_p I_p \frac{d^2y}{dz^2} \quad (10)$$

$E_p I_p$: Flexural rigidity of the pile in relation to the main axis of inertia

$y^{(n)}$: Derivative $n^{ième}$ of displacement perpendicular to the mean fiber with respect to z

$P(z)$: Soil reaction distributed along the pile in kN/m ($P = p \times B$)

B : Pile section width for a rectangular section ($B = D$ for a circular section)

$V(z)$: Shear force; by convention, shear force is counted positively and the derivative of the moment with respect to z is equal to $+V(z)$.

$M(z)$: Bending moment

In this work, the deferred Young's modulus ($E_{p,eff}$) was considered to take account of the effect of creep. If the soil reaction law can be considered as linear elastic: $P = E_s \cdot y$, the fundamental relationship describing the pile's behavior can be deduced.

If the ground reaction law can be considered linear elastic: $P = E_s \cdot y$ and we deduce the following fundamental relationship, which is the 4th-order linear differential equation of the pile deformation:

$$E_p I_p \frac{d^4y}{dz^4} + E_s(z) \cdot y = 0 \quad (11)$$

The reaction modulus distribution is assumed to be of the form: $E_s(z) = a \cdot z^n$ (Gibson's Sol).

We are in the case of an over-consolidated homogeneous clay, so $n = 0$

The reaction modulus is therefore constant $E_s(z) = E_s$

Equation (11) becomes :

$$E_p I_p \frac{d^4y}{dz^4} + E_s \cdot y = 0 \quad (12)$$

$$\frac{E_p I_p}{E_s} \cdot \frac{d^4 y}{dz^4} + y = 0 \quad (13)$$

Posing $l_0^4 = \frac{4E_p I_p}{E_s}$, we obtain the equation :

$$l_0^4 y^{(4)} + 4y(z) = 0 \quad (14)$$

$y^{(4)}$: 4th derivative of displacement perpendicular to the mean fiber with respect to z.

The general solution to this equation [17] is :

$$y(z) = e^{-z/l_0} \left(C_1 \cos \frac{z}{l_0} + C_2 \sin \frac{z}{l_0} \right) + e^{z/l_0} \left(C_3 \cos \frac{z}{l_0} + C_4 \sin \frac{z}{l_0} \right) \quad (15)$$

Where C_1, C_2, C_3, C_4 integration constants determined from the boundary conditions at the head and foot of the pile.

l_0 : Transfer or elastic length. It can be defined as the minimum pile length for which lateral head loading exists. The remainder of the plug beyond about three times this length is mechanically inactive.

$$l_0 = \sqrt[4]{\left(\frac{4E_p I_p}{E_s} \right)} \quad (16)$$

With this solution, the expressions for the bending moment M and the shear force V can be obtained at any soil level, and are given by the following relationships:

$$M(z) = \frac{2E_p I_p}{l_0^2} \left[e^{-z/l_0} \left(C_1 \sin \frac{z}{l_0} - C_2 \cos \frac{z}{l_0} \right) + e^{z/l_0} \left(-C_3 \sin \frac{z}{l_0} + C_4 \cos \frac{z}{l_0} \right) \right] \quad (17)$$

$$V(z) = \frac{2E_p I_p}{l_0^3} \left\{ e^{-z/l_0} \left[C_1 \left(\cos \frac{z}{l_0} - \sin \frac{z}{l_0} \right) + C_2 \left(\cos \frac{z}{l_0} + \sin \frac{z}{l_0} \right) \right] + e^{z/l_0} \left[-C_3 \left(\cos \frac{z}{l_0} + \sin \frac{z}{l_0} \right) + C_4 \left(\cos \frac{z}{l_0} - \sin \frac{z}{l_0} \right) \right] \right\} \quad (18)$$

These results can be applied when the pile sheet $l > 3l_0$ for a flexible pile or $l < l_0$ for a rigid pile, they are intended to be used for simple cases where the soil is relatively homogeneous and for given head loads. When the flexible piles are loaded at the head, the conditions at the tip do not come into play, and the positive exponential terms are negligible. We are then reduced to a system of two equations with two unknowns, and the conditions at the head make it possible to determine the two remaining constants C_1 and C_2 .

In the case of short (rigid) piles, where the length L is less than or equal to the transfer length l_0 , the tip

conditions come into play, and the general solution is equation (15):

$$y(z) = e^{-z/l_0} \left(C_1 \cos \frac{z}{l_0} + C_2 \sin \frac{z}{l_0} \right) + e^{z/l_0} \left(C_3 \cos \frac{z}{l_0} + C_4 \sin \frac{z}{l_0} \right)$$

To find the values of the constants, four conditions are sufficient: for a free pile, subjected to a horizontal force V_0 and a moment noted M_0 at $z = 0$, we have :

Free pile at head : $V(0) = V_0$ and $M(0) = M_0$

Free pile at tip : $V(L) = 0$ and $M(L) = 0$, as beam deformation may be negligible compared to ground deformation.

$$\frac{dy}{dz} = \frac{1}{l_0} e^{-z/l_0} \left((-C_2 - C_1) \sin \frac{z}{l_0} + (C_2 - C_1) \cos \frac{z}{l_0} \right) + \frac{1}{l_0} e^{z/l_0} \left((C_3 + C_4) \cos \frac{z}{l_0} + (C_4 - C_3) \sin \frac{z}{l_0} \right) \quad (19)$$

$$\frac{d^2 y}{dz^2} = \frac{M(z)}{E_p I_p} = \frac{2}{l_0^2} e^{-z/l_0} \left(C_1 \sin \frac{z}{l_0} - C_2 \cos \frac{z}{l_0} \right) + \frac{2}{l_0^2} e^{z/l_0} \left(C_4 \cos \frac{z}{l_0} - C_3 \sin \frac{z}{l_0} \right) \quad (20)$$

$$\frac{d^3 y}{dz^3} = \frac{2}{l_0^3} e^{-z/l_0} \left((C_2 - C_1) \sin \frac{z}{l_0} + (C_2 + C_1) \cos \frac{z}{l_0} \right) + \frac{2}{l_0^3} e^{z/l_0} \left((C_4 - C_3) \cos \frac{z}{l_0} + (-C_3 - C_4) \sin \frac{z}{l_0} \right) = \frac{V(z)}{E_p I_p} \quad (21)$$

According to the boundary conditions we have adopted above ($M(0) = M_0, V(0) = V_0$) and at the tip ($M(L) = 0, V(L) = 0$) lead respectively to the following system of equations (22 to 25) :

$$C_4 - C_2 = \frac{M_0 l_0^2}{2E_p I_p} \quad (22)$$

$$C_1 + C_2 - C_3 + C_4 = \frac{V_0 l_0^3}{2E_p I_p} \quad (23)$$

$$e^{-L/l_0} \left(C_1 \sin \frac{L}{l_0} - C_2 \cos \frac{L}{l_0} \right) + e^{L/l_0} \left(C_4 \cos \frac{L}{l_0} - C_3 \sin \frac{L}{l_0} \right) = 0 \quad (24)$$

$$e^{-L/l_0} \left((C_2 - C_1) \sin \frac{L}{l_0} + (C_2 + C_1) \cos \frac{L}{l_0} \right) + e^{L/l_0} \left((C_4 - C_3) \cos \frac{L}{l_0} - (C_3 + C_4) \sin \frac{L}{l_0} \right) = 0 \quad (25)$$

Solving this system of equations to determine the constants C_1, C_2, C_3 and C_4 was carried out using the Python programming language.

4. Parametric Study of the Model

The parametric study of pile behavior considering D_0 equal to 0.6 m, the soil rheological coefficient α equal to 0.5 and a pressiometric modulus of 2.5 MPa. The following results (Figure 8 to Figure 12) were obtained with V_0 equal to 0.1 MN, M_0 equal to 3MN.m and an average Young's modulus of the pile set at 10 GPa.

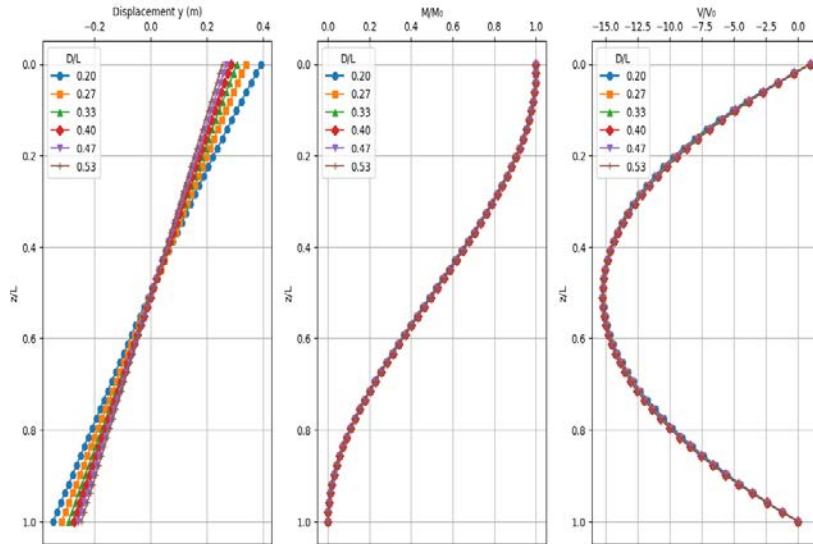


Figure 8. Influence of pile slenderness

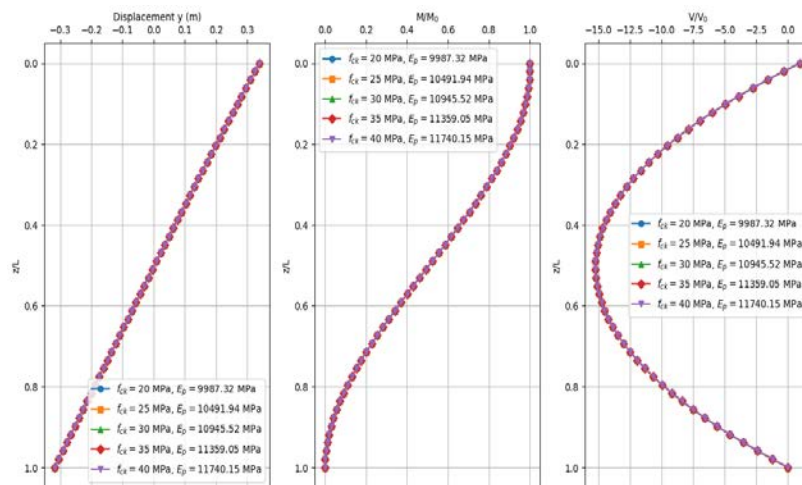


Figure 9. Influence of Young's modulus of pile

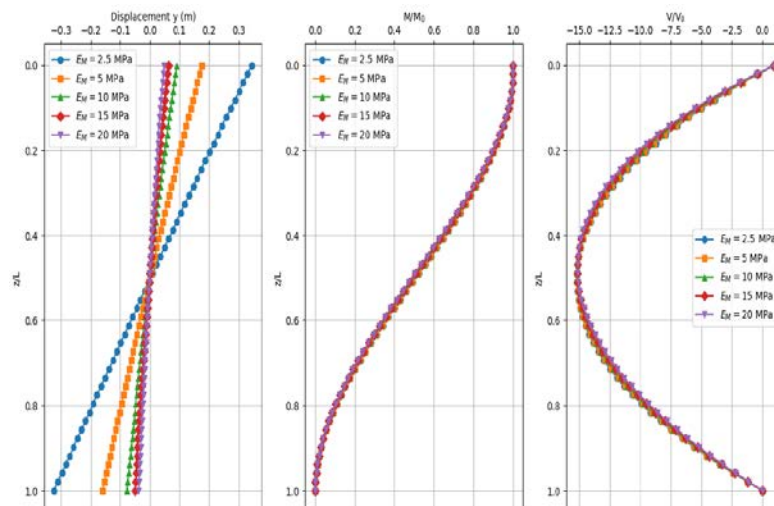


Figure 10. Influence of the Pressiometric Module

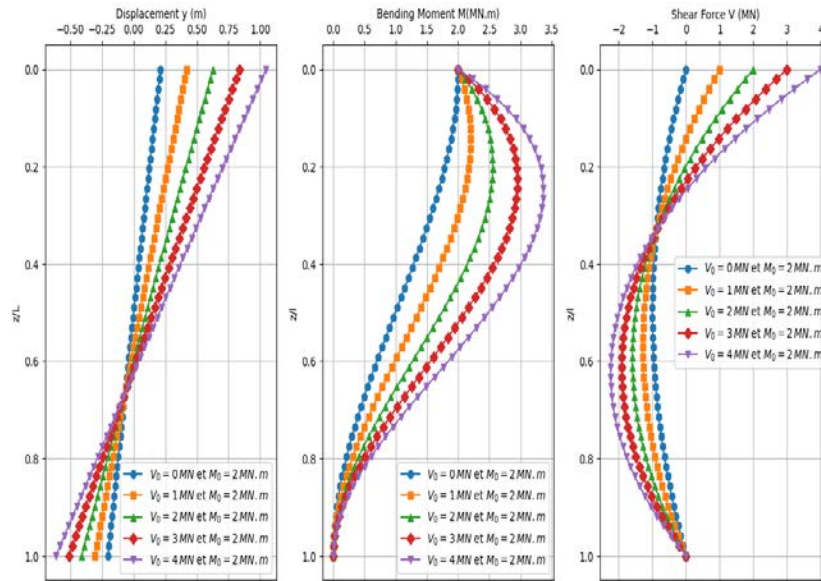


Figure 11. Influence of shear force

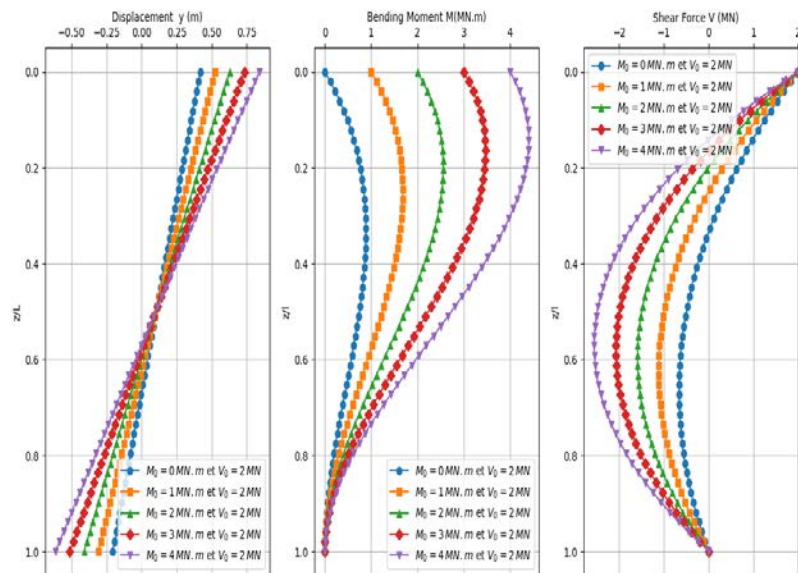


Figure 12. Influence of bending moment

According to the observations in Figure 8, the displacements of the rigid pile decrease as its diameter increases. The results show a slight variation in displacements for different slenderness values. These results highlight the additional rigidity provided by the massive pile. It should also be noted that displacements evolve linearly with depth. The displacements observed at the pile tip show that the position of the bending center is independent of slenderness.

Figure 8 also shows that the reduced moment and shear forces are almost insensitive to pile slenderness. Overall, a larger diameter pile can offer better overall stability to the structure due to its greater capacity to resist lateral loads, thus reducing the risk of failure or excessive inclination.

Figure 9 shows the insignificant impact of Young's modulus on the structural response of the rigid pile.

As shown in Figure 10, the displacements of the rigid pile decrease as the pressure modulus increases. The results show a significant variation in displacements for different values of pressure modulus. These results further highlight the additional rigidity provided by the pile. It

should also be noted that displacements evolve linearly with depth. The displacements observed at the pile tip show that the position of the bending center observed at mid-span does not change. From the observations in figure 10, we can see that displacements are linear and increase when the soil's pressure modulus is lower. A clay soil with a lower pressure modulus reacts by showing an increased capacity to deform and allow greater pile movements. On the other hand, a soil with a higher pressure modulus offers greater resistance, limiting deformation and displacement of the rigid pile. Lower values of soil pressure modulus indicate a greater capacity of the soil to deform under lateral load, resulting in greater displacements of the rigid pile. The results also show that pile stability may be more compromised in soils with lower pressuremeter moduli, as these soils tend to allow greater deformations, which can influence pile stability.

Figure 10 also shows that the reduced moment and shear forces are almost insensitive to the soil pressure modulus. Figure 11 and Figure 12 show that shear force has a significant influence on rigid pile displacements,

compared to moment. Both shear force and moment have a greater influence on the pile's ability to deform and modify its behavior in relation to the soil. An increase in applied load leads to a proportional increase in displacement in clay soil. Increased applied loads lead to increased pile displacements, a phenomenon often observed in structures subjected to heavy loads.

5. Conclusion

The aim of this study is to gain a better understanding of the behavior of piles in clay soil under lateral loads and head moments, taking into account soil-structure interaction, which is a complex phenomenon. After establishing the behavioral model, we studied the influence of certain parameters, such as soil reaction modulus, pile diameter, Young's modulus and pressure modulus, on the behavior of rigid piles.

Analytical and numerical approaches were used to predict pile response under lateral loads. The results showed the significant impact of the interaction model adopted and the forces applied at the head of the pile. The study shows that soil parameters have a greater influence on foundation behavior than pile parameters. The results also show that the forces applied at the head of the pile have a non-negligible influence on the behavior model.

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