Stock Returns and Forecast: Case of Tunisia with ARCH Model

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Abstract  The forecasts allow you to predict future values of historical time series data. With the possibilities of forecasting, you can make projections of future values based on the values of the past. Using projections, organizations can prepare for the changes in the economic or competitive conditions by analyzing historical time series data to predict performance and future trends. For example, in a supply chain, if the expected demand match the actual demand, significant efficiencies can be achieved in terms of production, distribution and return. Forecasts use various predictive methods based on mathematical algorithms that model the future demand based on historical time series data that can be obtained from queries and tables containing columns of date or time. The overall objective is to choose a method to generate a time series model best fit values of the past, identifying existing data patterns and projecting the model in the future to generate the forecast. The purpose of this article is to analyze the volatility behavior of the Tunisian stock returns series index TSR in daily frequency over January 1984 to June 2010 period. We will present various non linear models for the behavior of these series through the Arch models. However, these processes are based on the assumption of non autocorrelation endogenous variable in long term. In fact, these models have autocorrelation that decay very quickly when the delay increases.

Keywords: volatility, ARCH model, Tunisian stock exchange, forecasting


1. Introduction

The relationship between the volume of exchange and the stock price has attracted the attention of many academics. In fact, the majority of empirical studies that have been conducted have found a positive correlation between volatility and stock exchange, this work has been especially developed by Bollerslev, Gibson and Zhou in 2004, Ehrmmenn and Rigobon, Daniel Stavarek and Desislava Dimitrova in 2005. In most theoretical models that have been developed to explore the relationship between volume and dynamics of stock prices is generated due to the asymmetry of information. The following test can be used to predict future values of the Tunisian stock returns series index TSR; in fact, if the time series is relatively fixed with no general tendency to fluctuate in a part of the series compared with another part of the series, the moving average parameters, weighted moving average or exponential smoothing only provide the best fit model. If the time series shows a trend with a systematic upward or downward movement over time, the double exponential smoothing model provides the best fit.

2. Volatility Test

2.1. Shiller Test

Shiller retains the rational valuation formula as a model for determining the course of action. Therefore, the course of action is determinate by the core values, that is to say, the actual discounted value of expected future dividends:

\[ p_t = v_t = \sum_{i=1}^{n-1} \delta^i E_t D_{t+i} + \delta^n E_t P_{t+n} \]  \hspace{1cm} (2.1)

with \( P_{t+n} \) : the final anticipated rate at \( t+n \).

It is assumed that all investors have the same vision of the future. Therefore, all investors are the same anticipation of future dividends and it is also assumed that the discount factor \( \delta(0 < \delta < 1) \) is constant over all future periods. \( \delta \) is defined by \( 1/(1+k) \) were \( k \) is the required rate of return.

However assuring a required constant nominal return rate is unrealistic and well testing model estimate invariably a real constant discount rate and therefore the stock prices changes are also measured in real terms.

The data sets \( p_t^* \) will be calculated using the following formula:

\[ p_t^* = \sum_{i=1}^{n-1} \delta^i D_{t+i} + \delta^n p_{t+n} \]  \hspace{1cm} (2.2)
When calculating \( p_t^* \), the influence of the final prices \( p_{t+n} \) is pretty minimal since \( \delta^n \) is relatively low. However, if agents do not make systematic forecasting errors, so we will expect that these forecast errors in a large sample of data to be positive as negative and on average they will be almost zero. Therefore, it can be expected that the weighted sum of the prediction errors is relatively small and the general movements are correlated with \( p_t^* \).

With

\[
A_{t+i} = D_{t+i} - E_i D_{t+i}
\]

However, we can see that the correlation between \( p_t \) and \( p_t^* \) is low, and we reject the approach that the stock price are determined in an efficient market. However, the relationship between the variance of \( p_t \) and \( p_t^* \) where for \( x_t = p_t \) where \( p_t^* \) the variance of the sample is given by the following formula:

\[
\text{var}(x) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \quad (2.4)
\]

Where \( \bar{x} \) the sample mean
\( n \): number of observations

Forecast errors for investors can be represented by \( \mu_t \)

\[
\mu_t = p_t^* - p_t
\]

Where

\[
p_t^* = p_t + \mu_t \quad (2.5)
\]

If investors are rational, \( \mu_t \) will be independent of all the information available at time \( t \) makes their forecasts.

Will be as follows:

\[
\text{var}(p_t^*) = \text{var}(p_t) + \text{var}(\mu_t) + 2 \text{cov}(\mu_t, p_t) \quad (2.6)
\]

Informational efficiency implies that \( \text{cov}(p_t, \mu_t) \) is zero and we obtain:

\[
\text{var}(p_t^*) = \text{var}(p_t) + \text{var}(\mu_t) \quad (2.7)
\]

Since the variance of the forecast error is positive, we have:

\[
\text{var}(p_t^*) > \text{var}(p_t) \quad (2.8)
\]

\[
\text{VR} = \frac{\text{var}(p_t^*)}{\text{var}(p_t)} > 1
\]

Therefore, if the market product a stock price changes in accordance with the formula of rational assessment and if agents are rational to exploit the information and if the discount factor is constant, we will expect that the unequal variances are checked and the ratio variance or the ratio of the standard deviation SDR is greater than unity:

\[
SDR = \frac{\sigma(p_t^*)}{\sigma(p_t)} > 1
\]

### 2.2. Inequality Volatility

Inequality Shiller volatility is a pure consequence of the assumption that the real stock price is an optimal unbiased estimator of prices perfectly planned \( p_t^* \).

\[
p_t^* = p_t + \mu_t \quad (2.9)
\]

With \( \mu_t \) random error term and \( E(\mu_t | \Omega_t) = 0 \)

With \( \Omega_t \) all information

In other words, \( p_t \) is a sufficient statistic for predicting \( p_t^* \) and no other information than \( p_t \) can improve prediction of \( p_t^* \).

In this sense, \( p_t \) is optimum this implies that the conditional prediction error \( \text{E}(p_t^* - p_t | \Omega_t) \) be independent and therefore uncorrelated with \( \Omega_t \) since \( p_t \subset \Omega_t \) and \( p_t \) is independent of \( \mu_t \) means that \( \text{cov}(p_t, \mu_t) = 0 \) This is the informational efficiency. Using the definition of the covariance for the stationary series, we have the equation:

\[
\text{cov}(p_t^*, p_t) = \text{cov}(p_t + \mu_t, p_t)
\]

\[
= \text{cov}(p_t, p_t) + \text{cov}(p_t, \mu_t)
\]

\[
= \sigma^2(p_t) \quad (2.10)
\]

The definition of the correlation coefficient between \( p_t \) and \( p_t^* \) is given by this equation:

\[
\rho(p_t, p_t^*) = \frac{\text{cov}(p_t, p_t^*)}{\sigma(p_t)\sigma(p_t^*)} \quad (2.11)
\]

Replacing cov of (2.10) by (2.11), we obtained variances equality:

\[
\sigma(p_t) = \rho(p_t, p_t^*)\sigma(p_t^*) \quad (2.12)
\]

However, we know that the maximum value of \( \rho = 1 \) we have:

\[
\sigma(p_t) \leq \sigma(p_t^*) \quad (2.13)
\]

Volatility corresponds at the variation of assets prices. More fluctuation of underlying asset are rising and volatility will be high. Conversely, if these fluctuations are dropping volatility is low, it measures the price volatility of a financial asset. Thus, the higher the more risk you take will be important, however, the gain will be considerable.

### 3. Presentation of ARCH Model

#### 3.1. Model Specification

In 1982 Engle developed the ARCH models (Auto Regressive Conditional Heteroskedasticity) to allow the variance of a series depending on the available set of information, including time. This class of models is designed to overcome the shortcomings of traditional ARMA representations not adapted to financial issues. Financial series are indeed characterized by a variable volatility and asymmetry phenomena that can not be taken into account by modeling the type of ARMA. However, economic Viewpoint, it is particularly important to understand the volatility model of a series. Indeed,
investment decisions depend strongly not only of the evaluation of future returns, but also on the risks associated with various actions constituting portfolios. The estimate of the volatility of the profitability of an action provides a measure of risk attached to it. In addition, if the process followed by the volatility is specified correctly, it can provide useful in determining the process generating the returns information. ARCH models are based on an endogenous parameterization of the conditional variance. These processes have achieved significant success in financial applications partly due to the fact that they allow to rewrite the structural models for the choice of optimal portfolios, the relationship between asset returns and asset profitability and return on market portfolio.

Indeed, in ARCH model $\varepsilon_t$, is a process such as:

\[
E(\varepsilon_t / \varepsilon_{t-1}) = 0
\]

\[
V(\varepsilon_t / \varepsilon_{t-1}) = \sigma_t^2
\]  

(3.1)

With $\sigma_t^2$ is the conditional variance process $\varepsilon_t$. Thus, the conditional variance is a measure of volatility assets which may vary over time in this type of process, unlike the ARMA process where it is assumed constant.

Furthermore, the $\varepsilon_t$ process is not observable; it can match the innovation of a process of the ARMA series.

\[
\Phi(L)Y_t = \theta(L)\varepsilon_t
\]  

(3.2)

Or that of a general process:

\[
Y_t = f(X_{t-1}; b) + \varepsilon_t
\]  

(3.3)

With $f$ are a function of explanatory variables $X_{t-1}$ and a vector of unknown parameters $b$.

3.2. ARCH-M Process

Models ARCH-M are certainly the category of ARCH models most relevant from an economic point of view. Indeed, the risk assessment is a central point of financial economics. However, according to Engle, Lilien and Robins in 1987, the usual methods of measurement and risk prediction are often extremely simple and therefore unsuitable for the analysis of financial time series. Thus, the degree of uncertainty regarding the returns varies over time, the compensation received by agents for the risk averse end of the shareholding must also vary over time. Thus, it should not only measure the risk, taking into account its variation over time, but also to include this information as a determinant of profitability title or portfolio. The ARCH-M model takes account of this phenomenon by introducing the conditional variance in the mean equation.

Thus, a GARCH-M model can be written as follows:

\[
\Phi(L)Y_t = \Theta(L)\varepsilon_t + \alpha \sigma_t^2
\]  

(3.4)

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2
\]  

(3.5)

With $Y_t$ is a stationary process $\Phi(L)$ and $\Theta(L)$ are respectively polynomials and autoregressive moving average delay.

A variation of the conditional variance is therefore accompanied by a variation of the conditional mean of $Y_t$.

Therefore, a consistent estimate ARCH-M model requires a good $\sigma_t^2$ specification whereas for a GARCH $(p, q)$, consistent estimators of the parameters can be obtained even when the specification of $\sigma_t^2$ is poor. Furthermore, Pagan and Ullah in 1988, showed that the any specification error in the variance equation cause an eventual non convergence of estimators of the mean equation.

However, we can formulate the equation of the mean as follows:

\[
R_t = a + b\sigma_t^2 + \varepsilon_t
\]  

(3.6)

With $R_t$ refers to the profitability of earning ratio or portfolio and $b$ is the coefficient of relative risk aversion, there are many linear traditional evaluation models in which profitability is a function of $\sigma_t^2$ risk.

4. Estimation of ARCH Model

The usual estimation techniques (maximum likelihood, least squares, nonparametric methods) also applies to ARCH models. The maximum likelihood method and the least squares in two stages are the most used.

4.1. Maximum Likelihood

Most of the hazards related to financial series do not follow a normal distribution. However, the Gaussian density can be used to calculate the estimator even if the true distribution is not normal. Thus, the method of pseudo maximum likelihood is then used. A sufficient regularity condition to obtain convergence properties and asymptotic normality was established in 1982 by Weiss in the case of ARCH linear models. The most stringent of them seldom verified in practice is a condition of existence the moment of order 4 for $\varepsilon_t$ residues.

4.2. Least Squares

ARCH model can be seen as two successive ARMA models, one on the process, the second on the square innovations. It is this natural fact of introducing estimation procedure in two stages taking into account this particular structure.

Whether a regression model with ARCH errors:

\[
Y_t = f(X_{t}; b) + \varepsilon_t
\]  

(4.1)

\[
V(\varepsilon_t / \varepsilon_{t-1}) = h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2
\]  

(4.2)

The first step is to obtain consistent estimates of the parameters of the equation of the mean and the variance equation. The estimator $\hat{b}$ is obtained naturally by the OLS(Ordianry Least Squares) estimate from the regression of $Y_t$ on $f(X_{t}; b)$.

We put $\hat{b}$ this estimator. The estimated residuals are then given by:

\[
\hat{\varepsilon}_t = Y_t - X_{t_1} \hat{b}
\]  

(4.3)
By regressing $\hat{\varepsilon}_t$ on $1, \hat{\varepsilon}_{t-1}, ..., \hat{\varepsilon}_{t-q}$, we obtain a convergent estimators of the parameters in variance equation $\hat{\alpha}_0, \hat{\alpha}_1, ..., \hat{\alpha}_q$.

These two successive regressions were performed by the method of ordinary least squares, that is to say, without taking into account the phenomenon of heteroscedasticity. However, b estimators can be improved in a second step by regressing $Y_t$ on $f(X_t,b)$ by the generalized least squares (GLS) with:

$$\hat{h}_t = \hat{\alpha}_t^2 = \hat{\alpha}_0 + \hat{\alpha}_1 \hat{\varepsilon}_{t-1}^2 + ... + \hat{\alpha}_q \hat{\varepsilon}_{t-q}^2$$ (4.4)

We note that these estimators of the second stage $\hat{b}$

However, when the real conditionally distribution is normal we have:

$$V(\hat{\varepsilon}_t^2 / \varepsilon_{t-1}) = 2h_t^2$$ (4.5)

The estimators of the second stage are obtained by regressing $\varepsilon_t^2 / \hat{\varepsilon}_{t-1}^2, ..., \varepsilon_{t-q}^2$ by the GLS with $V(\varepsilon_t^2 / \varepsilon_{t-1}) = 2h_t^2$. It is important to note that this only affects the parameters of the medium: the parameter estimates of the variance are asymptotically efficient.

5. ARCH Modeling of Stock Exchange: the Case of Tunisia

The purpose of this study is to analyze the behavior of the volatility of stock returns series of the Tunisian index BVMT in daily frequency over the period January 1984 to December 2010.

Table 5.1. Estimation of the equation of the average AR (3)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000189</td>
<td>0.000173</td>
<td>1.02063</td>
<td>0.2539</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.142620</td>
<td>0.013496</td>
<td>9.863508</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-0.06543</td>
<td>0.013472</td>
<td>-5.2347</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.03878</td>
<td>0.05298</td>
<td>3.12582</td>
<td>0.0041</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.018094</td>
<td>Mean dependent VAR</td>
<td>0.000219</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.01936</td>
<td>SD dependent VAR</td>
<td>0.015138</td>
<td></td>
</tr>
<tr>
<td>S.E.of regression</td>
<td>0.015494</td>
<td>Akaike info criterion</td>
<td>-4.8134</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.21243</td>
<td>Schwatz criterion</td>
<td>-4.7936</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>189820.69</td>
<td>F-statistic</td>
<td>29.2109</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>2.03462</td>
<td>Prob(F-statistic)</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.2. Result of the ARCH test

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-Statistic</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000142</td>
<td>7.18E-04</td>
<td>14.8669</td>
<td>0.0000</td>
</tr>
<tr>
<td>RESID^2(-1)</td>
<td>0.34692</td>
<td>0.014019</td>
<td>27.8234</td>
<td>0.0000</td>
</tr>
<tr>
<td>RESID^2(-2)</td>
<td>-0.02486</td>
<td>0.013506</td>
<td>-2.0187</td>
<td>0.0892</td>
</tr>
<tr>
<td>RESID^2(-3)</td>
<td>0.09144</td>
<td>0.011984</td>
<td>6.98266</td>
<td>0.0000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.12790</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs*R-squared</td>
<td>702.933</td>
<td></td>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>F-statistic</td>
<td>260.6507</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3. Estimated model AR (3)-ARCH (3)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-Statistic</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000196</td>
<td>0.000189</td>
<td>1.07320</td>
<td>0.2396</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.2181</td>
<td>0.014191</td>
<td>15.2885</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-0.07619</td>
<td>0.01575</td>
<td>-5.0134</td>
<td>0.0000</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.04188</td>
<td>0.01189</td>
<td>2.97624</td>
<td>0.0021</td>
</tr>
<tr>
<td>C</td>
<td>9.52 E-02</td>
<td>1.18 E.05</td>
<td>81.0357</td>
<td>0.0000</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>0.21219</td>
<td>0.007899</td>
<td>21.13714</td>
<td>0.0000</td>
</tr>
<tr>
<td>ARCH(3)</td>
<td>0.15857</td>
<td>0.013016</td>
<td>12.6652</td>
<td>0.0000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.13622</td>
<td>0.011422</td>
<td>13.42134</td>
<td>0.0000</td>
</tr>
<tr>
<td>Obs*R-squared</td>
<td>702.933</td>
<td></td>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>F-statistic</td>
<td>261.571</td>
<td></td>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>Probability</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results from Table 5.3 show that the coefficients of the equation parameters of the variance is significantly different from zero and positive. Therefore, the coefficients satisfy the constraints ensuring positivity of the conditional variance. ARCH model (3) is a candidate model for the representation of the conditional variance of returns on the TSE index.

In view of Table 5.3, the coefficients of the variance equation are significant and positive. Moreover, we note that the sum of coefficients ARCH (1) and GARCH (1) is very close to 1. This reflects a persistence phenomenon in the conditional variance, frequently encountered in financial time series phenomenon.

6. Conclusion

Since the rejection of the hypothesis of efficient market volatility exists in the financial market, and it is known as changes in asset prices and returns. Why volatility is not only existing market information but also the behavior of investors, asset bubbles and several other factors. With influences, volatility is stabilizing financial markets, and also stabilizes the global economy. The influence of volatility can be reduced if one knows well and if we can anticipate. Although the methods are good and the reviews are accurate, volatility follows a random walk so it is very
difficult to follow. We have tried in this article to represent the evolution of the returns on the TSE Tunisian stock market index through ARCH model and the latter indicates that this series is highly volatile and has shown that large variations tend to be followed by strong and weak variations with small variations. So volatility changes over time. This observation suggested that ARCH process has been adapted to model the series.

References


