Validation of Results Obtained from Different Types of Fuzzy Controllers for Diagnosis of Inclined Edge Crack in Cantilever Beam by Vibration Parameters

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Abstract In this paper, the crack diagnosis using intelligent techniques (using membership functions in different fuzzy controllers) have been developed for inverse investigation of the vibration parameters (like modal frequencies and mode shapes) and crack parameters (like crack location, crack depth and crack inclination) of an inclined edge crack cantilever beam. The vibration parameters are calculated from finite element (using ANSYS) and experimental analysis which are used as inputs to the different fuzzy controllers. The different fuzzy controllers are designed by taking several types of membership functions to calculate the crack parameters. The calculated first three modal frequencies and mode shapes are used to generate the number of fuzzy rules with three output crack parameters. Finally, the proposed intelligent techniques are validated by comparing the results obtained from both FEA and experimental analysis. All the results are obtained from fuzzy controllers are in good agreement with experimental results.

Keywords: vibration parameters, crack parameters, inclined edge crack, FEA, Fuzzy, ANSYS


1. Nomenclature

L: Length of the cantilever beam
b: Width of the cantilever beam
t: Thickness of the cantilever beam
a: Edge Crack depth
L1: Crack location from fixed end of the cantilever beam
β: Relative crack location
α: Relative crack depth
θ: Crack inclination angle
RFNF: Relative First Natural Frequency
RSNF: Relative Second Natural Frequency
RTNF: Relative Third Natural Frequency
RFMD: Relative First Mode Difference
RSMD: Relative Second Mode Difference
RTMD: Relative Third Mode Difference
RCL: Relative Crack Location
RCD: Relative Crack Depth
CA: Crack Angle

2. Introduction

Early identification of cracks in engineering dynamic structure like beams during their life time is the main task to the researchers. Though vibration based crack analysis has been advanced for last three decades and, still there are various difficulties occur during application. There are various methods to calculate the problem of a cracked beam such as theoretical, analytical, experimental etc. FEM (Finite Element Method) is a general method to get the stiffness matrix of the cracked beam element. Many researchers are using vibration parameters like stiffness, natural frequencies and mode shapes to determine the position and depth of the crack in the beam. The natural frequencies decrease due to the cracks in the beam because of decrease the stiffness.

Shih et al. [1] determined fault identification in structures like beams and plates by using multi-criteria approach which includes two methods, one is modal flexibility and another is modal strain energy method. After getting results from above two methods are used in algorithms for identify single and multiple damages having different boundary conditions. Prasad et al. [2] investigated that the effect of crack position from free end to fixed end of the vibrating cantilever beam at each of the frequency on the resolve of crack growth rate. Mazanoglu and Sabuncu [3] have presented an algorithm for crack detection from searching over the frequency map and minimizing the measurement errors. Also a statistical approach called ‘recursively scaled zoomed frequencies (RSZF)’ is used for reducing the deviations. B. P. Nandwana and S. K. Maiti [4] have determined the crack position, crack depth and crack inclination by using finite element method and experimental method. The inclined
crack used as a rotational spring for vibrational analysis which helps to determine the crack location and depth, according to the changes of vibration signatures. Kalanad and Rao [5] have measured relative crack depth up to 0.9 times of total element depth and natural frequency of cracked beam more accurately using two dimensional finite element approaches. The function of frequency response and function of crack size and location are approached by means of surface-fitting techniques. Bing et al. [6] identified the multiple crack of beam by using a three-step-meshing method with less subdivision and more precision. It can be used to detection of multiple cracks of complicated structure. Kim et al. [7] described a computer based crack analysis system for concrete structures using Fuzzy logic theory. When they used this theory the approach to identify the crack the planned system delivered results like to those got by experts system. Saravanan et al. [8] proposed a method based on the vibration signals developed from the operating machines to effectively detect the conditions of unreachable moving components inside the machine. The proposed model was designed using fuzzy classifier to generate the rules automatically from the feature set. Boutros and Liang [9] established four condition monitoring indicators for identification of transient and gradual abnormalities using fuzzy logic method. They successfully examined and validated the fuzzy-based method in two different applications. Sugumaran and Ramachandran [10] described the use of decision tree of a fuzzy classifier for choosing top few features that will distinguish the fault condition of the bearing from given trained samples. Miguel and Blazquez [11] established a decision making module based on fuzzy logic for model based damage analysis applications.

The objective of this paper, intelligent techniques have been developed using different types of membership functions (MF) in fuzzy controllers for crack prognosis in a cantilever beam having inclined crack. The first three natural frequencies of the cracked cantilever beam structure have been calculated from both FEA (using commercial available ANSYS 12 [12] software) and experimental analysis. The results obtained from both the analysis have been presented. An inverse analysis has been carried out using fuzzy controllers for inclined crack identification. The results obtained from experimental analysis have been compared with the results obtained from both fuzzy controllers and FEA and a good agreement have been observed between the results.

3. Design Limitations

The continuous model of the beam has been discretized for simplification. According to B. P. Nandwana and S. K. Maiti [4], for inclined edge crack cantilever beams, \( L/b \geq 12 \), \( a \leq t/2 \) and \( \theta \leq 45^\circ \), the difference in the two extreme locations is less than 4% of the beam length. The relative crack position range \( 0.1 \leq \beta \leq 0.75 \) from fixed end and relative edged crack depth range \( 0.1 \leq \alpha \leq 0.5 \) are tested.

4. Vibration Analysis of Inclined Crack Cantilever Beam

The beam with an inclined edge crack is fixed at left end, free at right end and it has a uniform structure having constant rectangular cross-section. The Euler–Bernoulli beam model was assumed. The crack is considered as an open crack and the damping has ignored in this study. The free bending vibration of an Euler-Bernoulli beam of a constant rectangular cross section (Figure 1 and Figure 2) is given by the following governing equation:

\[
\frac{d^2 Y}{dx^2} + \lambda \rho Y = 0
\]

(1)

Where ‘\( Y \)’ is displacement, ‘\( A \)’ is the cross-sectional area, ‘\( \rho \)’ is the mass density, ‘\( f \)’ is natural frequency of the vibration of the beam, ‘\( I \)’ is the moment of inertia, ‘\( E \)’ is the Young’s modulus of elasticity and the \( x \)-axis is aligned with the axis of the beam. The equations for the each part are given below:

\[
\frac{d^2 Y_1}{d\beta^2} - \lambda^2 Y_1 = 0
\]

(2)

\[
\frac{d^2 Y_2}{d\beta^2} - \lambda^2 Y_2 = 0
\]

(3)

![Figure 1. Schematic diagram of inclined edge crack cantilever beam](image)

![Figure 2. Representation of inclined crack in cantilever beam](image)

Here \( \lambda^4 = (f^2 \rho AL^4)/EI \) and \( \beta = L_e/L \) are non-dimensional parameters. Solving eqs (2) and (3), we get displacements on each part of the beam

\[
Y_1(\beta) = A_1 \cos \beta + A_2 \sin \beta + A_3 \sin \beta A_4 \sin \beta
\]

(4)

\[
Y_2(\beta) = A_1 \cos \beta + A_2 \cos \beta + A_3 \sin \beta + A_4 \sin \beta + A_5 \sin \beta + A_6 \sin \beta
\]

(5)

where \( A_1 \) to \( A_6 \) are arbitrary constants to be calculated from the boundary conditions. The boundary conditions at the fixed and free ends, respectively, are

\[
Y_{1,0} = 0, \quad Y_{1,0}' = 0, \quad Y_{2,0} = 0, \quad Y_{2,0}' = 0
\]

(6)
The continuity of displacement, bending moment and shear forces at the crack location (say $\beta = L/L$) and jump condition in the slope can be written in the following form:

$$Y_1(\beta) = Y_2(\beta), \quad Y_1^{"}(\beta) = Y_2^{"}(\beta), \quad Y_1^{"\"}(\beta) = Y_2^{"\"}(\beta) \quad (7)$$

### 5. Finite Element Analysis (FEA)

The finite element analysis is carried out using ANSYS 12 finite element program for modal analysis of the uncracked and inclined edge crack cantilever beam to determine the natural frequencies at different crack location, crack depth and crack inclination. Figure 3 shows the finite element mesh model of the cantilever beam and Figure 4 shows a magnified view of three dimensional inclined open edge crack zone. The cracked beams of the present analysis have the following configurations.

- Length of the Beam ($L$) = 800mm
- Width of the beam ($b$) = 60mm
- Thickness of the Beam ($t$) = 6mm
- Relative crack depth ($\alpha$) = Varies from 0.1 to 0.5
- Relative crack location ($\beta$) = Varies from 0.1 to 0.75

The material properties are taken for analysis as given below.

- Young's modulus of elasticity ($E$) = 70GPa
- Poisson’s ratio ($\nu$) = 0.346
- Density ($\rho$) = 2710Kg/m$^3$

The first three natural frequencies and mode shapes are calculated by FEA.

### 6. Experimental Analysis

The experiment on cantilever beam has been done for verify the FEA and fuzzy logic results. The aluminum cantilever beam specimen with dimensions (800mm x 60mm x 6 mm) with and without a crack is subjected to number of experimentation with the experimental set up shown in Figure 6 and Figure 5 for determining the natural. An experimental set-up contains (1) Vibration Analyzer, (2) Accelerometer, (3) Vibration Indicator, (4) Vibration Exciter, (5) Power Amplifier, (6) Test Specimen-beam, (7) Power Distribution and (8) Function Generator are used for performing the experiment as shown Figure 5 in the schematic diagram.

![Figure 3. Finite element mesh model of the cracked beam](image)

![Figure 4. Magnified view at the crack zone](image)

### 6.1. Specifications of the Instruments Used in the Experimental Set up

1. **Vibration Analyzer**
   - Type: 3560L
   - Make: Bruel & kjaer
   - Frequency: 7 Hz to 20 KHz
   - Channels: 2 Inputs, 2 Tachometer
   - Input Type: Direct/CCLD

2. **Accelerometer**
   - Type: 4513-001
   - Make: Bruel & kjaer
   - Sensitivity: 10mv/g-500mv/g
   - Frequency Range: 1Hz-10KHz
   - Supply voltage: 24volts

3. **Vibration Indicator**
   - Software: PULSE LabShop Software Version 12
   - Make: Bruel & kjaer

4. **Vibration Exciter**
   - Type: 4808
   - Force rating: 112N (25 lbf) sine peak
   - Frequency: 5Hz to 10 kHz
   - First axial resonance: 10 kHz
(5) Power Amplifier
Type: 2719
Power Amplifier: 180VA
Make: Bruel & kjaer

(6) Test Specimen-beam
Material: Aluminum
Dimension: 800mmX60mmX6mm

(7) Power Distribution
220V power supply, 50Hz

(8) Function Generator
Model: FG200K
Frequency Range: 0.2Hz to 200 KHz
Output Level: 15Vp-p into 600 ohms

Before the experimental study the beams surface has been cleaned and organized for straightness. Subsequently, transverse inclined crack is created at different location from fixed end in different specimens with the help of Wire EDM machine. The natural frequencies corresponding to 1st, 2nd and 3rd mode are noted with different crack depth at different crack locations and different crack inclinations in the cracked cantilever beam.

6.2. Experimental Results
The mode shapes obtained from experimentation (Figure 5) for different relative crack locations, relative crack depths and crack angles have been compared with that of the finite element analysis (FEA) for both cracked and uncracked beam. The comparison results are shown in Figure 6 to Figure 11.

7. Analysis of the Fuzzy Controller
The different fuzzy controllers have developed for analysis of crack detection. Here only one fuzzy controller (Gaussian Fuzzy Controller) having six input and three output parameters as shown in Figure 12. The linguistic term used for the inputs are as follows; “RFNF”, “RSNF”, “RTNF”, “RFMD”, “RSMD” & “RTMD” and the linguistic term used for the outputs are as follows; “RCL”, “RCD” & “CA”. For each fuzzy controller there nine MFs have taken for each six input parameters (“RFNF”, “RSNF”, “RTNF”, “RFMD”, “RSMD” & “RTMD”) and nineteen MFs for “RCL”, nine MFs for “RCD” and
twenty nine MFs for “CA” have taken as shown in Figure 13, Figure 14 & Figure 15. More than one hundred fuzzy rules have developed and have found the output results close to the experimental results. The crisp values of RCL, RCD and CA are computed using the center of gravity method as given below.

Relative crack location = \( RCL_{1,2} = \int_{\text{location}} (\mu_{RCL_{1,2}}(\text{location}).d(\text{location})) \)

Relative crack depth = \( RCD_{1,2} = \int_{\text{depth}} (\mu_{RCD_{1,2}}(\text{depth}).d(\text{depth})) \)

Crack angle = \( CA_{1,2} = \int_{\text{angle}} (\mu_{CA_{1,2}}(\text{angle}).d(\text{angle})) \)

\( \mu \) and \( d \) are the membership functions and the degree of membership respectively.

\[ \begin{align*}
\text{Relative crack location} &= RCL_{1,2} = \int_{\text{location}} (\mu_{RCL_{1,2}}(\text{location}).d(\text{location})) \\
\text{Relative crack depth} &= RCD_{1,2} = \int_{\text{depth}} (\mu_{RCD_{1,2}}(\text{depth}).d(\text{depth})) \\
\text{Crack angle} &= CA_{1,2} = \int_{\text{angle}} (\mu_{CA_{1,2}}(\text{angle}).d(\text{angle}))
\end{align*} \]

8. Discussion

The results found from finite element, fuzzy and experimental analysis is discussed in the following section.
The each fuzzy controller is designed with fuzzy Gaussian, Trapezoidal and Bell Shape membership functions. An experimental set-up (as shown in Figure 5) has been developed to validate the results from the various methodologies adopted in the current analysis by performing experiments on an Aluminum beam specimen. The results from FEA, different fuzzy controller and experimental analysis are compared as shown in Table 1. Thus, the intelligent fuzzy controller can be successfully used for structural health monitoring.

Figure 15. Gaussian MF for Relative Crack Location, Relative Crack Depth and Crack Angle vs. Degree of Membership Respectively

9. Conclusion

The conclusions derived from the various analyses as mentioned above are depicted below.

- The results from finite element analysis are good agreement with the results obtained from fuzzy logic analysis.
- The analysis of vibration characteristics of the cracked beam shows a variation of mode shapes and natural frequencies for the cracked and un-cracked beam.
- The results obtained from finite element analysis, fuzzy analysis are compared with the experimental results. They show a very good agreement.
- This technique can be used as a condition monitoring tool for vibrating fault structures like beams and plates.

References