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Some Reduction Formulae Associated with Gauss and Fox-Wright Hypergeometric Functions

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Abstract In this paper, we describe some reduction formulae for Gauss' hypergeometric function and Fox-Wright hypergeometric function associated with suitable convergence conditions using series rearrangement technique.

Keywords: *Generalized hypergeometric function, Fox-Wright generalized hypergeometric function, Pfaff-Kummer's linear transformation*

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1. Introduction and Basic Notations

In the present paper, we shall use the following standard notations:

$$\mathbb{N} := \{1,2,3,\ldots\}, \mathbb{N}_0 := \{0,1,2,3,\ldots\} = \mathbb{N} \cup \{0\}$$

$$\mathbb{Z}_0^- := \{0,-1,-2,-3,\ldots\},$$

$\mathbb{Z}^- := \{-1,-2,-3,\ldots\} = \mathbb{Z}_0^- \setminus \{0\}$ and $\mathbb{Z} = (\mathbb{Z}_0^- \cup \mathbb{Z})$.

Here, as usual, $\mathbb{Z}$ denotes the set of integers, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^+$ denotes the set of positive real numbers and $\mathbb{C}$ denotes the set of complex numbers.

The Pochhammer symbol (or the shifted factorial) $(\lambda)_\nu$ $(\lambda, \nu \in \mathbb{C})$ is defined in terms of the familiar Gamma function by

$$(\lambda)_\nu := \frac{\Gamma(\lambda+\nu)}{\Gamma(\lambda)} = \begin{cases} 1; & (\nu = 0; \lambda \in \mathbb{C} \setminus \{0\} \\ \lambda(\lambda+1)\ldots(\lambda+n-1); & (\nu = n \in \mathbb{N}; \lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-) \\ \frac{(-1)^n k!}{(k-n)!}; & (\lambda = -k; \nu = n; n,k \in \mathbb{N}_0; 0 \le n \le k) \\ 0; & (\lambda = -k; \nu = n; n,k \in \mathbb{N}_0; n > k) \\ \frac{(-1)^n}{(1-\lambda)_n}; & (\nu = -n; n \in \mathbb{N}; \lambda \ne 0, \pm 1, \pm 2, \ldots). \end{cases} \tag{1.1}$$

it being understood *conventionally* that $(0)_0 = 1$, and assumed tacitly that the Gamma quotient exists.

In the Gaussian hypergeometric series ${}_2F_1(a,b;c;z)$, there are two numerator parameters a, b and one denominator parameter c. A natural generalization of this series is accomplished by introducing any arbitrary number of numerator and denominator parameters. The non-terminating hypergeometric series [[5], pp.42-43]

$$ {}_pF_q\left[\begin{matrix} \alpha_1, \alpha_2, \ldots, \alpha_p; \\ \beta_1, \beta_2, \ldots, \beta_q; \end{matrix} z\right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n (\alpha_2)_n \ldots (\alpha_p)_n}{(\beta_1)_n (\beta_2)_n \ldots (\beta_q)_n} \frac{z^n}{n!}, \tag{1.2}$$

is known as the generalized Gauss and Kummer series, or simply, *generalized hypergeometric series*. Here p and q are positive integers or zero (interpreting an empty product as unity), and we assume that the variable z, the numerator parameters $\alpha_1, \alpha_2, \ldots, \alpha_p$ and the denominator parameters $\beta_1, \beta_2, \ldots, \beta_q$ take on complex values, provided that

$$\beta_j \ne 0, -1, -2, \ldots; \quad j = 1, 2, \ldots, q. \tag{1.3}$$

Convergence conditions [[5], p.43]:

Suppose that none of the numerator parameters is zero or negative integer (otherwise the question of convergence will not arise), and with the usual restriction (1.3), the ${}_pF_q$ series in the definition (1.2)

(i) converges for $|z| < \infty$, if $p \le q$,

(ii) converges for $|z| < 1$, if $p = q+1$.

Furthermore, if we denote

$$\omega = \sum_{j=1}^{q}\beta_j - \sum_{i=1}^{p}\alpha_i,$$

it is known that the ${}_pF_q$ series, with $p = q+1$, is

(a) absolutely convergent for $|z|=1$, if $\Re(\omega) > 0$,

(b) conditionally convergent for $|z|=1$, $z \neq 1$, if $-1 < \Re(\omega) \le 0$.

Fox-Wright generalized hypergeometric function of one variable:

The Fox-Wright Ψ function of one variable ([[3], p.389]; see also [4,6,7]) is given by

$$
\begin{aligned}
&{}_p\Psi_q\left[\begin{matrix}(\alpha_1,A_1),\ldots,(\alpha_p,A_p);\\(\beta_1,B_1),\ldots,(\beta_q,B_q);\end{matrix}\, z\right]\\
&= \sum_{n=0}^{\infty}\frac{\Gamma(\alpha_1+A_1n)\ldots\Gamma(\alpha_p+A_pn)}{\Gamma(\beta_1+B_1n)\ldots\Gamma(\beta_q+B_qn)}\frac{z^n}{n!}\\
&= \frac{\Gamma(\alpha_1)\ldots\Gamma(\alpha_p)}{\Gamma(\beta_1)\ldots\Gamma(\beta_q)}\sum_{n=0}^{\infty}\frac{(\alpha_1)_{nA_1}\cdots(\alpha_p)_{nA_p}}{(\beta_1)_{nB_1}\cdots(\beta_q)_{nB_q}}\frac{z^n}{n!}, \qquad (1.4)\\
&= \frac{\Gamma(\alpha_1)\ldots\Gamma(\alpha_p)}{\Gamma(\beta_1)\ldots\Gamma(\beta_q)}\ {}_p\Psi_q^{*}\left[\begin{matrix}(\alpha_1,A_1),\ldots,(\alpha_p,A_p);\\(\beta_1,B_1),\ldots,(\beta_q,B_q);\end{matrix}\, z\right], \qquad (1.5)\\
&= \frac{1}{2\pi\rho}\int_L \frac{\Gamma(\zeta)\prod_{i=1}^{p}\Gamma(\alpha_i - A_i\zeta)}{\prod_{j=1}^{q}\Gamma(\beta_j - B_j\zeta)}(-z)^{-\zeta}\,d\zeta, \qquad (1.6)
\end{aligned}
$$

where $\rho^2=-1$, $z \in \mathbb{C}$; parameters $\alpha_i, \beta_j \in \mathbb{C}$; coefficients $A_i, B_j \in \mathbb{R} = (-\infty,+\infty)$ in case of series (1.4) (or $A_i, B_j \in \mathbb{R}^{+}(0,+\infty)$ in case of contour integral (1.6)), $A_i \neq 0$ $(i=1,2,\ldots,p)$, $B_j \neq 0$ $(j=1,2,\ldots,q)$. In equation (1.4), the parameters α_i, β_j and coefficients A_i, B_j are adjusted in such a way that the product of Gamma functions in numerator and denominator should be well defined [1,2].

$$\Delta^{*} = \left(\sum_{j=1}^{q}B_j - \sum_{i=1}^{p}A_i\right), \qquad (1.7)$$

$$\delta^{*} = \left(\prod_{i=1}^{p}|A_i|^{-A_i}\right)\left(\prod_{j=1}^{q}|B_j|^{B_j}\right), \qquad (1.8)$$

$$\mu^{*} = \sum_{j=1}^{q}\beta_j - \sum_{i=1}^{p}\alpha_i + \left(\frac{p-q}{2}\right), \qquad (1.9)$$

$$\sigma^{*} = (1+A_1+\ldots+A_p)-(B_1+\ldots+B_q) = 1-\Delta^{*}. \qquad (1.10)$$

Case(I): When contour (L) is a left loop beginning and ending at $-\infty$, then ${}_p\Psi_q$ given by (1.4) or (1.6) holds the following convergence conditions

i) when $\Delta^{*} > -1, 0<|z|<\infty, z\neq 0,$

ii) when $\Delta^{*} = -1, 0<|z|<\delta^{*},$

iii) when $\Delta^{*} = -1, |z| = \delta^{*}$, and $\Re(\mu^{*}) > \frac{1}{2}.$

Case(II): When contour (L) is a right loop beginning and ending at $+\infty$, then ${}_p\Psi_q$ given by (1.4) or (1.6) holds the following convergence conditions

i) when $\Delta^{*} < -1, 0<|z|<\infty, z\neq 0,$

ii) when $\Delta^{*} = -1, |z| > \delta^{*},$

iii) when $\Delta^{*} = -1, |z| = \delta^{*}$, and $\Re(\mu^{*}) > \frac{1}{2}.$

Case (III): When contour (L) is starting from $\gamma - i\infty$ and ending at $\gamma + i\infty$, where $\gamma \in \mathbb{R}$, then ${}_p\Psi_q$ is also convergent under the following conditions

i) when $\sigma^{*} > 0, |\arg(-z)| < \frac{\pi}{2}\sigma^{*}, 0<|z|<\infty, z\neq 0,$

ii) when $\sigma^{*} = 0$, $\arg(-z)=0$, $0<|z|<\infty$, $z \neq 0$ such that $-\gamma\Delta^{*} + \Re(\mu^{*}) > \frac{1}{2}+\gamma,$

iii) when $\gamma = 0$, $\sigma^{*} = 0$, $\arg(-z)=0$, $0<|z|<\infty$, $z\neq 0$ such that, $\Re(\mu^{*}) > \frac{1}{2}.$

Next we collect some results that we will need in the sequel.

Decomposition identity:

The idea of separation of a power series into its even and odd terms exhibited by the elementary identity

$$\sum_{r=0}^{\infty}\Phi(r) = \sum_{r=0}^{\infty}\Phi(2r) + \sum_{r=0}^{\infty}\Phi(2r+1), \qquad (1.11)$$

is atleast as old as the series themselves and concerned power series is absolutely convergent.

Pfaff-Kummer's linear transformation [[5],p.33(eq.19)]:

$${}_2F_1\left[\begin{matrix}a,b;\\d;\end{matrix}\, z\right] = (1-z)^{-a}\,{}_2F_1\left[\begin{matrix}a,d-b;\\d;\end{matrix}\, \frac{-z}{1-z}\right], \qquad (1.12)$$

($a,b \in \mathbb{C}$; $d \in \mathbb{C}\setminus\mathbb{Z}_0^{-}$ and $|\arg(1-z)| < \pi$).

Binomial theorem:

$$(1-z)^{-b} = {}_1F_0\left[\begin{matrix}b;\\-;\end{matrix}\, z\right] = \sum_{r=0}^{\infty}\frac{(b)_r z^r}{r!}, \qquad (1.13)$$

($|z|<1$; $b \in \mathbb{C}\setminus\mathbb{Z}_0^{-}$),

$$(1-z)^{m} = {}_1F_0\left[\begin{matrix}-m;\\-;\end{matrix}\, z\right] = \sum_{r=0}^{m}\frac{(-m)_r z^r}{r!}, m \in \mathbb{N}_0.$$

In sections 2 and 3, we obtain some reduction formulae for Gauss' hypergeometric function and Fox-Wright hypergeometric function associated with suitable convergence conditions by using binomial theorem, decomposition identity and Pfaff-Kummer's linear transformation.

2. Some Reduction Formulae for Gauss' Hypergeometric Function

The following reduction formulae associated with suitable convergence conditions hold true

$$2\sqrt{(1+x)}\ {}_2F_1\left[\begin{matrix} b,1-b; \\ 1/2; \end{matrix} -x\right]$$
$$=\left(\sqrt{(1+x)}+\sqrt{x}\right)^{2b-1}+\left(\sqrt{(1+x)}-\sqrt{x}\right)^{2b-1}, \quad (2.1)$$
$$\left(\left|\frac{x}{1+x}\right|<1;1-2b\in \mathbb{C}\setminus \mathbb{Z}_0^-\right).$$

$$2(2b-1)\sqrt{x}\ {}_2F_1\left[\begin{matrix} b,1-b; \\ 3/2; \end{matrix} -x\right]$$
$$=\left(\sqrt{(1+x)}+\sqrt{x}\right)^{2b-1}-\left(\sqrt{(1+x)}-\sqrt{x}\right)^{2b-1}, \quad (2.2)$$
$$\left(\left|\frac{x}{1+x}\right|<1;1-2b\in \mathbb{C}\setminus \mathbb{Z}_0^-\right).$$

$$2\ {}_2F_1\left[\begin{matrix} -b,b; \\ 1/2; \end{matrix} -x\right]$$
$$=\left(\sqrt{(1+x)}+\sqrt{x}\right)^{2b}+\left(\sqrt{(1+x)}-\sqrt{x}\right)^{2b}, \quad (2.3)$$
$$\left(\left|\frac{x}{1+x}\right|<1;-2b\in \mathbb{C}\setminus \mathbb{Z}_0^-\right).$$

$$4(b-1)\sqrt{(x+x^2)}\ {}_2F_1\left[\begin{matrix} b,2-b; \\ 3/2; \end{matrix} -x\right]$$
$$=\left(\sqrt{(1+x)}+\sqrt{x}\right)^{2b-2}-\left(\sqrt{(1+x)}-\sqrt{x}\right)^{2b-2}, \quad (2.4)$$
$$\left(\left|\frac{x}{1+x}\right|<1;2-2b\in \mathbb{C}\setminus \mathbb{Z}_0^-\right).$$

Proof of reduction formula (2.1):
Suppose right hand side of equation (2.1) is denoted by Ω. Then by using binomial theorem (1.13), we obtain

$$\Omega=\left(\sqrt{(1+x)}+\sqrt{x}\right)^{2b-1}+\left(\sqrt{(1+x)}-\sqrt{x}\right)^{2b-1}$$
$$=\left(\sqrt{(1+x)}\right)^{2b-1}\left\{\begin{matrix} {}_1F_0\left[\begin{matrix} 1-2b; \\ -; \end{matrix} -\sqrt{\left(\frac{x}{1+x}\right)}\right] \\ +\ {}_1F_0\left[\begin{matrix} 1-2b; \\ -; \end{matrix} \sqrt{\left(\frac{x}{1+x}\right)}\right] \end{matrix}\right\} \quad (2.5)$$
$$=\left(\sqrt{(1+x)}\right)^{2b-1}$$
$$\times\sum_{r=0}^{\infty}\left\{\frac{(1-2b)_r}{r!}\left(\sqrt{\left(\frac{x}{1+x}\right)}\right)^r\left[(-1)^r+1\right]\right\}.$$

Now using decomposition identity (1.11) in equation (2.5), we get

$$\Omega=2\left(\sqrt{(1+x)}\right)^{2b-1}{}_2F_1\left[\begin{matrix} (1-2b)/2,(2-2b)/2; \\ 1/2; \end{matrix} \frac{x}{1+x}\right]. \quad (2.6)$$

Further using Pfaff-Kummer's linear transformation (1.12) in equation (2.6), and simplifying further, we arrive at left hand side of equation (2.1).

Proof of reduction formula (2.2):
Suppose right hand side of equation (2.2) is denoted by Δ. Then by using binomial theorem (1.13), we get

$$\Delta=\left(\sqrt{(1+x)}+\sqrt{x}\right)^{2b-1}-\left(\sqrt{(1+x)}-\sqrt{x}\right)^{2b-1}$$
$$=\left(\sqrt{(1+x)}\right)^{2b-1}\left\{\begin{matrix} {}_1F_0\left[\begin{matrix} 1-2b; \\ -; \end{matrix} -\sqrt{\left(\frac{x}{1+x}\right)}\right] \\ -\ {}_1F_0\left[\begin{matrix} 1-2b; \\ -; \end{matrix} \sqrt{\left(\frac{x}{1+x}\right)}\right] \end{matrix}\right\} \quad (2.7)$$
$$=\left(\sqrt{(1+x)}\right)^{2b-1}\sum_{r=0}^{\infty}\left\{\frac{(1-2b)_r}{r!}\left(\sqrt{\left(\frac{x}{1+x}\right)}\right)^r\left[(-1)^r-1\right]\right\}.$$

Now using decomposition identity (1.11) in equation (2.7), we get

$$\Delta=2\sqrt{x}(2b-1)(1+x)^{b-1}{}_2F_1\left[\begin{matrix} -b+1,-b+3/2; \\ 3/2; \end{matrix} \frac{x}{1+x}\right]. \quad (2.8)$$

On using Pfaff-Kummer's linear transformation (1.12) in equation (2.8), and simplifying further, we have left hand side of equation (2.2).

Proof of reduction formula (2.3):
Suppose right hand side of equation (2.3) is denoted by Λ. Then by using binomial theorem (1.13), we have

$$\Lambda=\left(\sqrt{(1+x)}+\sqrt{x}\right)^{2b}+\left(\sqrt{(1+x)}-\sqrt{x}\right)^{2b}$$
$$=\left(\sqrt{(1+x)}\right)^{2b}\left\{\begin{matrix} {}_1F_0\left[\begin{matrix} -2b; \\ -; \end{matrix} -\sqrt{\left(\frac{x}{1+x}\right)}\right] \\ +\ {}_1F_0\left[\begin{matrix} -2b; \\ -; \end{matrix} \sqrt{\left(\frac{x}{1+x}\right)}\right] \end{matrix}\right\} \quad (2.9)$$
$$=\left(\sqrt{(1+x)}\right)^{2b}\sum_{r=0}^{\infty}\left\{\frac{(-2b)_r}{r!}\left(\sqrt{\left(\frac{x}{1+x}\right)}\right)^r\left[(-1)^r+1\right]\right\}.$$

Now using decomposition identity (1.11) in equation (2.9), we get

$$\Lambda=2(1+x)^b{}_2F_1\left[\begin{matrix} -b,-b+1/2; \\ 1/2; \end{matrix} \frac{x}{1+x}\right]. \quad (2.10)$$

On using Pfaff-Kummer's linear transformation (1.12) in equation (2.10), and simplifying further, we arrive at left hand side of equation (2.3).

Proof of reduction formula (2.4):
Suppose right hand side of equation (2.4) is denoted by Ξ. Then by using binomial theorem (1.13), we obtain

$$\Xi=\left(\sqrt{(1+x)}+\sqrt{x}\right)^{2b-2}-\left(\sqrt{(1+x)}-\sqrt{x}\right)^{2b-2}$$
$$=\left(\sqrt{(1+x)}\right)^{2b-2}\left\{\begin{matrix} {}_1F_0\left[\begin{matrix} 2-2b; \\ -; \end{matrix} -\sqrt{\left(\frac{x}{1+x}\right)}\right] \\ -\ {}_1F_0\left[\begin{matrix} 2-2b; \\ -; \end{matrix} \sqrt{\left(\frac{x}{1+x}\right)}\right] \end{matrix}\right\}$$

$$=\left(\sqrt{(1+x)}\right)^{2b-2}\sum_{r=0}^{\infty}\left\{\begin{array}{c}\frac{(2-2b)_r}{r!}\left(\sqrt{\left(\frac{x}{1+x}\right)}\right)^r\\ \times\left[(-1)^r-1\right]\end{array}\right\}. \quad (2.11)$$

Now using decomposition identity (1.11) in equation (2.11), we get

$$\begin{aligned}\Xi &= 4\sqrt{x}(b-1)(1+x)^{b-3/2}\\ &\times {}_2F_1\left[\begin{array}{c}-b+2,-b+3/2;\ \frac{x}{1+x}\\ 3/2;\end{array}\right].\end{aligned} \quad (2.12)$$

Applying Pfaff-Kummer's linear transformation (1.12) in equation (2.12), and simplifying further, we get left hand side of equation (2.4).

3. Applications of Reduction Formulae in Fox-Wright function

As an application of formulae (2.1)-(2.4), we obtain the following two formulae for Fox-Wright hypergeometric function associated with convergence conditions

$${}_1\Psi_1^*\left[\begin{array}{c}(b,1/2);\\ (b,-1/2);\end{array} z\right]=\frac{2}{\sqrt{(z^2+4)}}\left(\frac{z+\sqrt{(z^2+4)}}{2}\right)^{2b-1} \quad (3.1)$$

and

$${}_1\Psi_1^*\left[\begin{array}{c}(b,1/2);\\ (1+b,-1/2);\end{array} z\right]=\left(\frac{z+\sqrt{(z^2+4)}}{2}\right)^{2b}, \quad (3.2)$$

$$\left(\left|\frac{z^2}{4+z^2}\right|<1; b\in\mathbb{C}\setminus\{0,\pm\frac{1}{2},\pm1,\pm\frac{3}{2},\pm2,\pm\frac{5}{2},\pm3,\dots\}\right).$$

Proof of reduction formula (3.1):

Suppose left hand side of equation (3.1) is denoted by ζ. Then by using the definitions of ${}_p\Psi_q^*$ (1.4) and (1.5), we obtain

$$\zeta = {}_1\Psi_1^*\left[\begin{array}{c}(b,1/2);\\ (b,-1/2);\end{array} z\right]=\sum_{n=0}^{\infty}\frac{(b)_{n/2}z^n}{(b)_{-n/2}n!}. \quad (3.3)$$

Now using decomposition identity (1.11) in equation (3.3), we have

$$\zeta=\sum_{n=0}^{\infty}\frac{(b)_n z^{2n}}{(b)_{-n}(2n)!}+\sum_{n=0}^{\infty}\frac{(b)_{(2n+1)/2}z^{2n+1}}{(b)_{-(2n+1)/2}(2n+1)!}.$$

Solving further, we obtain

$$\begin{aligned}\zeta &= {}_2F_1\left[\begin{array}{c}b,1-b;\ \frac{-z^2}{4}\\ 1/2;\end{array}\right]\\ &+\frac{z\,\Gamma(b+1/2)}{\Gamma(b-1/2)}\,{}_2F_1\left[\begin{array}{c}b+1/2,-b+3/2;\ \frac{-z^2}{4}\\ 3/2;\end{array}\right].\end{aligned} \quad (3.4)$$

Now applying the reduction formulae (2.1), (2.4), and simplifying further, we get the right hand side of equation (3.1).

Proof of reduction formula (3.2):

Suppose left hand side of equation (3.2) is denoted by Θ. Then by using the definitions of ${}_p\Psi_q^*$ (1.4) and (1.5), we get

$$\Theta = {}_1\Psi_1^*\left[\begin{array}{c}(b,1/2);\\ (1+b,-1/2);\end{array} z\right]=\sum_{n=0}^{\infty}\frac{(b)_{n/2}z^n}{(1+b)_{-n/2}n!}. \quad (3.5)$$

On using decomposition identity (1.11) in equation (3.5), we get

$$\Theta=\sum_{n=0}^{\infty}\frac{(b)_n z^{2n}}{(1+b)_{-n}(2n)!}+\sum_{n=0}^{\infty}\frac{(b)_{(2n+1)/2}z^{2n+1}}{(1+b)_{-(2n+1)/2}(2n+1)!}.$$

Solving further, we get

$$\begin{aligned}\Theta &= {}_2F_1\left[\begin{array}{c}b,-b;\ \frac{-z^2}{4}\\ 1/2;\end{array}\right]\\ &+\frac{z\,(b)_{1/2}}{(1+b)_{-1/2}}\,{}_2F_1\left[\begin{array}{c}b+1/2,-b+1/2;\ \frac{-z^2}{4}\\ 3/2;\end{array}\right]\\ &= {}_2F_1\left[\begin{array}{c}b,-b;\ \frac{-z^2}{4}\\ 1/2;\end{array}\right]\\ &+zb\,{}_2F_1\left[\begin{array}{c}b+1/2,-b+1/2;\ \frac{-z^2}{4}\\ 3/2;\end{array}\right].\end{aligned} \quad (3.6)$$

Now applying the reduction formulae (2.2), (2.3), and solving further, we obtain the right hand side of equation (3.2).

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