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Fixed Point Theorems for Occasionally Weakly Compatible Mappings in Dislocated-Metric Spaces

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Abstract In this paper we prove some fixed point theorems for one and two pairs of selfmaps which are occasionally weakly compatible and satisfy a "max" and $(\psi-\phi)$ contractive conditions. Also, some existing results are derived as corollaries from theorems of this paper in the framework of dislocated metric spaces.

Keywords: *occasionally weakly compatible, dislocated metric, contraction condition, common fixed point*

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1. Introduction

Hitzler and Seda in [18] introduced the concept of a dislocated metrics as a generalization of metrics where the self distance for any point need not to be equal to zero. They generalize the celebrated Banach contraction principle in dislocated metric spaces. Since then, many research papers have dealt with fixed point theory for single-valued mappings in dislocated metric spaces as a larger class than that of metric spaces (see, e.g., [11,13,14,15,19,20,21]).

Al-Thagafi and Shahzad [2] defined the concept of occasionally weakly compatible mappings which is more general than the concept of weakly compatible maps. Bhatt et al. [3] have given application of occasionally weakly compatible mappings in dynamical system. Motivated by the works of many authors for occasionally weakly compatible maps in metric spaces, in this paper we give some fixed point theorems for occasionally weakly compatible mappings satisfying $(\psi-\phi)$ -weakly contractive condition in the setting of dislocated metric spaces. Our theorems unify and generalize various known results from metric spaces to dislocated metric spaces.

2. Preliminaries

Definition 2.1 [18] Let X be a non-empty and let $d: X\times X\to R^{+}$ be a function, called a distance function if for all $x,y,z\in X$, satisfies:

$$d_1 : d(x,x)=0$$

$$d_2 : d(x,y)=d(y,x)=0 \Rightarrow x=y$$

$$d_3 : d(x,y)=d(y,x)$$

$$d_4 : d(x,y)\le d(x,z)+d(z,y)\ .$$

If d satisfies the condition d_1-d_4, then d is called a metric on X. If it satisfies the conditions d_1, d_2 and d_4 it is called a quasi-metric. If d satisfies conditions d_2, d_3 and d_4 it is called a dislocated metric (or simply d -metric). If d satisfies only d_2 and d_4 then d is called a dislocated quasi-metric (or simply dq-metric) on X. A nonempty set X with dq-metric d, i. e., (X,d) is called a dislocated quasi-metric space.

Definition 2.2 [18] A sequence (x_n) in d -metric space (X,d) is called Cauchy if for all $\varepsilon>0$, $\exists n_0\in N$ such that $\forall m,n\ge n_0$, $d(x_m,x_n)<\varepsilon$.

Definition 2.3 [18] A sequence (x_n) dislocated converges or d -converges to x if $\lim_{n\to\infty} d(x_n,x)=0$. In this case x in called a d -limit of (x_n) and we write $x_n\to x$.

Definition 2.4 [18] A d -metric space (X,d) is complete if every Cauchy sequence in it is d -convergent.

Lemma 2.5 [18] Every subsequence of d -convergent sequence to a point x_0 is d -convergent to x_0.

Definition 2.6 [18] Let (X,d) be a d -metric space. A mapping $T: X\to X$ is called contraction if there exists $0\le\lambda<1$ such that:

$$d(Tx,Ty)\le\lambda d(x,y) \text{ for all } x,y\in X.$$

Lemma 2.7 [18] Let (X,d) be a d -metric space. If $f: X\to X$ is a contraction function, then $f^{n}(x_0)$ is a Cauchy sequence for each $x_0\in X$.

Lemma 2.8 [18] d-limits in a d-metric space are unique.

Definition 2.9 [9] Let F and S be mappings of a set X into itself. Then, F and S are said to be weakly compatible if they commute at their coincidence point; that is $Fx = Sx$ for some $x \in X$ implies $SFx = FSx$.

Definition 2.10 [9] Two self-maps f and g of a set X are occasionally weakly compatible (owc) iff there is a point z in X which is a coincidence point of f and g at which f and g commute.

Example 2.11 Let $X = [0,1]$ with dislocated metric $d(x,y) = x + y$. Define $f, g : X \to X$ by $fx = 2x$, $gx = x^2$. Then $C(f,g) = \{0,2\}$, $f(g(0)) = g(f(0))$ and $f(g(2)) \neq g(f(2))$. Thus the pair (f,g) is occasionally weakly compatible but not weakly compatible.

3. Main Results

After recalling some definitions and lemmas in dislocated metric space, we state the following theorems.

Theorem 3.1 Let (X,d) be a dislocated-metric space and f and g are occasionally weakly compatible self-mappings of X, satisfying the condition:

$$d(fx, fy) \leq c \max \begin{Bmatrix} d(gx, gy), d(gx, fy), d(gy, fx), d(gy, fy), \\ d(gx, fx), d(gx, gx), d(gy, gy) \end{Bmatrix} \quad (1)$$

for all $x, y \in X$ and $0 \leq c < \frac{1}{2}$. Then f and g have a unique common fixed point.

Proof. Since f and g are occasionally weakly compatible there exists a point z in X such that $fz = gz, fgz = gfz$. We claim that fz is the unique common fixed point of f and g. Let show that fz is a fixed point of f. Consider:

$$\begin{aligned} &d(fz, ffz) \\ &\leq c \max \begin{Bmatrix} d(gz, gfz), d(gz, ffz), d(gfz, fz), d(gfz, ffz), \\ d(gz, fz), d(gz, gz), d(gfz, gfz) \end{Bmatrix} \\ &= c \max \{ d(fz, ffz), d(ffz, ffz), d(fz, fz) \} \\ &\leq 2cd(fz, ffz) \end{aligned}$$

From this inequality and since $0 \leq c < \frac{1}{2}$, we have $d(fz, ffz) = 0$. Thus $ffz = fz$ and from $ffz = fgz = gfz = fz$ we see that fz is a common fixed point of f and g.

Uniqueness. Suppose that z and v are two common fixed point of f and g such that $fz = gz = z$ and $fv = gv = v$ and $z \neq v$.

By condition (1) have:

$$\begin{aligned} &d(z,v) = d(fz, fv) \\ &\leq c \max \begin{Bmatrix} d(gz, gv), d(gz, fv), d(gv, fz), d(gv, fv), \\ d(gz, fz), d(gz, gz), d(gv, gv) \end{Bmatrix} \\ &= c \max \{ d(z,v), d(z,v), d(v,z), d(v,v), d(z,z) \} \\ &\leq 2cd(v,z) \end{aligned}$$

since $0 \leq c < \frac{1}{2}$ we have $d(v,z) = 0$. This implies $z = v$. Thus fixed point is unique.

Example 3.2 Let $X = [0,1]$ with dislocated metric $d(x,y) = x + y$. Define $f, g : X \to X$ by $fx = \frac{1}{4}x$ for $x \in [0,1]$ and $gx = \frac{x}{3}\, for\, x \in [0,1)\, or\, \frac{1}{4}\, for\, x = 1$. Then $C(f,g) = \{0,1\}$, $f(g(0)) = g(f(0))$ and $f(g(1)) \neq g(f(1))$, so the pair (f,g) is occasionally weakly compatible. Also for all $x, y \in X$ and $0 \leq c < \frac{1}{2}$ have,

$$\begin{aligned} d(fx, fy) &= \frac{1}{4}(x+y) \\ &\leq \frac{1}{3}(x+y) \\ &= d(gx, gy) \\ &\leq c \max \begin{Bmatrix} d(gx, gy), d(gx, fy), d(gy, fx), d(gy, fy), \\ d(gx, fx), d(gx, gx), d(gy, gy) \end{Bmatrix} \end{aligned}$$

Thus all conditions of theorem are satisfied and 0 is the unique common fixed point of f and g.

Corollary 3.3 Let (X,d) be a dislocated-metric space and f and g are occasionally weakly compatible self-mappings of X, satisfying the condition:

$$\begin{aligned} d(fx, fy) \leq\ & \alpha d(gx, gy) + \beta d(gx, fy) \\ & + \gamma d(gy, fx) + \delta d(gy, fy) \\ & + \eta d(gx, fx) + \mu d(gx, gx) + \theta d(gy, gy) \end{aligned}$$

for all $x, y \in X$ and nonnegative constant $\alpha, \beta, \gamma, \delta, \eta, \mu, \theta$ with $0 \leq \alpha + \beta + \gamma + \delta + \eta + \mu + \theta < \frac{1}{2}$.

Then f and g have a unique common fixed point.

Proof. This theorem can be obtained as corollary of theorem 3.1 since we have that;

$$\begin{aligned} d(fx, fy) \leq\ & \alpha d(fx, gy) + \beta d(gx, fy) \\ & + \gamma d(gx, gy) + \delta d(gy, fy) \\ & + \eta d(gx, fx) + \mu d(gx, gx) + \theta d(gy, gy) \\ \leq\ & (\alpha + \beta + \gamma + \delta + \eta + \mu + \theta) \\ & \max \begin{Bmatrix} d(fx, gy), d(gx, fy), d(gx, gy), d(gy, fy), \\ d(gx, fx), d(gx, gx), d(gy, gy) \end{Bmatrix} \end{aligned}$$

for all $x, y \in X$ and constant $\alpha, \beta, \gamma, \delta, \eta, \mu, \theta$ non negative with $0 \le \alpha+\beta+\gamma+\delta+\eta+\mu+\theta < \frac{1}{2}$.

For the following theorem which involve two pairs of self mappings each owc, we use the class of function $\Psi;\Phi$ where $\Psi = \{\psi : [0,\infty) \to [0,\infty)\}$ such that ψ is a continuous non decreasing with $\psi(t) = 0 \, iff \, t = 0$, $\Phi = \{\phi : [0,\infty) \to [0,\infty)\}$ such that ϕ is a continuous function with $\phi(t) = 0 \, iff \, t = 0$ and denote

$$M(x,y) = \max\left\{\begin{array}{l} d(Sx,Ty), d(Sx,Gy), d(Ty,Fx), \\ \frac{k}{4}\left[d(Sx,Fx) + d(Ty,Gy)\right] \end{array}\right\}.$$

Fixed point results that we are proving can be considered as continuation or generalization of many results given by [11,13,14,16].

Theorem3.4 Let (X,d) be a dislocated metric space and F, G, S and T be self-mappings of X. The pairs (F,S) and (G,T) are owc, and satisfy the condition:

$$\psi(d(Fx,Gy)) \le \psi(M(x,y)) - \phi(M(x,y)) \qquad (2)$$

for all $x, y \in X$ and $0 < k < \frac{1}{4}$ where $\psi \in \Psi, \phi \in \Phi$.

Then there exists a unique common fixed point of F, G, S and T.

Proof. Since the pairs (F,S) and (G,T) are owc, there are points $x, y \in X$ such that $Fx = Sx$ and $FSx = SFx$, $Gy = Ty$ and $GTY = TGy$. We claim that $Fx = Gy$. Consider that

$$\begin{aligned} M(x,y) &= \max\left\{\begin{array}{l} d(Sx,Ty), d(Sx,Gy), d(Fx,Ty), \\ \frac{k}{4}\left[d(Sx,Fx) + d(Ty,Gy)\right] \end{array}\right\} \\ &= \max\left\{\begin{array}{l} d(Fx,Gy), d(Fx,Gy), d(Fx,Gy), \\ \frac{k}{4}\left[d(Fx,Fx) + d(Gy,Gy)\right] \end{array}\right\} \\ &= d(Fx,Gy) \end{aligned}$$

By the condition of theorem have:

$$\psi(d(Fx,Gy)) \le \psi(d(Fx,Gy)) - \phi(d(Fx,Gy))$$

This inequality is a contradiction unless $d(Fx,Gy) = 0$, thus we have $Fx = Gy$, i.e. $Fx = Sx = Gy = Ty$. Firstly observe that

$$\begin{aligned} M(Fx,y) &= \max\left\{\begin{array}{l} d(SFx,Ty), d(SFx,Gy), d(FFx,Ty), \\ \frac{k}{4}\left[d(FFx,Gy) + d(Ty,Gy)\right] \end{array}\right\} \\ &= \max\left\{\begin{array}{l} d(FFx,Gy), d(FFx,Gy), d(FFx,Gy), \\ \frac{k}{4}\left[d(FFx,Gy) + d(Gy,Gy)\right] \end{array}\right\} \\ &= d(FFx,Fx) \end{aligned}$$

Suppose that $F^2x \ne Fx$, then inequality (2) gives:

$$\begin{aligned} \psi\left(d\left(F^2x,Fx\right)\right) &= \psi(d(FFx,Gy)) \\ &\le \psi(M(Fx,y)) - \phi(M(Fx,y)) \\ &= \psi(M(FFx,Fx)) - \phi(M(FFx,Fx)) \end{aligned}$$

which is a contradiction. Hence $d(FFx,Fx) = 0$ and so $FFx = Fx = SFx$.

Thus Fx is a fixed point of F and S.

In the same way, we observe that

$$\begin{aligned} M(x,Gy) &= \max\left\{\begin{array}{l} d(Sx,TGy), d(Sx,GGy), d(Fx,TGy), \\ \frac{k}{4}\left[d(Sx,Fx) + d(TGy,GGy)\right] \end{array}\right\} \\ &= \max\left\{\begin{array}{l} d(Gy,GGy), d(Gy,GGy), d(Gy,GGy), \\ \frac{k}{4}\left[d(Gy,Gy) + d(GGy,GGy)\right] \end{array}\right\} \\ &= d(Gy,GGy) \end{aligned}$$

If suppose that $GGy \ne Gy$, from condition (2) we have,

$$\begin{aligned} \psi(d(GGy,Gy)) &= \psi(d(Fx,GGy)) \\ &\le \psi(M(x,Gy)) - \phi(M(x,Gy)) \\ &= \psi(d(GGy,Gy)) - \phi(d(GGy,Gy)) \end{aligned}$$

which is a contradiction, unless $d(Gy,GGy) = 0$. Therefore, we get $GGy = Gy = TGy$, and $Fx = Sx = Gy = Ty$ is a common fixed point of F, G, S and T.

Uniqueness. If we assume that there exists two common fixed points u and z of F, G, S and T. For $u \ne z$ again from the condition of theorem we get

$$\begin{aligned} M(u,z) &= \max\left\{\begin{array}{l} d(Su,Tz), d(Su,Gz), d(Fu,Tz), \\ \frac{k}{4}\left[d(Su,Fu) + d(Tz,Gz)\right] \end{array}\right\} \\ &= d(u,z) \end{aligned}$$

So

$$\begin{aligned} \psi(d(u,z)) &= \psi(d(Fu,Gz)) \\ &\le \psi(M(u,z)) - \phi(M(u,z)) \\ &= \psi(d(u,z)) - \phi(d(u,z)) \end{aligned}$$

which is a contradiction unless $d(u,z) = 0$ and as a result $u = z$. Therefore u is the unique common fixed point of F, G, S and T.

Corollary 3.5 Let (X,d) be a dislocated metric space and F, S and T be self-mappings of X. The pairs (F,S) and (F,T) are owc, and satisfy the condition:

$$\psi(d(Fx,Fy)) \le \psi(M(x,y)) - \phi(M(x,y))$$

for all $x, y \in X$ and $0 < k < \frac{1}{4}$ where $\psi \in \Psi, \phi \in \Phi$.

Then there exists a unique common fixed point of F, S and T.

Proof. This is clear if in theorem3.4 we put $G = F$.

Corollary 3.6 Let (X, d) be a dislocated-metric space and F and G are occasionally weakly compatible self-mappings of X, satisfying the condition:

$$\psi(d(Fx, Fy)) \le \psi(M(x, y)) - \phi(M(x, y))$$

for all $x, y \in X$ and $0 \le 4k < 1$ and functions $\psi \in \Psi$, $\phi \in \Phi$. Then F and G have a unique common fixed point.

Proof. The proof is taken from theorem 3.4 if we take in it $S = T = i$ (identity map)

Example 3.7 Let $X = [0,1]$ with dislocated metric $d(x, y) = x + y$. Define $f, g: X \to X$ by $fx = x^2$, $gx = 2x$. Then $C(f, g) = \{0, 2\}$, $f(g(0)) = g(f(0))$ and $f(g(2)) \ne g(f(2))$. Thus the pair (f, g) is occasionally weakly compatible but not weakly compatible, and for functions $\psi; \phi$ as $\psi(t) = \frac{t}{2}$ and $\phi(t) = \frac{t}{4}$ we observe that,

$$\begin{aligned}\psi(d(fx, fy)) &= \frac{1}{2}d(fx, fy) = \frac{1}{2}(x^2 + y^2) \le \frac{1}{2}(x + y) = \\ &= \frac{1}{4}(2x + 2y) = \frac{1}{4}d(gx, gy) \le \frac{1}{4}M(x, y) = \\ &= \frac{1}{2}M(x, y) - \frac{1}{4}M(x, y) \\ &= \psi(M(x, y)) - \phi(M(x, y))\end{aligned}$$

for all $x, y \in X$.

Thus all conditions of theorem hold and 0 is the unique common fixed point of f and g.

Corollary 3.8 Let (X, d) be a dislocated metric space and F, G, S and T be self-mappings of X. The pairs (F, S) and (G, T) are owc, and satisfy the condition:

$$d(Fx, Gy) \le M(x, y) - \phi(M(x, y))$$

for all $x, y \in X$ and $0 < k < \frac{1}{4}$ where $\phi \in \Phi$.

Then there exists a unique common fixed point of F, G, S and T.

This corollary is taken from theorem3.4 If we take the function ψ as $\psi(t) = t$.

Corollary 3.9 Let (X, d) be a dislocated-metric space and F and S are occasionally weakly compatible self-mappings of X, satisfying the condition:

$$\psi(d(Fx, Fy)) \le \psi(M(x, y)) - \phi(M(x, y))$$

for all $x, y \in X$ and $0 \le k < \frac{1}{4}$ where $\psi \in \Psi, \phi \in \Phi$. Then F and S have a unique common fixed point.

This corollary is taken from theorem if we put in it $G = F$ and $T = S$.

Corollary 3.10 Let (X, d) be a dislocated metric space and F, G, S and T be self-mappings of X. The pairs (F, S) and (G, T) are owc, and satisfy the condition:

$$d(Fx, Gy) \le rM(x, y)$$

for all $x, y \in X$, $0 \le r < 1$ and $0 < k < \frac{1}{4}$.

Then there exists a unique common fixed point of F, G, S and T.

This corollary is taken from above corollary3.8 if we take in it the function $\phi(t) = (1 - r)t$ for $0 \le r < 1$.

Let be the class L of functions $p: R^+ \to R^+$ which are Lebesgue integrable functions and summable nonnegative such that $\int_0^{\varepsilon} p(t)dt > 0$ for each $\varepsilon > 0$. Now we give the following fixed point theorems for occasionally weakly compatible mappings satisfying contractive conditions of integral type.

Theorem 3.11 Let (X, d) be a dislocated metric space and F, G, S and T be self-mappings of X. The pairs (F, S) and (G, T) are owc, and satisfy the condition:

$$\int_0^{d(Fx,Gy)} p(r)dr \le \int_0^{M(x,y)} p(r)dr - \int_0^{M(x,y)} h(r)dr \quad (3)$$

for all $x, y \in X$ and $0 < k < \frac{1}{4}$ where $p, h \in L$.

Then F, G, S and T have a unique common fixed point.

Proof. If we take $\psi(r) = \int_0^r p(s)ds$ and $\phi(r) = \int_0^r h(s)ds$ then we see that the functions ψ, ϕ are functions from $\Psi; \Phi$. So on this conditions, we can use theorem 3.4 and the self mappings F, G, S and T have a unique common fixed point.

Corollary 3.12 Let (X, d) be a dislocated metric space and F, G, S and T be self-mappings of X. The pairs (F, S) and (G, T) are owc, and satisfy the condition:

$$\int_0^{d(Fx,Gy)} p(s)ds \le \lambda \int_0^{M(x,y)} p(s)ds$$

for all $x, y \in X$, $0 < \lambda < 1$ and $0 < k < \frac{1}{4}$ where $p \in L$.

Then F, G, S and T have a unique common fixed point.

Proof. If we take $h(r) = (1 - \lambda)p(r)$ then from theorem 3.11 we conclude that F, G, S and T have a unique common fixed point.

Remark 3.13 These theorems are an extension of many results on fixed point given in dislocated metric spaces by authors [11,13,14,15,16,17,19,22,23] for occasionally weakly compatible mappings without imposing conditions on the space or mappings such as completeness, closedness and continuity.

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