Dynamic Problem in Thermoelastic Solid Using Dual-Phase-Lag Model with Internal Heat Source

Praveen Ailawalia¹*, Shilpy Budhiraja²

¹Department of Applied Sciences, Baddi University of Emerging Sciences and Technology, Makhnumajra, Baddi, Solan, H.P(INDIA)
²Research Scholar Punjab Technical University, Jalandhar, Punjab(INDIA)

*Corresponding author: praveen2117@rediffmail.com

Received September 18, 2013; Revised February 28, 2014; Accepted March 04, 2014

Abstract The dual-phase lag heat transfer model is employed to study the problem of isotropic generalized thermoelastic medium with internal heat source. The force is acting along the interface of isotropic generalized thermoelastic medium and the elastic layer of depth h. The normal mode analysis is used to obtain the exact expressions for displacement components, force stress and temperature distribution. The variations of the considered variables through the horizontal distance are illustrated graphically. The results are discussed and depicted graphically.

Keywords: Dual-phase-lag model, thermoelasticity, temperature distribution, normal-mode


1. Introduction

Thermoelasticity theories which involve finite speed of thermal signals (second sound) have created much interest during the last three decades. The conventional coupled dynamic thermoelasticity theory (CTE) based on the mixed parabolic-hyperbolic governing equations of (Biot [1]; Chadwick [2]) predicts an infinite speed of propagation of thermoelastic disturbance. To remove the paradox of infinite speed of propagation of thermoelastic disturbance, several generalized thermoelasticity theories have been developed, which involve hyperbolic governing equations. Among these generalized theories, the extended thermoelasticity theory (ETE) proposed by Lord and Shulman[3] involving one relaxation time (called single-phase-lag-model) and the temperature-rate-dependent theory of thermoelasticity (TRDTE) proposed by Green and Lindsay [4] involving two relaxation times are two important models of generalized theory of thermoelasticity. Experimental studies (Kaminski [5], Mitra et al. [6], Tzou [7,8]) indicate that the relaxation times can be of relevance in the cases involving a rapidly propagating crack tip, a localized moving heat source with high intensity, shock wave propagation, laser technique etc. Because of the experimental evidence in support of finiteness of heat propagation speed, the generalized thermoelasticity theories are considered to be more realistic than the conventional theory in dealing with practical problems involving very large heat fluxes at short intervals like those occurring in laser units and energy channels. For a review of the relevant literature, see (Chandrasekharaiah [9], Ignaczak [10]).

Green and Naghdi [11,12,13] formulated three different models of thermoelasticity among which, in one of these models, there is no dissipation of thermoelastic energy. This model is referred to as the G-N model of thermoelasticity without energy dissipation (TEWOED). Problems concerning generalized thermoelasticity theories and G-N theory have been studied by many authors (RoyChoudhuri and Debnath [14], RoyChoudhuri [15,16,17], Dhaliwal and Rokne [18,19], RoyChoudhuri [20], Chandrasekharaiah and Marthy [21], Chandrasekharaihaiah and Srinath [22], RoyChoudhuri and Banerjee [23], RoyChoudhuri and Bandyopadhyay [24], RoyChoudhuri and Dutta [25], Tzou [7,8] and Ozisik and Tzou [26] have developed a new model called dual-phase-lag model for heat transport mechanism in which Fourier’s law is replaced by an approximation to a modification of Fourier’s law with two different time translations for the heat flux and the temperature gradient. According to this model, classical Fourier’s law \( \mathbf{q} = -\mathbf{K} \nabla T \) has been generalized as \( \mathbf{q}(P, t + \tau_q) = -K^* \mathbf{V} \nabla T(P, t + \tau_0) \), where the temperature gradient \( \mathbf{V} \nabla T \) at a point P of the material at time \( t + \tau_0 \) corresponds to the heat flux vector \( \mathbf{q} \) at the same point at the time \( t + \tau_q \). Here \( K^* \) is the thermal conductivity of the material. The delay time \( \tau_q \) is interpreted as that caused by the microstructural interactions (small-scale heat transport mechanisms occurring in microscale) and is called the phase-lag-of the temperature gradient. The other delay time is \( \tau_0 \) interpreted as the relaxation time due to the fast transient effects of thermal inertia (small scale effects of heat transport in time) and is called the phase-lag of the heat flux. If \( \tau_q = \tau \) and \( \tau_0 = 0 \), Tzou [7,8] refers to the
model as the single phase-lag model. The case \( \tau_0 \neq \tau_q (\neq 0) \) corresponds to the dual phase-lag model of the constitutive equation connecting the heat flux vector and the temperature gradient. The case \( \tau_q = \tau_0 (\neq 0) \) becomes identical with the classical Fourier’s law. Further for materials with \( \tau_q > \tau_0 \), the heat flux vector is the result of a temperature gradient and for materials with \( \tau_0 > \tau_q \), the temperature gradient is the result of a heat flux vector. For a review of the relevant literature, see (Chandrasekharaiah [27]). A hyperbolic thermoelastic model was developed in this same reference, taking into account the phase-lag of both temperature gradient and heat flux vector and also the second order term in \( \tau_q \) in Taylor’s expansion of the heat flux vector and the first order term in \( \tau_q \) in Taylor’s expansion of the temperature gradient in the generalization of classical Fourier’s law. It may be pointed out that ETE was formulated by taking into account the thermal relaxation time, which is in fact the phase-lag of the heat flux vector (single-phase-lag model).


The present paper is concerned with the investigation related to the effect of dual phase-lag subjected to a normal force at the interface of elastic layer and isotropic generalized thermoelastic medium with internal heat source. The normal mode method is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented graphically.

2 Formulation of the Problem and Fundamental Equations

We consider a elastic layer (depth \( h \)) lying over the surface of a linear homogeneous isotropic, thermally conducting elastic half space. All quantities considered are functions of the time variable \( t \) and of the coordinates \( x \) and \( y \). A rectangular coordinate system \((x, y, z)\) with \( y \)-axis pointing vertically downward is considered. The region \( y > 0 \) is occupied by isotropic generalized thermoelastic elastic medium (medium I) and the region \( h < y < 0 \) represents the elastic layer (medium II). The plane \( y = 0 \) represents the interface of medium I and medium II.

The field equations and constitutive relations for a homogeneous, generalized thermoelastic solid in the absence of incremental body forces and heat sources are:

\[
\begin{align*}
t^0_{ij} &= \lambda^0 \delta_{ij} + \mu^0 \left( \delta_{ij} + \delta_{ji} \right) - \sigma T \delta_{ij}, \\
e_{ij} &= \left( \frac{1}{2} \right) (u_{ij} + u_{ji}),
\end{align*}
\]

\[
(\lambda^0 + \mu^0) \left( \nabla \delta(u_i^0) \right) + \mu^0 \nabla^2 u^0 - \nabla T = \rho^0 \frac{\partial^2 u^0}{\partial t^2},
\]

Then the heat conduction equation in the context of dual phase lag thermoelasticity proposed by Tzou in this case takes the form

\[
K^0 \left(1 + \tau^0_0 \frac{\partial}{\partial t^0} \right) \nabla^2 T = \rho C_e (1 + \tau^q_0 \frac{\partial}{\partial t^q} \nabla T)
\]

\[
+ \theta T (1 + \tau^q_0 \frac{\partial}{\partial t^q} \nabla T) + (1 + \tau^q_0 \frac{\partial}{\partial t^q} \nabla T) T
\]

For medium I (isotropic generalized thermoelastic medium) replace \( \delta \) by I and for medium II (elastic layer of depth \( h \)) replace \( \delta \) by II.

3. Solution of the Problem

If we restrict our analysis parallel to \( xy \) plane and \( \partial/\partial y \), the displacement components have the following form

\[
u^1_x = u^1_x (x, y, t), \quad \nu^1_y = u^1_y (x, y, t), \quad w^1_z = 0,
\]

From Equations (2) and (5), we obtain the strain components

\[
e_{xx} = \frac{\partial u^1_x}{\partial x}, \quad e_{yy} = \frac{\partial u^1_y}{\partial y}, \quad e_{xy} = \frac{1}{2} \left( \frac{\partial u^1_x}{\partial y} + \frac{\partial u^1_y}{\partial x} \right),
\]

\[
e_{zz} = e_{xz} = e_{zx} = 0,
\]

To facilitate the solution, following dimensionless quantities are introduced:

\[
\{x', y'\} = \frac{x}{c_1}, \quad \{u^1_x', u^1_y'\} = \frac{p^1_c \omega u^1_x}{9 T_0}, \quad \{u^1_x, u^2_x\} = \frac{p^1_c \omega u^1_x}{9 T_0},
\]

\[
T = \frac{T}{T_0}, \quad q = \frac{T}{c T_0}, \quad t = \omega t,
\]

\[
\tau^0_0 = \omega \tau^0_0, \quad \tau^q_0 = \omega \tau^q_0, \quad a = \frac{1}{\lambda^0} Q_0,
\]

where

\[
e_{2} = \frac{\lambda^0 + 2 \mu^0}{\rho^1}, \quad \omega = \frac{p^1_c e_{c_1}^2}{K^0}.
\]

Equation (3), with the help of equations (1) and (5)- (7) may be recast into the dimensionless form after suppressing the primes as:

\[
\frac{\partial^2 u^1_x}{\partial x^2} + \xi_{12} \frac{\partial^2 u^1_y}{\partial y^2} + \xi_{11} \frac{\partial^2 u^1_x}{\partial x \partial y} - \xi_{11} \frac{\partial^2 u^1_x}{\partial x^2} = \frac{\partial T}{\partial x^2},
\]

\[
\frac{\partial^2 u^1_x}{\partial y^2} + \xi_{11} \frac{\partial^2 u^1_y}{\partial x \partial y} + \xi_{12} \frac{\partial^2 u^1_x}{\partial y^2} - \xi_{11} \frac{\partial^2 u^1_y}{\partial y^2} = \frac{\partial^2 T}{\partial y^2},
\]

\[
\frac{\partial^2 u^1_x}{\partial x^2} + \xi_{12} \frac{\partial^2 u^1_y}{\partial y^2} + \xi_{11} \frac{\partial^2 u^1_x}{\partial x \partial y} - \xi_{11} \frac{\partial^2 u^1_y}{\partial x \partial y} = \frac{\partial T}{\partial x \partial y}.
\]
The heat conduction equation is given by

$$\nabla^2 T = \left( \frac{(1 + \tau_0 \frac{\partial}{\partial t})}{(1 + \tau_0 \frac{\partial}{\partial t})} \right) \left\{ \frac{\partial T}{\partial t} + \xi_{21} \frac{\partial \phi}{\partial t} - \xi_{22} Q \right\}. \tag{10}$$

Using the expression relating displacement components \(u_1^1(x, y, t), u_2^1(x, y, t)\), to the scalar potential functions \(\phi(x, y, t)\) and \(\psi(x, y, t)\)

\[
u_1^1 = \frac{\partial \phi}{\partial x}, \nu_2^1 = \frac{\partial \phi}{\partial y}, \nu_1^2 = \frac{\partial \psi}{\partial x}, \nu_2^2 = \frac{\partial \psi}{\partial y}, \tag{11}\]

in equations (8) - (10), we obtain

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial t^2} \right) \phi - T = 0, \tag{12}\]

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial t^2} \right) \psi = 0, \tag{13}\]

\[
\nabla^2 T = \left( \frac{(1 + \tau \frac{\partial}{\partial t})}{(1 + \tau_0 \frac{\partial}{\partial t})} \right) \left\{ \frac{\partial T}{\partial t} + \xi_{21} \frac{\partial \phi}{\partial t} - \xi_{22} Q \right\}. \tag{14}\]

where

\[
\xi_{11} = \frac{\lambda_1 + 1}{\rho_1 c_1^2}, \xi_{12} = \frac{\mu_1}{\rho_1 c_1^2}, \xi_{13} = \frac{\lambda_1}{\rho_1 c_1^2}, \xi_{21} = \frac{\delta + \tau_0^2}{\rho_1 K_1^2}, \xi_{22} = \frac{\lambda_1 + 1}{K_1 T_0^2}, \tag{16}\]

4. Normal Mode Analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

\[
[\phi, \psi, T, u^1_{11}(x, y, t)] = [\phi^0, \psi^0, T^0, u^0_{11}](y)e^{i(\omega t + \xi_k y)}, \tag{15}\]

\[
Q = Q_0e^{i(\omega t + \xi_k y)}, \tag{16}\]

where \([\phi^0, \psi^0, T^0, u^0_{11}]\) are the magnitude of the functions, \(\omega\) is the complex time constant and \(\xi_k\) is the wave number in \(x\)-direction and \(Q_0\) is the magnitude of stable internal heat source.

Using (15) - (16), in equations (12) - (14) we obtain,

\[
[D^2 + a^2 - \omega^2 \phi - T = 0, \tag{17}\]

\[
\xi_{24} [D^2 + a^2 - \omega^2 \phi - T = \xi_{22} \psi = 0, \tag{18}\]

where

\[
\xi_{23} = (a^2 + \omega \phi), \xi_{24} = \xi_{21} + \omega \phi = \frac{(1 + \tau_0 \omega)}{(1 + \tau_0 \omega)} \tag{19}\]

Eliminating \(T\) from equations (17) - (18), we obtain,

\[
[D^4 + \lambda_1 D^2 + \lambda_2] \bar{\psi}(y) = -\epsilon \xi_{22} Q_0. \tag{20}\]

where, \(D = \frac{d}{dy}\),

\[
\lambda_1 = (a^2 + \omega^2 + \xi_{23} + \xi_{24}), \lambda_2 = (a^2 + \omega^2)\xi_{23} + a^2 \xi_{24}. \tag{25}\]

The solution of equation (20) is given by:

\[
\bar{\psi}(y) = \sum_{j=1}^{2} S_j(a, \omega)e^{-k_j y} - l_1, \tag{21}\]

In a similar way, we get

\[
\bar{T}(y) = \sum_{j=1}^{2} a^*_j S_j(a, \omega)e^{-k_j y} + l_2, \tag{22}\]

The solution of equation (19) is given by:

\[
\bar{\psi}(y) = S_3(a, \omega)e^{-k_3 y}, \tag{23}\]

where

\[
l_1 = \epsilon \xi_{22} Q_0, l_2 = l_1(a^2 + \omega^2), k_3^2 = \frac{\xi_{23} a^2 + \omega^2}{\xi_{21}}, a^*_j = (k_j^2 - a^2 - \omega^2), j = 1, 2, \tag{24}\]

where \(S_j(a, \omega), R_j(a, \omega)\) are some parameters depending on \(a\) and \(\omega\). \(k_j^2 (j = 1, 2)\) are the roots of the characteristic equation (20).

Neglecting thermal effect in equation (1), we obtain elastic layer of depth \(h\). Adopting the same approach, the displacement components \(u_1^1, u_2^1\) and stresses \(\sigma_{11}, \sigma_{22}\), in the elastic layer (i.e for \(-h \leq y \leq 0\)) is given by

\[
u_1^1 = \{\alpha_S a e^{-k_4 y} + \alpha_S e^{-k_5 y} + \alpha_S e^{-k_6 y}\}, \tag{25}\]

\[
u_2^1 = \{-k_4 S e^{-k_4 y} + k_4 S e^{-k_5 y} + \alpha_S e^{-k_6 y}\}, \tag{26}\]

\[
u_1^2 = \{\alpha_S a e^{-k_4 y} + \alpha_S e^{-k_5 y} + \alpha_S e^{-k_6 y}\}, \tag{27}\]

\[
u_2^2 = \{-k_4 S e^{-k_4 y} + k_4 S e^{-k_5 y} + \alpha_S e^{-k_6 y}\}, \tag{28}\]

where

\[\alpha_S = -\xi_{34} a^2 + \xi_{31} k_4 a + \tau a k_4 (\xi_{34} - \xi_{31}), \tag{29}\]

\[\tau a = -2 a k_4 \xi_{33}, \xi_{34} = -\xi_{33} (a^2 + k_4^2), \tag{30}\]

\[
\xi_{31} = \frac{\lambda_{11} + 2 \mu_{11}}{\rho_1 c_1^2}, \xi_{32} = \frac{\lambda_{11} + 2 \mu_{11}}{\rho_1 c_1^2}, \tag{31}\]

\[
\xi_{33} = \frac{\lambda_{11}}{\rho_1 c_1^2}, \tag{32}\]
5. Applications

The boundary conditions at the interface $y = 0$ subjected to an arbitrary normal force $P_1$ are

(i) $t_{22}^H = -P_1 e^{(\omega t + \xi)}$ at $y = -h$,
(ii) $t_{22}^H = 0$ at $y = 0$,
(iii) $t_{22}^H - t_{22}^L = 0$ at $y = 0$,
(iv) $t_{22} = t_{22}^L$ at $y = 0$,
(v) $u_2 = u_2^L$ at $y = 0$,
(vi) $\frac{\partial T}{\partial y} = 0$ at $y = 0$,

where $P_1$ is the magnitude of mechanical force. Using equations (1) and (7) on the non-dimensional boundary conditions and then using (21) - (23), we get the expressions of displacement, force stress and temperature distributions for isotropic generalized thermoelastic medium as,

$$u_1^L = \left( \sum_{j=1}^{2} \frac{\alpha_j S_j(a, \omega) e^{k_j y + k_j S_j e^{-k_j y}} e^{(\omega t + \xi)}}{\tau_0} \right) - 1,$$

$$u_2^L = -\sum_{j=1}^{2} \frac{\alpha_j S_j(a, \omega) e^{k_j y} + \alpha_j S_j e^{-k_j y} e^{(\omega t + \xi)}}{\tau_0},$$

$$t_{22}^L = \sum_{j=1}^{2} b_j^* S_j(a, \omega) e^{k_j y} + N_j S_j e^{-k_j y} e^{(\omega t + \xi)} + N_{2j},$$

$$t_{21}^L = \sum_{j=1}^{2} c_j^* S_j(a, \omega) e^{k_j y} + N_j S_j e^{-k_j y} e^{(\omega t + \xi)},$$

$$T = \sum_{j=1}^{2} a_j S_j(a, \omega) e^{k_j y} e^{(\omega t + \xi)} + t_2^L,$$

where

$$b_j^* = (k_j^2 - a_j^2 \xi_j^2 - a_j^2),$$

$$c_j^* = -\sum_{j=1}^{2} \alpha_j k_j \xi_j,$$

$$N_1 = \tau_0 \alpha_j (\xi_j^2 - 1),$$

$$N_2 = (a_j^2 \xi_j^3 + 1),$$

$$N_3 = -(a_j^2 + k_j^2).$$

Invoking the boundary conditions (29) at the surface $y = 0$, we obtain a system of seven equations, and applying the inverse of matrix method, we have the value of seven constants $S_j$, $j = 1, 2, 3, \ldots, 7$.

$$S_1 = \frac{\Delta_1}{\Delta}, S_2 = \frac{\Delta_2}{\Delta}, S_3 = \frac{\Delta_3}{\Delta}, S_4 = \frac{\Delta_4}{\Delta},$$

$$S_5 = \frac{\Delta_5}{\Delta}, S_6 = \frac{\Delta_6}{\Delta}, S_7 = \frac{\Delta_7}{\Delta},$$

where $\Delta_i, \Delta_i, i = 1, 2, 3, \ldots, 7$ are defined in appendix A.

6. Numerical Results

For computational work, to illustrate the analytical procedure presented earlier, we consider now a numerical example. The results depict the variations of displacements, force stress and temperature distribution. For this purpose, sand stone is considered as the thermoelastic material body for which we have the physical constants as follows

$$\rho = 2.30 \text{gm/(cm)}^3, \omega_0 = 0.4 \times 10^{-5} / (\text{degC}),$$

$$\lambda = 0.8 \times 10^{11} \text{dyne/cm}^2,$$

$$K^* = 0.6 \times 10^{-2} \text{cal/cm}^2 \cdot \text{cmsec} \cdot \text{degC},$$

$$C_E = 0.23 \text{cal/ gmdegC},$$

$$T_0 = 296k.$$

The physical constants for elastic layer are given by Bullen [36],

$$\mu^I = 0.884 \times 10^{11} \text{dyne/cm}^2,$$

$$\mu^I = 1.2667 \times 10^{11} \text{dyne/cm}^2, \rho^I = 2.6 \text{gm/cm}^3.$$

The computations are carried out in the range $0 \leq x \leq 10$ and on the surface $y = 1.0$. The numerical values for normal displacement $u_2$, normal force stress $t_{22}$ and temperature distribution $T$ are shown in figure 1 - figure 6 for mechanical force with $\omega = \omega_0 + \xi$, $\omega_0 = 2.3, a = 2.1$ for a

a. Isotropic generalized thermoelastic medium with internal heat source ($\tau_q = 2$ and $\tau_\phi = 2$) by solid line.

b. Isotropic generalized thermoelastic medium with internal heat source ($\tau_q = 4$ and $\tau_\phi = 2$) by dashed line.

c. Isotropic generalized thermoelastic medium with internal heat source ($\tau_q = 6$ and $\tau_\phi = 2$) by solid line with centered symbol (*)

d. Isotropic generalized thermoelastic medium with internal heat source ($\tau_q = 8$ and $\tau_\phi = 2$) by dashed line with centered symbol (*)

e. Isotropic generalized thermoelastic medium with internal heat source ($\tau_q = 6$ and $\tau_\phi = 2$) by solid line.

f. Isotropic generalized thermoelastic medium with internal heat source ($\tau_q = 8$ and $\tau_\phi = 2$) by dashed line.

g. Isotropic generalized thermoelastic medium with internal heat source ($\tau_q = 3$ and $\tau_\phi = 4$) by solid line with centered symbol (*)

h. Isotropic generalized thermoelastic medium with internal heat source ($\tau_q = 4$ and $\tau_\phi = 4$) by dashed line with centered symbol (*)

Figure 1. Variations of Normal displacement $u_2$ with distance $x$ for phase-lag of heat flux

Figure 2. Variations of Normal force stress $t_{22}$ with distance $x$ for phase-lag of heat flux
7.1. Effect of Phase Lag of the Heat Flux ($\tau_q$)

Figure 1 to Figure 3 gives the effect of the phase-lag of the heat flux $\tau_q$ (with fixed value of $\tau_\theta = 2$).

Figure 1 depicts the variations of normal displacement $u_2$ with distance $x$. The behaviour of normal displacement $u_2$ with reference to $x$ is same i.e. oscillatory for $\tau_q = 2$, $\tau_q = 4$ and $\tau_q = 8$ with difference in their magnitude, whereas for $\tau_q = 4$ and $\tau_q = 6$ show opposite oscillatory pattern in the entire range which show the impact of phase-lag of the heat flux.

The variations of normal force stress $t_{22}$ with distance $x$ is depicted in figure 2. The values of normal force stress $t_{22}$ for $\tau_q = 2$ and $\tau_q = 4$ show similar patterns with different degree of sharpness. i.e. the values for $\tau_q = 2$ and $\tau_q = 4$ increases and decreases alternately with distance $x$. It is also noticed that the variations of normal force stress $t_{22}$ for $\tau_q = 6$ show oscillatory pattern. Figure 3 shows the variations of temperature distribution $T$ with distance $x$.

The pattern observed for $\tau_q = 2$ and $\tau_q = 4$ are opposite in nature with fluctuating values which clearly reveals the effect of phase lag of the heat flux. The trend of variations for $\tau_q = 6$ and $\tau_q = 8$ are similar in nature in the entire interval, i.e. the values for $\tau_q = 6$ and $\tau_q = 8$ increases and decreases alternately with distance $x$.

7.2. Effect of Phase-lag of Temperature Gradient ($\tau_\theta$)

Figure 4 to Figure 6 gives the effect of the phase-lag of temperature gradient $\tau_\theta$ (with fixed value of $\tau_q=4$).

Figure 4 depicts the variations of normal displacement $u_2$ with distance $x$. The variations of normal displacement $u_2$ are comparable amongst themselves for $\tau_\theta = 1$ and $\tau_\theta = 3$. These variations are oscillatory in nature and opposite to the variations of normal displacement $u_2$ for $\tau_\theta = 2$ and $\tau_\theta = 4$.

Figure 5 depicts the variations of normal force stress $t_{22}$ with distance $x$. The pattern observed for $\tau_\theta = 1$ and $\tau_\theta = 4$ are opposite in nature near the point of application of the source but with increase in $x$, both assume similar pattern with fluctuating values. It is noticed that the variations of normal force stress $t_{22}$ for $10 \tau_\theta = 2$ show oscillatory pattern about the origin. Also the pattern observed for $\tau_\theta = 3$ and $\tau_\theta = 4$ are opposite in nature with fluctuating values.

The variations of temperature distribution $T$ with distance $x$ is depicted in figure 6. The pattern observed for $\tau_\theta = 3$ and $\tau_\theta = 4$ are opposite in nature with fluctuating values. which clearly reveals the effect of phase-lag of temperature gradient, whereas for $\tau_\theta = 1$ and $\tau_\theta = 3$ shows similar oscillatory pattern with different degree of sharpness in magnitude. Further temperature distribution $T$ shows small variations close to zero value in the whole range for $\tau_\theta = 2$.

8. Conclusion

Appreciable effect of dual-phase-lag (DPL) model i.e. effect of phase-lag of heat flux ($\tau_q$) and effect of phase-lag of temperature gradient ($\tau_\theta$) is observed on the components of displacement, force stress and temperature distribution. The variations of Normal displacement $u_2$ are uniform in nature in comparison to Normal force stress $t_{22}$.
and Temperature distribution $T$ under the effect of $T$. The normal mode analysis used in this article is applicable to wide range of problems in different branches. This method gives exact solutions without any assumed restrictions on either the temperature or stress distributions.

**Appendix A**

$$\Delta = \zeta_1 (e^{k_1 a_1^*} (N_3 k_4 + r_1 a_1) - e^{k_2 a_2^*} (N_3 k_4 + r_1 a_1) - N_3 k_4 a_1^* (r_2 k_2 - c_2 k_4) + N_3 k_4 a_2^* (r_3 k_1 - c_1 k_4)),$$

$$\Delta_1 = \zeta_1 \zeta_2, \Delta_2 = \zeta_1 \zeta_3, \Delta_3 = \zeta_1 \zeta_4, \Delta_4 = \zeta_1 \zeta_5,$$

$$\Delta_5 = \zeta_6 \zeta_7, \Delta_6 = \zeta_9 - \eta_1 + \eta_2, \Delta_7 = \eta_4 - \eta_3 + \eta_6,$$

$$\zeta_1 = r_2 (\text{targ}_e^{k_4} - \eta_2 \text{targ}_e^{k_2} - \eta_4 \text{targ}_e^{k_4} - k_4 \text{targ}_e^{k_3} + k_4 \text{targ}_e^{k_3} + k_2 \text{targ}_e^{k_3}) + r_2 (\text{targ}_e^{k_3} - \eta_2 \text{targ}_e^{k_2} - \eta_4 \text{targ}_e^{k_3} + k_4 \text{targ}_e^{k_3} + k_2 \text{targ}_e^{k_3}) + r_2 (\text{targ}_e^{k_3} - \eta_2 \text{targ}_e^{k_2} - \eta_4 \text{targ}_e^{k_3} + k_4 \text{targ}_e^{k_3} + k_2 \text{targ}_e^{k_3})$$

**References**


[33] Kh. Lotfy, (2010). Transient disturbance in a half-space under generalized magnetothermoelasticity with a stable internal heat source under three theories. Multidiscipline Modeling in Mat. and

