A Quantum Scalar-Vector-Spinor Model as Vortex-Field Theory for Approaching Physical Unification without Dark Sectors

Fred Y. Ye*

European Academy of Sciences and Arts; International Joint Informatics Laboratory (IJIL), Nanjing University, University of Illinois, Nanjing -Champaign, China – USA
*Corresponding author: yye@nju.edu.cn, blueyye@gmail.com

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Abstract The basic mathematical cliffs (scalar, vector, spinor) and the basic physical measures (mass-energy, wave-momentum, spin-information) are applied as logic foundations and linked as basic equations. While the local scalar-vector-spinor relations among mathematical cliffs and physical measures described electroweak and strong interactions, the global mathematical-physical equations interpreted gravity and repulsion, where a quantum scalar-vector-spinor (SVS) model approaches physical unification without dark sectors. Locally, Maxwell equations and Yang-Mills Fields are naturally included. Globally, Einstein-Friedmann equations characterize the total distribution of energy-momentum in space-time. A modified gravity explains ‘dark matter’ and a scalar energy with phase transitions produces ‘dark energy’. It is suggested to maintain three core principles as fundamental principles in physics, i.e. the action principle (Hamilton principle) which determines the dynamic mechanism of physical processes, the duality principle (Heisenberg principle) which produces quantum effects, and the equivalence principle (Einstein principle) which explains universal equilibrium. The verifications and developments are discussed, where three kinds of Higgs are expected, with predicting heavier Higgs around 17.58TeV and lighter Higgs around 233.7MeV, and the cosmological constant problem is naturally solved. While Newton and Einstein theories are included, the vortex-field theory balances mathematical structure and physical essence with combining micro-world and macro-universe well.

Keywords: standard model, quantum SVS model, vortex-field theory, cosmological constant, dark matter, dark energy, physical unification

1. Introduction

Based on the quantum field theory and the general relativity [1], we have particle standard model and cosmological standard model [2], where most experiments and observations matched the theories very well, though the two standard models are difficult to unify into a harmony theoretical framework. Meantime, the dark sectors, i.e. dark matter and dark energy [3,4], became the biggest issues in contemporary physics.

The presence of dark matter is implied in a variety of astrophysical observations, including rotation curves of galaxies and gravitational lenses, where all the effects that could not be explained unless more matter was present than Newton-Einstein system predicted. However, dark matter is only a hypothetical form of matter, which is thought to be non-baryonic in nature, possibly being composed of some undiscovered subatomic particles such as axions [5], or being nothing as we never find anything in dark matter detection [6].

Furthermore, dark energy is an unknown form of energy, which is thoroughly hypothesized to permeate all of space, for interpreting the universal accelerating expansion [7,8]. The density of dark energy is very low, about $7 \times 10^{-36}$ g/cm$^3$, much less than the density of ordinary matter or dark matter within galaxies. However, it dominates the mass-energy of the universe because it is uniform across all the space.

According to Planck satellite’s observation reports [9,10], the total mass-energy of the universe contains 4.9% ordinary matter and energy, 26.8% dark matter and 68.3% dark energy (negative pressure). Facing the unimaginable data, we need both new mathematical framework and physical ideas beyond two standard models, for which we have to find a unified mathematical language at first and then re-think the world structure as well as universe evolution, so that we could push the physical progress.
2. Brief Review

There are many hypotheses beyond standard models of particles and cosmology for approaching physical unification, in which we mention and review three kinds of representative ones. For each kind of the hypotheses, we conclude its advantages and costs for reaching its physical images. These hypotheses can be classified as string theory and M-theory [11,12,13]; LQG and CDT [14,15,16,17]; as well as MOND and TeSeV [18,19,20,21,22,23,24].

2.1. String theory and M-theory

Among all hypotheses beyond the standard models, string theory and M-theory are the most famous ones, though this kind of models are never verified by physical experiments and never forecast verifiable physics. Originated from string theory and developed to become M-theory (even F-theory), this kind of hypotheses focused on and characterized by supersymmetry and duality.

We can characterize two typical properties of M-theory. 1) The ‘theory’ contains strings and branes (membranes) as structural bases in 11 dimensions of space-time, with using compactification to explain how the extra dimensions reduce to the four space-time dimensions we observe. 2) There are dualities and identifications within the ‘theory’ that allow it to reduce to special cases of the string theories known, and ultimately into the physics we observe in our universe. Roughly speaking, fermions are the constituents of matter, while bosons mediate interactions between particles. In the hypotheses with supersymmetry, each boson has a counterpart which is a fermion, and vice versa. When supersymmetry is imposed as a local symmetry, one automatically obtains a quantum mechanical hypothesis that includes gravity. Such a hypothesis is called a supergravity, while a hypothesis of strings that incorporates the idea of supersymmetry is called a superstring. There are several different versions of superstring ‘theories’ which are all subsumed within the M-theory framework, including type I, type IIA, type IIB, Supergravity, Heterotic-O and Heterotic-E. At low energies, the superstring hypotheses are approximated by supergravity in ten space-time dimensions. Similarly, M-theory is approximated at low energies by supergravity in eleven dimensions.

Theoretically, the advantages of M-theory include 1) mathematical harmony and 2) logical concordance. However, for reaching its bright image, we have to pay following costs: 1) we need supersymmetric particles with duality between bosons and fermions, and 2) we need 11 dimensions (for M-theory) or even 12 dimensions (for F-theory). Over the past nearly 30 years of research, superstring or M-theory has developed greatly, and then the M-theory became the mainstream of contemporary theoretical physics. However, M-theory is only an overall hypothesis embracing not only quantum gravity but also matter and forces, based on the idea that present particles and hypothetical supersymmetric partners are vibrating strings. We may say that string theory and M-theory look beautiful, but we cannot say that they are true. The biggest issue of M-theory focuses on its verification, where it forecasts billions ways to real world, leading to unknown reality. Even there is no any mastered equations, though there were mathematical Seiberg-Witten equations based on Yang-Mills field [12,13]. One never believe that the string theory and M-theory characterize reality, and one never know any real physics beyond 4-dimensional space-time. Therefore, string theory and M-theory remain only tantalizing conjectures.

2.2. The LQG and CDT

With merging quantum mechanics into general relativity, loop quantum gravity (LQG) [14] and causal dynamical triangulations (CDT) [15] are hypotheses of quantum gravity, making them to be possible candidates as theory of everything. Its goal is to unify gravity in a common theoretical framework with the other three fundamental forces of nature, beginning with relativity and adding quantum features, leading to two main versions, LQG and CDT, relating each other [16,17].

The proponents of LQG are Ashtekar, Rovelli and Smolin, whose works are based on Ashtekar’s discovery that general relativity could be expressed in language like that of a gauge field. LQG is a mathematical formalism that defines a tentative quantum hypothesis of space-time, which can be defined as a Schrödinger quantization of a canonical formalism. The space of quantum states is defined as a Hilbert space K of complex-valued Schrödinger wave function on gravitational connection, and the quantum dynamics of space-time is governed by the Wheeler-De Witt equation, or the Einstein-Schrödinger equation.

The CDT originates from Euclidean quantum gravity, proposed by physicist Stephen Hawking and developed systematically by Ambjorn, Jurkiewicz and Loll. Its basic idea is that space-time geometry is made by piling up a large number of blocks, each of which represents a simple causal process. The causality means that the space-time geometry contains information about which events cause which other events. CDT approximates space-time as a mosaic of triangles, which have a built-in distinction between space and time. On a small scale, space-time takes on a fractal shape. There are a few simple rules that govern how the blocks can be piled up and a simple formula that gives the quantum-mechanical probability of quantum space-time.

Theoretically, the advantages of LQG and CDT include 1) co-evolution of space-time and 2) emergence of space-time and matter. However, for reaching the bright object, we have to pay costs: 1) we have to accept discrete space-time, and 2) we must accept space-time blocks. Moreover, discrete space-time in the LQG and CDT resemble artificial suppositions only. A recent research revealed that the LQG may also link with M-theory via H-duality [18].

2.3. The MOND and TeVeS

This kind of hypotheses originated from 1983 when Milgrom published his Modified Newtonian dynamics (MOND) [19], in which he proposed a modification of Newton's laws to account for observed properties of galaxies. It is an alternative to the hypothesis of dark matter in terms of explaining why galaxies do not appear
to obey the currently understood laws of physics. Since Milgrom proposed his original MOND, the hypothesis has developed to become Tensor-Vector-Scalar (TeVeS) theory by Sanders [20,21] and Bekenstein [22,23], as well as analogy from Moffat [24]. In its relativistic version according to Bekenstein, important physical principles, such as action principle, equivalence principle etc. are kept. An important alternative is that geometric metric $g_{\mu\nu}$ is replaced by physical metric $\bar{g}_{\mu\nu}$.

Based on the physical metric and dynamic scalar and 4-vector fields, involving one free function, and length scale and two positive dimensionless parameters, the advantages of TeVeS include 1) predicting gravitational lensing in agreement with the observations without dark matter; and 2) keep concordance to general relativity (GR) with passing the usual solar system tests. For reaching the bright image, we have to pay following costs: 1) we have to modify GR; and 2) we must introduce two or three parameters. This is feasible, while TeVeS provides a specific formalism for constructing cosmological models that resemble near true.

On other hypotheses, we can mention spinor and twistor theory (STT) [25], as well as HUFT [26]. STT came from Penrose whose (and Rindler) introduced spinor (invented by mathematician Cartan, originated from the root of a vector) calculus in four-dimensional Lorentzian space-time, and then proposed twistor hypothesis arose from a desire to unify and account for the various occurrences of complex numbers, holomorphic functions, and conformally invariant calculus in general relativity and space-time geometry. Similar to superstring-theory, STT had its unique value primarily in mathematics rather than in physics, and its predictions of the space curvature $K<0$ and the cosmological constant $\Lambda>0$ seem troublesome, as astronomical observation suggests strongly that $K=0$ and $\Lambda=0$. HUFT extended GR with including both GR and quantum theory (QT), which is also an independent hypothesis covering universe. However, HUFT needs spin-force and scalar-force as basic interactions and forecasts 19 or 26 dimensions and more vector-bosons than standard model, which have no present experimental supports.

Therefore, we explore along the way combining TeVeS with spinor mathematics based on particle standard model and cosmological standard model, and we also consider cosmological phase transitions [27], for approaching physical unification without dark sectors.

3. Theoretical Foundations

An improved physics demands an innovative mathematical language. With referring to the idea of three worlds from Penrose [25], we also set three worlds: the natural world, the mathematical world and the physical world. Now we begin new design based on nine basic concepts: (1) three concepts for describing the natural world: space ($s$), time ($t$) and their combination or order ($r$); (2) three concepts in the mathematical worlds: scalar ($\phi$), vector ($A$) and spinor ($B$); and (3) three concepts for quantifying the physical world: mass-energy ($E$), wave-momentum ($p$) and spin-information ($S$). Some new concepts may introduce when there is theoretical need such as field, entropy and velocity. The choose of basic physical concepts should match with physical measures for theoretical analysis and practical application, so that it is reasonable to choose energy $E$ and momentum $p$ as basic physical measures based on known physics. Meantime, $E$ and $p$ will merge into action, so they are better concepts than some worse concepts such as inertia and force.

For matching relativity, mass-energy and wave-momentum are dynamically linked by quantitative relation

$$E^2 - p^2 c^2 = m^2 c^4 \tag{1}$$

in which $c$ is speed of light. When we use 4-momentum form $p^\mu = (E, p)$ and natural unit, the Eq. (1) will become $p_\mu p^\mu = m^2$. When wave stopped and matter kept, we knew its famous static result as Einstein formula $E = mc^2$.

Meanwhile, the wave-momentum ($p$) follows De Broglie’s matter wave

$$p = h / \lambda = h / c \tag{2}$$

where $h$ is Planck constant, $\lambda$ is wave length and $v$ is wave frequency. When pure dynamic wave of zero mass happened, we have $E = hv$.

The Eqs. (1) and (2) defined basic relations among basic physical measures $E$ and $p$, with linking matter and wave as a foundation of wave-particle duality.

3.1. Fundamental Framework

The three worlds and nine basic concepts are philosophically constructed a triad relation as shown as Figure 1, where the natural world exists there, we want to describe the natural reality approached by measurements in the physical world while the mathematics provides ideal and abstracting methodology.

![Figure 1. Nine basic concepts distributed in the three worlds](image-url)
constructs a natural existence of the world and others are artificial ones for measuring the world.

In the integrated and synthetic framework, there are three integrated and synthetic cliffs (multi-vectors) M, N, and P, with three mathematical operations (addition, multiplication and calculus), and there are three links among them, where we marked with L1, L2 and L3. For describing the relations among three worlds, we need to follow past knowledge and physical principles.

The L1 denotes the linkage between mathematical world and physical world, where we need a key concept field (F), which describes the reality. The L2 denotes the linkage between physical world and natural world, where we need a key concept mass (m), which indicates matter. The L3 denotes the linkage between natural world and mathematical world, where we need another key concept connection (ω), which links the geometry of space-time to mathematical cliffs. Modern mathematics has developed rich connection theories and modern physics has set up quantum field theory, which contribute the foundations for further exploration with considering that Higgs quantum field theory, which contribute the foundations of maximum and minimum dualities, which may connect micro- and macro-world. Quantum mechanism orignated from Heisenberg principle, where duality is a key. Now we can define quantum mechanism as canonical commutation relation between different components of cliffs as well as different types of cliffs in different worlds, including the relations between scalar and vector linking with space-time, such as scalar energy E (linking to vector time t) and vector momentum p (linking to scalar space s), which construct similar canonical commutation relations

\[ [t, E] = tE - Et = h/i \]  \hspace{1cm} \hspace{1cm} (5)  
\[ [s, p] = sp - ps = h/i \] \hspace{1cm} \hspace{1cm} (6)

where i is the imaginary unit and h is the reduced Planck's constant (h = \hbar/2\pi).

Since energy is a scalar, the time has to be a vector, with direction from past to future. Meanwhile, as momentum is a vector, the space should be a scalar.

P2. Duality principle (Heisenberg principle). This principle produces quantum effects, which fits duality of particle and wave in quantum theory, as well as combination of maximum and minimum dualities, which may connect micro- and macro-world. Quantum mechanism orignated from Heisenberg principle, where duality is a key. Now we can define quantum mechanism as canonical commutation relation between different components of cliffs as well as different types of cliffs in different worlds, including the relations between scalar and vector linking with space-time, such as scalar energy E (linking to vector time t) and vector momentum p (linking to scalar space s), which construct similar canonical commutation relations

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Since energy is a scalar, the time has to be a vector, with direction from past to future. Meanwhile, as momentum is a vector, the space should be a scalar.

P3. Equivalence principle (Einstein principle). This principle balances mathematical structure and physical essence, which claims the equivalence of space-time curvature and energy-momentum distribution in Einstein general relativity, leading gravity equation of tensor form as

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (R - 2\Lambda) = \kappa T_{\mu\nu} \] \hspace{1cm} \hspace{1cm} (7)

in which \Lambda is a cosmological constant. The equation will be replaced by equivalent spinor form in this article.

P1 originated from analytical mechanics, which is a foundation of physics, linking with all physical theories. P2 came from quantum mechanics, which is a basic consideration in micro-physics. P3 proposed by Einstein in his general relativity, which contributes a key to macro-physics.
Meanwhile, both micro-physics and macro-physics will apply same Clifford mathematical language via scalars, vectors and spinors \cite{30}.

### 3.3. Phenomenological Foundations

There is a basic fact in the universe, i.e. universal vortex phenomena from micro-level to macro-scale, as shown as Figure 2.

![Universal vortexes from micro-particle to macro-galaxy (from Ye \cite{28})](Image)

The Figure 2 describes various vortexes from micro-level to macro-scale, where we see, from left to right, the imaged spinning particle, real river vortex, real atmospheric vortex and the typical galaxy (NGC3370). These phenomenological vortexes mean that the vortexes are universal phenomena, so that we should have their mathematical and physical description of phenomenological vortexes at different scales in the universe. The basic rule can be 'one cliff, one vortex' in mathematics and physics. Therefore, everything in the universe is vortex, and we need mathematical structure and physical measurement for describing the basic fact.

Meanwhile, the experiments reveal that matter consists of quarks and leptons (as fermions) combined by bosons. The quarks come in triplets of 'color' with carrying color index c=1, 2, 3, and both the quarks and the leptons come in doublets of weak isospin with family index b=1, 2, 3. All fermions are left-handed and can be arranged into spinor fields, which are eigenstates of the chiral projection operators \( P_L \) or \( P_R \), so that we introduce the total wave function as

\[
\psi = (\psi_L, \psi_R, \psi^L, \psi^R)^T,
\]

\[
\psi_L = (\psi_{L,1}, \psi_{L,2}, \psi_{L,3}), \quad \psi_R = (\psi_{R,1}, \psi_{R,2}, \psi_{R,3}).
\]

\[
P_L \psi^C = \psi^C_R, \quad P_R \psi^C = P_R \psi^C_L = 0
\]

where the subscripts L and R denote left- and right-handed fields respectively and C indicates the charge conjugate field of the antiparticle, e and n mean electrons and neutrinos while q and l stand for quarks and leptons, respectively.

New theory would be based on above foundations. A big issue in current physics focuses on that there are different mathematical languages in two standard models, where particle standard model applies the language of gauge field while cosmological standard model uses the language of tensor analysis. Now we need a unified mathematical language at first, for revealing mathematical structure and physical essence of the universe.

### 4. Mathematical Structure

Physical progress depends strongly on the innovation of mathematical methods. Here, we found a mathematical methodology developed since 1960s, and we combine geometric calculus \cite{31,32} with spinor analysis \cite{33,34,35,36}, proposing a concise mathematical framework for approaching reality.

In Clifford algebra, a compound number is called a cliff or a multi-vector \( M_k \) (k = 0, 1, 2, 3, 4), where \( M_k \) is a cliff of grade k, k=0 corresponds to scalar, k=1 to vector, k=2 to bivector, k=3 to trivector as pseudovector, and k=4 to pseudoscalar (closure to scalar). \( M_k \) is functional of space-time, on which any cliff or compound number consists of three parts: complex scalar \( \phi = M_0 + iM_4 \), complex vector \( A = M_1 + iM_3 \), and bivector \( B = M_2 \), writing as

\[
M = (\phi, A, B).
\]

So one cliff or compound number \( M \) is consist of scalar \( \phi \), vector \( A \) and spinor \( B \). Writing two cliffs \( M_1 \) and \( M_2 \), they construct a mathematical ring with two operators + (addition and subtraction) and * (multiplication and division), and there exists the third mathematical operation, i.e. calculus (differential and integral).

\[
M_1 = (\phi_1, A_1, B_1)
\]

\[
M_2 = (\phi_2, A_2, B_2)
\]

where \( \phi \) stands for a complex scalar while \( A \) indicates a complex vector and \( B \) denotes a spinor.

While \( \phi \), \( A \) and \( B \) abide by mathematical rules according to scalar, vector and spinor operations respectively, cliffs \( M_k \) will follow the Clifford algebraic rules, where the addition (and subtraction) happen in same cliffs when negative element exists

\[
M_1 + M_2 = (\phi_1 + \phi_2, A_1 + A_2, B_1 + B_2)
\]

There are exchange law and combination law in addition (and subtraction)

\[
M_1 + M_2 = M_2 + M_1
\]

\[
(M_1 + M_2) + M_3 = M_1 + (M_2 + M_3)
\]

Multiplication (and division) can happen in same and different cliffs. However, exchange law and combination law do not generally exist here, where the multiplication is consisted of interior (dot) product and exterior (wedge) product as

\[
M_1 \ast M_2 = M_1 \cdot M_2 + M_1 \wedge M_2
\]

in which dot product did grade-reducing and wedge product did grade-increasing.

Generally, the algebra is a neither communicative nor associative system. If there is asymmetric communicative relation

\[
M_1 \ast M_2 = -M_2 \ast M_1
\]

the mathematical system is anti-communicative ring.

The revised conjugation of \( M \) is denoted as \( \bar{M} \)

\[
\bar{M} = (\phi, -A, B)
\]

and then any real variable \( x \) associates its conjugate \( x' \) as

\[
x' = Bx\bar{B}
\]

The entanglement will happen between \( M \) and \( \bar{M} \), which is different from anti-particle, where anti-particle is
described by \(-M = (-\phi, -A, \overline{B})\).

Three Pauli matrices \(\sigma^k = (\sigma^1, \sigma^2, \sigma^3)\) and unit matrix \(\sigma^0\) are applied as orthonormal basis

\[
\begin{align*}
\sigma^0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
\sigma^1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\
\sigma^2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \\
\sigma^3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\end{align*}
\] (19)

and they are extended as spinor basis on spinor indices \(K, L\), i.e. \(\sigma\)-type spinor matrices as

\[
\begin{align*}
\sigma_{KL}^0 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \sigma_{0KL}^0, \\
\sigma_{KL}^1 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -\sigma_{1KL}^1, \\
\sigma_{KL}^2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = \sigma_{2KL}^2, \\
\sigma_{KL}^3 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -\sigma_{3KL}^3
\end{align*}
\] (20)

where \(A'\) or \(B'\) denotes complex conjugate of \(A\) or \(B\).

Also, \(\gamma^{\mu} (\mu = 0, 1, 2, 3)\) are applied by four Dirac matrices as four orthonormal basis in Pauli-Dirac representation as

\[
\begin{align*}
\gamma^0 &= \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} = -\gamma_0, \\
\gamma^k &= \begin{pmatrix} 0 & -\sigma^k \\ \sigma^k & 0 \end{pmatrix} = -\gamma_k, \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}
\end{align*}
\] (21)

or in chiral representation as

\[
\begin{align*}
\gamma^0 &= \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix} = -\gamma_0, \\
\gamma^k &= \begin{pmatrix} 0 & -\sigma^k \\ -\sigma^k & 0 \end{pmatrix} = -\gamma_k, \gamma^5 = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}
\end{align*}
\] (22)

in which \(I\) is 2×2 identity matrix, \(\gamma^0\) is time-like base, \(\gamma^k (k = 1, 2, 3)\) space-like bases, and their \(\gamma\)-type spinor matrices as

\[
\begin{align*}
\gamma_{KL}^0 &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma_{0KL}^0 & 0 \\ 0 & -\sigma_{0KL}^0 \end{pmatrix}, \\
\gamma_{KL}^k &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \sigma_{0KL}^k \\ \sigma_{0KL}^k & 0 \end{pmatrix}
\end{align*}
\] (23)

Spinors and tensors can be translated each other [34,35], where only difference is the purely formal one of replacing each tensor index by a pair of spinor indices. The spinors are more fundamental than tensors in the description of space-time structure and spins, as spinors can be used to describe particles with spins 1/2, 3/2, ... in addition to those with spins 0, 1, 2..., whereas tensors can describe only the latter kind of spins. Therefore, we replace tensors with spinors for theoretical perfection, but we also keep tensors for computational convenience.

The world metric \(g_{\mu\nu}\), as geometric metric also links spinor via tensor, as follows

\[
\begin{align*}
g^{\mu\nu} &= \gamma^\mu \gamma^\nu = \sigma_{KL}^0 \sigma_{PQ}^0 = g_{KL}^0 \sigma_{PQ}^0 \quad (24) \\
g_{\mu\nu} &= \gamma_\mu \gamma_\nu = \sigma_{KL}^1 \sigma_{PQ}^1 = g_{KL}^1 \sigma_{PQ}^1
\end{align*}
\]

where space-time metric \(g^{\mu\nu}\) or \(g_{\mu\nu}\) is naturally generated.

A spinor \(B\) has its complex conjugate \(\overline{B}\), While the label \(K'\) is regarded as the complex conjugate of the label \(K\), there exists

\[
\overline{B^K} = \overline{B}^{K'}.
\] (26)

The raising and lowering spinor indices is similar to that of tensors in raising and lowering space-time indices as

\[
\overline{B^K} = \sigma_{KL}^0 B_L, B_K = B^K \sigma_{KL}^0.
\] (27)

The scalar and vector partial differential operator gives normal calculus as

\[
\begin{align*}
\nabla &= \sigma^\mu \partial_\mu = \sigma_{KL}^0 \partial_{KL}, \partial_\mu = \gamma^\mu \partial_\mu = \gamma_{KL}^0 \partial_{KL}.
\end{align*}
\] (28)

The covariant derivative defines as

\[
\begin{align*}
\nabla_{\mu} &= \partial_{\mu} - ig A_\mu, D_{KL} = (\partial_{KL} - i g A_{KL}).
\end{align*}
\] (30)

When \(\sigma\) are spinor affine connections, the curvature spinor is constructed as

\[
\begin{align*}
\Omega_{PQKL, CD} &= \Omega_{PQ, \mu} \gamma^\mu_{KL} \gamma^\nu_{CD} \gamma^{\nu' \mu'}_{KL} \gamma^{\nu' \mu'}_{CD}, \\
\Omega_{PQ, \mu} &= \Gamma_{PQ, \mu} - \Gamma_{PQ, \nu} \gamma_\nu \gamma_\mu, \\
+ \Gamma_{KL, \nu} \gamma_\nu \gamma_\mu - \Gamma_{KL, \nu} \gamma_\nu \gamma_\mu.
\end{align*}
\] (31)

The operator of dual indices spinor derivative defines as

\[
\nabla_{KL} = \nabla_K \partial_L = \sigma_{KL}^0 \gamma_{KL}.
\] (33)

When a scalar \((k)\) times a cliff \((M)\), it is normal calculation. However, when a vector \((A)\) times a cliff \((M)\), it scales the cliff (and reverse its direction if negative). A vector \((A)\) times a k-vector \((M_k)\) is completely characterized by the geometric product (omitting symbol*)

\[
AM_k = A \cdot M_k + A \wedge M_k
\] (34)

or generalizing Eqs. (6) and (10) then producing a (k-1)-vector and a (k+1)-vector.
where the inner product becomes a grade lowering operation while the outer product is a grade raising operation. These concepts may link with Dirac's bra–ket notation $< \mid | \mid >$ [36], where matrix formulation and wave formulation are equivalence of both the basis and the operators, carrying time-dependence. The left part $< |$, called “bra”; represents a row vector, while the right part $| >$, called “ket”; denotes a column vector; and $< | >$ represents an inner product, producing a scalar, while the $| > \mid$ denotes an outer product, producing a tensor. The interaction picture allows for operators to act on the state vector at different times and forms the basis for quantum field theory and other methods.

Physically, one cliff $M$ is just one state. A cliff $M$ characterizes a particle or a rotated thing, and the conjugation $\bar{M}$ measures its entangled particle or duality thing. While scalar $C$, vector $A$ and spinor $B$ respectively revealed local information, $M$ and $\bar{M}$ described global information.

The differential process can also be viewed as an geometric calculus operator $\nabla$ acting on functional linearly. A differential operation acting on a geometric functional $F(M)$ is equivalent to the geometric product $\nabla F = \nabla \cdot F + \nabla \times F$ (37)

When $S$ is a $k$-dimensional surface, if we interpret an element of directed area $dS$ as a $k$-vector-valued measure on boundary $\partial S$, we get the generalized Stokes theorem between $\partial S$ of a physical system. The local and global considerations quantum mechanism [30] as mass-energy $E$, wave-momentum $p$ and spin-information $S$, which also construct eigenvalues of a physical system. The local and global considerations are based on mathematical methodology and physical metrics.

The mathematical methodology also supplies the quantum mechanism [30] as

$$[x, Y] = xY - Yx = \hbar / i$$ (39)

The Eq.(39) means the micro-error of measurement essentially. For any operator $X$ on Hilbert space, when $<x><\psi > = <x|M|\psi >$ and $(Ax)^2 = <x|^2 - <x>x$, if $<xY+Yx> - 2<x><Y> = 0$ for any measure $Y$, we have Heisenberg uncertainty relation

$$(Ax)^2 (\Delta Y)^2 \geq \hbar / 2.$$ (40)

Its dynamic extension includes probability distribution [30]

$$i(\phi) = \text{pro}(\phi) = \frac{\hbar}{\pi e^{\psi \phi}} \left( \frac{\hbar}{\pi (e^{\psi \phi} \pm 1)} \right)$$ (41)

where $\text{pro}(\phi)$ denotes probability density in statistical distribution or wave function.

The physical metric applies to replace geometrical metric. Similarly to TeVeS theory, the SVS theory also applies physical metric $\tilde{g}_{\mu \nu}$, but it generates from geometric metric $g_{\mu \nu}$ with including the combination of vector $A$ and scalar $\phi$ naturally

$$\tilde{g}_{\mu \nu} = e^{-2\phi}(g_{\mu \nu} + A_\mu A_\nu) - e^{2\phi} A_\mu A_\nu$$ (42)

and its inverse physical metric as

$$\tilde{g}^{\mu \nu} = e^{2\phi}(g^{\mu \nu} + A^\mu A^\nu) + e^{-2\phi} A_\mu A_\nu$$ (43)

While the local relations among components of mathematical cliffs $(\phi, A, B)$ link with the physical world $P$, the global relations between $M$ and $P$ cover all elements of the system, where the $\phi$, $A$ and $B$ act as the local realities and the $M$ and $P$ link as the global realities. The active system may be a particle, an atom, a star, a galaxy and even the cosmos.

Statically, the situation resembles that one cliff describes one state and one state is one vortex (one cliff, one state, one vortex). Dynamically, imaginal functional $i(\phi)$ produces probability density. In inner $M$, there are respective $\phi$, $A$ and $B$ for describing local features, while there are outer relations linking with $P$ for describing global features.

Structurally, every vortex is formally same, so that a micro-vortex such as particle and a macro-vortex such as galaxy resemble similar except scaling size. Thus, we can describe any vortex with mathematical cliff $M=(\phi, A, B)$, where different vortex has different values of $\phi$, $A$ and $B$.

The wave function $\psi$ can be introduced with relation to complex scalar and differential operator

$$\phi' = \psi\phi\sigma_3\partial = \overline{\psi}\phi\sigma_3\partial = \psi' = U\psi; \overline{\psi'} = \overline{\psi'} = \overline{\psi}U'$$ (44)

where $U$ is matrix as $U = \exp(i\sigma^\mu A_\mu)$.

This is a harmonic physics. If $\phi$ and $\psi$ merged into a complex scalar $z = \phi + i\psi$, or scalar $\phi$ linked with wave function as $\psi = \phi + i\psi$, there might generate special cases of bi-scalars interactions.

When mathematical $M = (\phi, A, B)$ reveals that everything can be characterized by three eigenvalues $(\phi, A, B)$, the eigenvalues of physical particles are...
characterized as (E, p, S). Dirac equations for particles originated from Eq.(1) become
\[ \gamma^\mu (\partial_\mu - iqA_\mu) + m\psi = 0 \tag{45} \]
We know \( q = e/\hbar c \) and \( m = m_0 c/\hbar \) when \( h \neq 1 \) and \( c \neq 1 \). Its spinor form is
\[ [\gamma^{KL'}(\partial_{KL'} - iqA_{KL'}) + m]\psi = 0 \tag{46} \]
with Lagrangian density as
\[ L = \mathcal{G}(\gamma^{KL'}, i\partial_{KL'} + m)\psi - q\mathcal{G}^{KL'}\psi A_{KL'}. \tag{47} \]
The 4-component spinor is composed of two 2-component spinors as
\[ \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}; \quad P_L \psi = \frac{1-\gamma^5}{2} \psi_L; \quad P_R \psi = \frac{1+\gamma^5}{2} \psi_R \tag{48} \]
the Dirac equations are always correct for describing fermions.
In present physical theories, electroweak, strong and even gravitational interactions are related to local symmetries and described by Abelian and non-Abelian gauge fields [37]. Following physical system supposes that electroweak and strong fields relate to the local relations while gravitational and repulsive interactions concern to the global equations.

5.1. Local Relations Based on P1 and P2: Gauge Fields

In the mathematical world, there are scalar \( \phi \), vector \( A \) and spinor \( B \). We know that the electrical field \( E \) and magnetic field \( H \) as \( E + iH \) may link \( \phi \) and \( A \) as follows
\[ H = \nabla A; \quad E = -\nabla \phi - \frac{\partial A}{\partial t}. \tag{49} \]
The \( E \) and \( H \) also produce density \( \rho \) and current \( J \) as
\[ \rho = \nabla E; \quad J = \frac{\partial H}{\partial t} + \nabla E. \tag{50} \]
There is a mathematical inner relation when we apply natural unit system as \( c = h = 1 \) as
\[ \nabla A + \frac{\partial \phi}{\partial t} = 0 \rightarrow \nabla A + \phi A \rightarrow \partial_{KL'}A_{KL'} = 0 \tag{51} \]
where \( A_\mu = (\phi, A) \) and constant \( c \) can be defined as \( \max(ds/dt) \) i.e. speed of light.
Equivalently, we introduce electromagnetic field
\[ E_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{52} \]
the electromagnetic Lagrangian becomes exactly
\[ L_{EM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J^\mu A_\mu \tag{53} \]
leading to Maxwell equations (with ignoring factor \( 4\pi/c \) in natural unit) via \( A \) and \( \phi \).
\[ \partial_\mu E^{\mu\nu} = J^\nu \quad \text{or} \quad \nabla E = J \tag{54} \]
Its spinor form is
\[ \nabla_{PQ} E^{KL'PQ'} = J^{KL'} \tag{55} \]
When the field Eq. (52) extends to Yang-Mills gauge fields with including vector fields and spinor fields
\[ F_{KL'PQ} = \partial_{KL'} A_{PQ} - \partial_{PQ} A_{KL'} - g e_{abc} A_{KL'} A_{PQ} \tag{56} \]
\[ G_{KL'PQ} = \partial_{KL'} B_{PQ} - \partial_{PQ} B_{KL'} - g_q f_{abc} B_{KL'} B_{PQ} \tag{57} \]
where \( g_e \) and \( g_q \) are electroweak and strong coupling constants respectively in vector and spinor fields, while we use \( g \) as main coupling constant in scalar field.
The total Lagrangian includes complex scalar field (equivalent to Higgs field), gauge vector field (equivalent to electroweak Yang-Mills field), spinor field (as extended Dirac field) covering all fermions (quarks and leptons) and bosons, as well as coupling field (as Yukawa items), which can be called as Higgs-Yang-Mills-Dirac-Yukawa Lagrangian [38] as follows
\[ L = L_\phi + L_\gamma + L_\Delta + L_{\text{c.t.}} = L_{HH} + L_{YM} + L_D + L_Y \tag{58} \]
\[ L_{HH} = (\partial_{KL'} \phi) (\partial_{KL'} \phi') - g (\phi' \phi) = L_{\phi} \tag{58a} \]
\[ L_{YM} = -\frac{1}{4} F_{KL'PQ}' F^{aKL'PQ'} - \frac{1}{4} g_{ab} F_{KL'PQ}' G^{bKL'PQ'} \tag{58b} \]
\[ = a_D = \text{Re} (\bar{\psi} D \psi) - m \bar{\psi} \psi = L_{\text{c.t.}}; \quad \bar{L}_s = L_{\text{c.t.}} + L_{\text{c.t.}} \tag{58c} \]
\[ L_Y = -g \bar{\psi}_L \psi_R - g (\bar{\psi}_L \psi_R) = L_{\text{c.t.}}. \tag{58d} \]
This is a gauge field framework that combines spinor fields, vector Bosons and scalar Higgs together, including both electroweak field stemmed by U(1)×SU(2) with three-generation index \( a \) and strong gluon field stemmed by SU(3) with including 8 duplicates (index b) in 3 generations with a color-index c (\( c = 1, 2, 3 \) or b, g, r).
The gauge invariance may keep via connection \( \omega \) and matrix \( U \)
\[ \phi \rightarrow \phi' = e^{i\omega} \phi, \quad \overline{\phi} \rightarrow \overline{\phi}' = e^{-i\omega} \tag{60a} \]
\[ A_{KL'} \rightarrow A'_{KL'} = A_{KL'} - \partial_{KL'} \omega, \quad \overline{A}_{KL'} \rightarrow A'_{KL'} = \overline{A}_{KL'} + \partial_{KL'} \omega \tag{60b} \]
\[ B_{KL} \rightarrow B'_{KL} = UB_{KL} \hat{U} - g^{-1}UB_{KL} \hat{U}. \] (60e)

Totally, the results match particle standard model, and will be consistent with CPT theorem.

5.2. Global Relations Based on P1 and P3: Gravity and Repulsion

Since Einstein general relativity (GR) concerns geometric metric tensor and energy-momentum tensor, its spinor form should extend to include scalar and vector in above framework. Now we consider global relations between mathematical M (φ, A, B) and physical P (E, p, S), so that we extend spinor metric with including scalar and vector in at first.

Similarly to TeVeS, we use physical metric \( \hat{g}_{\mu \nu} \) replacing geometric metric \( g_{\mu \nu} \). Differentiating from artificial TeVeS, SVS provides a natural form for embedding scalar \( \phi \) and vector A into spinors because scalar-vector-spinor have to link together

\[ \hat{g}_{KL'PQ'} = e^{2\phi} (g_{KL'PQ'} + A_{CD'}A_{CD}) - e^{2\phi} A_{CD'}A_{CD'}. \] (61)

The total Lagrangian L and action S will be

\[ L = L_\phi + L_\nu + L_s; S = \int \sqrt{-g} d^4x = S_\phi + S_\nu + S_s \] (62)

\[ L_\phi = -k_2 \frac{2}{\kappa_G} \sigma^2 \left( \frac{g_{KL'PQ'} - A_{KL'PQ'}}{2} + \frac{k_0 \sigma^2 f(\sigma)}{2} \right) \] (62a)

\[ L_\nu = -k_2 \frac{32\pi G}{\kappa_G} \left( g_{KL'PQ'} g_{CD'EF'} A_{KL,CD} A_{PQ,EF} \right) \] (62b)

\[ L_s = \frac{1}{16\pi G} \left( g_{KL'PQ'} R_{KL'PQ'} + 2\Lambda \right) \] (62c)

where \( k_0, k_1 \) and \( k_2 \) are three coupling constants, \( k_0 \) is a dimensionless constant, \( k_2 \) is \( k_0 \), \( k_1 \) is \( c_0 \) to the limit \( k_0 \rightarrow 0 \) corresponds to general relativity (i.e. \( k_2 \rightarrow 0 \) and \( k_1 \rightarrow \infty \), which means to ignore relations of \( L_\phi \) and \( L_\nu \) to \( L_s \) as well as their interactions, so that only spinor Lagrangian or equivalent tensor Lagrangian is kept, yielding GR). While scalar \( \phi \) is dimensionless, the dimensions of \( \sigma^2 \) are those of \( G^1 \). The \( \phi, \mu \) and \( \sigma^2 \) are linked by modified Poisson equation

\[ \nabla \mu(\nabla \phi / |a_0|) = 4\pi G \rho_0 \kappa_0 G \sigma^2 = \mu. \] (63)

The interpolation function \( \mu(\chi) \) is a linear function interpolating between the Newtonian regime \( \mu(\chi)=1 \) and the MOND regime \( \mu(\chi)=\chi \), so it may get the form \( \mu(\chi) = \chi/|1+\chi| \).

The total energy-momentum spinor (or tensor) also include three parts as

\[ \tilde{T}_{KL'PQ'} = T_{\phi KL'PQ'} + T_{\nu KL'PQ'} + T_{s KL'PQ'}. \] (64)

According to action principle (P1) and equivalence principle (P3), we have

\[ \sqrt{-g} \frac{\partial S}{\partial g^{KL'PQ'}} = \frac{2}{\sqrt{-g}} \delta S_{\phi} + \frac{2}{\sqrt{-g}} \delta S_{\nu} + \frac{2}{\sqrt{-g}} \delta S_{s} \] (65)

\[ = -T_{\nu KL'PQ'} - T_{s KL'PQ'} - T_{s KL'PQ'} = -\tilde{T}_{KL'PQ'}. \]

The matter action \( S_m \) links with matter current \( j^{KL'PQ'} \)

\[ \frac{2}{\sqrt{-g}} \delta S_m = -j^{KL'PQ'}. \] (66)

Via the variation of \( g_{KL'PQ'} \), we can obtain following form of field equations [24]

\[ \tilde{G}_{KL'PQ'} = G_{KL'PQ'} - g_{KL'PQ'} \Lambda + H_{KL'PQ'}. \] (67)

where \( \kappa = 8\pi G / c^4 = 8\pi / M^2_\text{Pl} \) is linking parameter, \( g_{KL'PQ'} \) is equivalent to Einstein tensor and contributed by spinor (as matter), \( \Lambda \) term is contributed by scalar (as the effect of dark energy), and \( H_{KL'PQ'} \) is contributed by vector (as the effect of dark matter), which is also an interactive couplings as

\[ H_{KL'PQ'} = k_2 \left( g_{KL'PQ'} A_{KL,AB} \Lambda A_{PQ,CD} \right) - k_1 A_{AB} A_{CD'}. \] (68)

The energy-momentum spinor for an ideal fluid will maintain

\[ \tilde{T}_{KL'PQ'} = \tilde{p}n_{KL'PQ'} - \tilde{\rho}g_{KL'PQ'} + \tilde{p}n_{KL,'PQ'} \] (69)

where \( \tilde{\rho} \) stands for the proper energy density, \( p \) for the isotropic pressure, \( y \) for the 4-velocity, \( \tilde{\rho} g_{KL'PQ'} = \tilde{n}u_{KL'PQ'} = 1 \) and \( \tilde{u}_{KL'} = \tilde{e}^\mu A_{KL} \), following Bekenstein [22]. As scaling variable \( a \) linking radium \( r = GM/a \) in natural unit system \( c = h = 1 \), we have Friedmann equation in time distribution as

\[ \tilde{G}_{00} = 3(\ddot{a} + k / \dot{a}^2) \] (70)

where \( \ddot{a} = a / (\tilde{\rho} + \tilde{\rho}_0) \) is scale factor, \( a_0 = a(t_0) \) denotes the initial value and \( a = a(t) \) means present one,

\[ y(\mu) = -\mu f(\mu) - \mu_f(\mu) / 2, \text{ andk indicates curvature index with } k = +1, 0 \text{ or -1 corresponding to closed, flat or open spatially geometries. As the space distribution } \tilde{G}_{11} = \tilde{G}_{22} = \tilde{G}_{33} = -2(\ddot{a} / \dot{a}) - (\ddot{a} + k) / \dot{a}^2, \text{ we have} \]

\[ \frac{\ddot{a}}{\dot{a}} = -4\pi G / 3 \tilde{\rho} + 3 \tilde{\rho}_0 + \Lambda \tilde{e}^\phi \] (71)

As the components of density and pressure associated with the scalar, vector and spinor fields, we have
\[ \rho = \rho_\phi + \rho_\chi + \rho_s \]  
\[ \tilde{\rho} = \rho_\phi + \rho_\chi + \rho_s \]  
(72)  
(73)

Recall the definition of dimensionless density parameters \( \Omega \), we have
\[ \Omega_\phi = \frac{8\pi G \rho_\phi}{3H^2}, \quad \Omega_\chi = \frac{8\pi G \rho_\chi}{3H^2}, \quad \Omega_s = \frac{8\pi G \rho_s}{3H^2}, \]  
(74)

where subscript \( \phi \) (\( = \delta e = \Lambda \)) indicates the source from scalar (marks dark energy), \( \chi \) (\( = \delta m \)) from vector (marks dark matter), \( s \) from spinor (marks matter). Since \( s = b + r \), \( b \) as baryons and \( r \) as radiation (photons plus relativistic neutrinos), we understand the basic construction of the universe, for fitting observational data with \( \Omega_{\text{m}} \approx 0.268, \Omega_\text{b} \approx 0.049, \Omega_\phi \approx 0.01 \).

Recall the critical density (when \( K = \frac{\rho_0}{\rho_\text{crit}} = 0 \) and \( \Lambda = 0 \)) as
\[ \rho_\text{c} = \frac{3H_0^2}{8\pi G} = 1.878 \times 10^{-29} \text{g} \cdot \text{cm}^{-3} \sim (3 \times 10^{-33} \text{eV})^4. \]
For dynamic expanding cosmos, it has to include total density with deducing dynamic item \( 3k/8\pi G a^2 \)
\[ \rho_\Lambda > \rho_\text{m} + \rho_\delta m + \rho_\chi - \rho_\text{c} = \rho - 3k/8\pi G a^2. \]  
(75)

Based on the Friedmann equations, we can discuss following relations
\[ \begin{align*}
\begin{cases}
\tilde{a} = \tilde{a} = 0 & \text{static universe.} \\
\tilde{a} \neq \tilde{a} & \text{dynamic universe.} \\
\tilde{a} = e^{-\phi} a; \tilde{a} = \tilde{a} / d t = e^{-\phi} (a - a \phi); d \tilde{t} = e^\phi d t \\
\tilde{p} = -\tilde{p} & \text{inflationary era.} \\
\tilde{p} \sim \tilde{p} & \text{3\tilde{p} = \tilde{p} \to radiation era.} \\
\tilde{p} \to 0, \tilde{p} \sim \tilde{a}^{-3} & \text{matter era.} \\
k = 1 & \text{closed universe.} \\
k \sim \tilde{a} & k = 0 \to \text{flat universe.} \\
k = -1 & \text{open universe.}
\end{cases}
\end{align*} \]
(76)

In all discussions, it is convenient to consider Robertson–Walker (RW) metric
\[ \begin{align*}
\ddot{g}_{KL} p Q dK L dP Q' &= d\tilde{x}^2 = d\tilde{r}^2 - \hat{a}(\tilde{r})^2 [d\theta^2 + \sin^2 \theta d\phi^2] \\
&= d\tilde{x}^2 = d\tilde{r}^2 - \hat{a}(\tilde{r})^2 [d\theta^2 + \sin^2 \theta d\phi^2]
\end{align*} \]
(77)

where \( (t, r, \theta, \phi) \) is comoving coordinate system. The \( k_0 \) and \( k_1 \) link with initial scale factor as
\[ a_0 = \frac{\sqrt{3k_0}}{4\pi k_1}. \]  
(78)

Another issue concerns the cosmological phase transitions. Before thermodynamic process, there were two transitions, the first transition from a symmetric high temperature phase with massless gauge bosons to the Higgs phase at temperature about 100 GeV, and the second transition from a quark-gluon plasma to a confinement phase with no free quarks and gluons at temperature about 200 MeV (according to QCD). Cosmological observations suggested so-called ‘cosmological principle’ (the Universe is homogeneous and isotropic), which is a fact that we need to follow. Only homogeneous and isotropic thermodynamic phase transition could produce the results. Therefore, the phase transitions should include cosmological phase transitions [26] and thermodynamic phase transition.

With considering Clausius–Clapeyron equation for characterizing a thermodynamic phase transition between two phases of matter of a single constituent, we havewhen we introduce entropy \( S \) or information
\[ \frac{dp}{dT} = \frac{Q}{T} \frac{\Delta S}{\Delta V}; S = -\frac{dF}{dT} = -I \]
(79)

where \( dp = -\tilde{p} + \rho \) and \( Q \) is the specific latent heat, \( T \) is the temperature, \( \Delta V \) is the specific volume change of the phase transition, and \( \Delta S \) is the specific entropy change in the phase transition. Since \( p \) is positive pressure from gravity and \( \tilde{p} \) is negative pressure as repulsion as ‘dark energy’, when \( \tilde{p} > p \), the cosmos expands.

Therefore, the outer relations are mastered by Einstein-Friedmann equations totally with Clausius-Clapeyron equations in thermodynamic phase transition. Under the above quantum SVS framework, Einstein-Friedmann and Clausius-Clapeyron equations may naturally describe the Universe. After cosmos expanded, when matter density became rare, thermodynamic phase transition happened, and the Clausius-Clapeyron equation describes the process with emitting heat homogeneously, for pushing the universal expansion till reaching the equilibrium of next phase transitions.

When we define \( \Phi \) as grand system ensemble based on system ensemble \( i(\phi) \) with probability density \( \rho(\phi) \), the free energy \( F \) in the system can be calculated as
\[ F = -k_B T \ln \Omega = -k_B T \int \frac{h}{\pi e^{\phi/k_B T} \pm 1} d\phi; \]
\[ dF = dE - SdT - T dS. \]
As information is equivalent to negative entropy, it is possible to penetrate classical and quantum information theory [39].

Einstein-Friedmann equations and Clausius-Clapeyron equations are remained in the vortex-field theory meantime, since they fit the phenomenological evidences very well. The improving model of modified gravity with multistage phase transitions resembles suitable for describing the universe, where modified gravity characterized and mastered dark matter and multistage phase transitions as energy push the universal accelerating expansion.

6. Possible Verifications

The particle standard model is also a harmonic successful model that predicts the Higgs boson, which had
been experimentally discovered in 2012 and verified in 2013 by LHC at energy 125.6 GeV/c² [3,4]. It is well known that the Higgs mechanism describes how the weak SU(2) gauge symmetry is broken and how fundamental particles obtain mass, which was also the last particle predicted by the micro-particle standard model to be observed, although efforts to confirm that it has all of the properties predicted by the standard model are ongoing. Other great successes of the micro-particle standard model included the prediction of the W and Φ bosons, the gluon, and the top and the charm quarks, before they had been observed. However, the worst disadvantage in the micro-particle standard model is the complete absence of gravity, and it predicts neutrinos to be massless while the observed evidence of neutrino oscillations implies that neutrinos have tiny mass [5].

Various coupling constants will provide possible verifications in SVS framework. The constant c (speed of light) originates from the coupling or combination of scalar φ and vector A as Eq. (51), and Planck constant originates from the exchange production of scalar φ and vector A as Eq. (39). The gravitational constant G implies in linking parameter in Eq. (7) as κ = 1/8πG. Since TeVeS passed the usual solar system test and predicted gravitational lensing in agreement with the observations without dark matter [22], SVS would do so similarly.

Based on Cosmic Microwave Background (CMB) [40] and Big Bang Nucleosynthesis (BBN) [41,42], if we agree the cosmological principle (the universe is homogeneous and isotropic), the spatial distribution of matter in the universe never produces observable irregularities in the large-scale structure. Although the universe is inhomogeneous at smaller scales, it is statistically homogeneous on the large-scale scope. The CMB is isotropic, which was verified by Cobe 1992, WMAP 2005 and Planck 2013, though there was inhomogeneous at smaller scales. Certainly there are strong observational and phenomenological evidences for supporting that we need new hypothesis beyond standard models, in which dark matter best-fitsΛCDM (Lambda Cold Dark Matter) model and dark energy [43] keeps mystery. Now both dark matter and dark energy are included in model effects in vortex-field theory.

6.1. Micro-coupling Constants \( g_\alpha \)

In the SVS model, it is feasible to provide an origin of natural coupling constants.

There are four micro-couplings, \( g, g_e, g_w \) and \( g_q \) respectively, as main coupling (with Higgs), electrical coupling, electroweak coupling and strong coupling respectively. The coupling \( g \) as a variable links with any particle mass \( m_\phi \) via Higgs, which determines the mass caused by Higgs generally

\[
g = \lambda \frac{8\pi^2}{m^2_H} \tag{81}
\]

where key parameter \( \lambda \) relates to \( v \sim 246 \text{GeV} \). Since \( m_\phi \sim 125 \text{GeV} \) [44,45], we have

\[
\lambda = \frac{m^2_H}{2v^2} = \frac{125^2}{2 \times 246^2} \sim 0.1291 \tag{82}
\]

According to standard model, there is a relation among in electroweak field where \( g=g' \) as [46]

\[
e^2 = \frac{g^2}{\sqrt{g^2 + g_w^2}} = g_w \sin \theta_W \rightarrow \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g_w^2} \tag{83}
\]

in which weak mixing angle (Weinberg angle)

\[
\cos \theta_W = m_w / m_Z \text{ or } \tan \theta_W = g'/g_w \ . \text{ When we have} \ m_w = 80.379 \text{GeV and} \ m_Z = 91.187 \text{GeV, we know that} \ 
\theta_W \sim 28.75^\circ \text{ and } g_w \sim 0.653 \text{, so that we obtain}
\]

\[
e = g_w \sin \theta_W = 0.653 \times 0.4809 \sim 0.3141 \tag{84}
\]

\[
g_e = g_w \cos \theta_W = 0.653 \times 0.8767 \sim 0.5725 \tag{85}
\]

\[
g' = g_w \tan \theta_W = 0.653 \times 0.5486 \sim 0.3582 \tag{86}
\]

The Higgs coupling \( g_q \) can be also computed by Eq. (81), where the mass of a quark such as charm quark \( m_c=1.275 \text{GeV} \), so that we can estimate

\[
g_{qc} = \lambda \frac{8\pi^2 m^2_c}{m^2_H} = 0.1291 \times \frac{8 \times 1.275^2}{125^2} \sim 0.0001. \tag{87}
\]

This mean the most mass of hadrons came from inner quark confinement (\( m^2_c/E^2 \)), where Higgs coupling contributed little.

In the coupling view, since basic particles exist in three generations, it is possible to exist three Higgs for corresponding to the three generations. Except \( m_h \sim 125 \text{GeV} \), there may be heavier Higgs and lighter Higgs. According to 0.0001 contribution to heavy (t) quark and light (u) quark, we estimate heavier Higgs \( m_{h_{uu}} = (0.1291 \times 8 \times 173^2/0.0001)^{1/2} \approx 17.58 \text{TeV} \) and lighter Higgs \( m_{h_{uu}} = (0.1291 \times 8 \times 23^2/0.0001)^{1/2} \approx 233.7 \text{MeV} \). The Fermi coupling constant \( g_F \) and fine structure constant \( \alpha \) will keep as

\[
F = \frac{1}{4\sqrt{2}} \frac{g_w^2}{m^2_W} \sim 1.166 \times 10^{-5}\text{GeV}^{-2}; \tag{88}
\]

\[
\alpha = \frac{e^2}{4\pi} \sim 7.297 \times 10^{-3} \approx 1/137 \tag{89}
\]

The issues for natural and reasonable understanding on various micro-couplings constants resemble clear. We know what relations are among various \( g_i \), which links Wolfenstein parameters \( \rho \) and \( \eta \) by

\[
\rho = \rho (1 - \lambda^2 / 2) \approx 0.12; \eta = \eta (1 - \lambda^2 / 2) \approx 0.36. \tag{89}
\]

The CKM global fit and Higgs interaction with particles show in Figure 3.

The \( \lambda \) also determines CKM matrix and links Wolfenstein parameters \( \rho \) and \( \eta \) as

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
1 - \lambda^2 / 2 & \lambda & A\lambda^3 (\rho - i\eta) \\
-\lambda & 1 - \lambda^2 / 2 & A\lambda^2 \\
A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} \tag{90}
\]
in which $A \sim 0.811$. If Higgs mechanism is universal rule for contributing mass of all particles, i.e. neutrino Higgs may exist, the mass of neutrinos will become similarly understandable.

While time reversal and CP violation may happen, the CPT invariance remains.

6.2. Macro-density Parameters $\Omega_i$

Recall $c = \hbar = 1$, we have Friedmann equation Eq. (70). According to Eq. (74), we have

$$\Omega_m = \frac{8\pi G \rho_m}{3H^2}, \Omega_{dm} = \frac{8\pi G \rho_{dm}}{3H^2},$$

$$\Omega_\Lambda = \frac{8\pi G \rho_\Lambda}{3H^2}, \Omega_m + \Omega_{dm} + \Omega_\Lambda = 1$$

where $\Omega_\Lambda = \Omega_{\phi}$ stands for the smooth components of the scalar field, $\Omega_{dm}$ denotes density parameter of oscillating ‘dark matter’, and $\Omega_m = \Omega_b + \Omega_r = \Omega_{osc}$ indicates matter (baryons and radiation) density. The results will match with cosmological parameters [47].

According to Bekenstein and Sanders [20, 21, 22, 23], the scalar $\phi$ determines the cosmological evolution. According to the analysis from Sanders [21], we will have similar results as shown as Figure 4.

In Figure 4, the left side came from observation and the right side is simulation of TeVeS by Sanders. This is meaningful and valuable reference.

Supposing that the cosmic mass-energy is consists of matter ($\Omega_m$), dark matter ($\Omega_{dm}$) and dark energy ($\Omega_\Lambda$), when we apply $\rho_\Lambda = \Lambda/(8\pi G)$ to indicate vacuum energy density i.e. dark energy, we see via Eq. (75) with Planck mass $M_{Pl} = G^{-1/2}$

$$\rho_\Lambda - \rho_c = \rho - 3kM_{Pl}^2/8\pi a^2$$

When $\rho - 3kM_{Pl}^2/8\pi a^2 \to \epsilon > 0$, the $\rho_\Lambda - \rho_c \sim 7 \times 10^{-30}$ g/cm$^3 \to \epsilon > 0$, which maintains the universal evolution, and there is no cosmological constant problem.
Combining cosmological state equation $p = p(\rho) = w\rho$ with the three unknown functions of the time, $p$, $\rho$ and $R$, the cosmic matter (energy) density $\rho$ and isotropic pressure $p$ is linked by relativistic equation of state

$$p = p(\rho) = (\alpha - 1)c^2 \rho = w\rho; \quad \rho = 3H(\rho + p) = -3(1 + w)\rho \frac{\dot{\rho}}{\dot{a}} \sqrt{3 - 4\alpha} \tag{93}$$

in which the cosmic state is determined by $w$, and $w$ may link to redshift $z$ as \[48\]

$$w(z) = \frac{\sqrt{3}(1 + z)d\ln H}{d\ln z - 1} \tag{94}$$

Therefore, we may consider cosmological redshift as another important measurement for verification.

### 6.3. Cosmological Redshift

Since an accelerating expansive universe implied the cosmological redshift, exploring the redshift became a meaningful work in current research. For light signals, as $ds^2 = 0$, redshift factor $z$ has been defined as

$$1 + z = \frac{a_0}{a} \tag{95}$$

where the $1 + z$ is also called as cosmological redshift factor, and it relates with Hubble parameter $H$ as

$$H = \frac{d\ln a}{dt} = \frac{d\ln a_0}{1 + z} = -\frac{1}{1 + z} \frac{dz}{dt} \tag{96}$$

Let $\Omega_m - \Omega_\Lambda$, $\Omega_{dm} = \Omega_\Lambda$ and $\Omega_m = \Omega_\Lambda$ stand for the densities of matter, dark matter and dark energy respectively. We will have

$$H^2 = H_0^2[\Omega_m + \Omega_\Lambda + \Omega_{dm}] = H_0^2[\Omega_m^0(1 + z)^3 + \Omega_\Lambda^0 + \Omega_{dm}^0(1 + z)^2] \tag{97}$$

where the indices 0 indicate the today values.

Combining Eq. (98) and Eq. (99), we obtain

$$\frac{dz}{dt} = -(1 + z)H_0 \left[\Omega_m(t)(1 + z)^3 + \Omega_{dm}(t)(1 + z)^2 + \Omega_\Lambda(t)\right]^{1/2} \tag{98}$$

$$\frac{dt}{dz} = -\frac{1}{1 + z}H_0 \left[\Omega_m^0(1 + z)^3 + \Omega_\Lambda^0 + \Omega_{dm}^0(1 + z)^2\right]^{-1/2}. \tag{99}$$

Therefore, we see that the redshift will change with time. By substitution $\Omega_{dm}^0 = 1 - \Omega_m^0 - \Omega_\Lambda^0$, we find the redshift difference $\Delta z$ and time difference $\Delta t$ between today ($z=0$) and the time at redshift $z$ will be respectively

$$\Delta z = \int_0^1 (1 + z) \left[\left(1 + \Omega_m^0(t)\right)z(1 + z)^2\right]^{1/2} dt \tag{100}$$

$$\Delta t = \int_0^1 \frac{dz}{1 + z} \left[\left(1 + \Omega_m^0z\right)(1 + z)^2 - z(2 + z)\Omega_\Lambda^0\right]^{-1/2} \tag{101}$$

Therefore, it is feasible to measure the cosmological redshift for understand the universal evolution.

Since CMB was discovered in 1965, big bang cosmology was generally accepted, where the universe started in a hot, dense state and had been expanding over time. The CMB had contributed an observation of the redshirt measured by radiation, and a large structure of galaxies at redshift $z \sim 3$ had also been observed and discussed \[46\]. The measurement was realized by the COBE satellite, while more accurate measurement was achieved by the WMAP experiment. The final estimated CMB temperature is $T_0 = 2.728$K. As $T$ underwent evolutionary process, the cosmological redshift was set up by 21cm cosmology \[49\]. Here we predict that there will be redshift deceleration effect in the cosmic expanding direction, as shown in Figure 5.

![Figure 5. The cosmological redshift measurement (synthesizing figures in \[49\])](image)
Around CMB, dark matter and dark energy could be discussed in unified framework. During the 1980s, most researches focused on cold dark matter with critical density in matter, around 95% CDM (Cold Dark Matter) and 5% baryons, as these showed success at forming galaxies and clusters of galaxies, but problems remained. Notably, the model required a Hubble constant lower than preferred by observations, and the model under-predicted observed large-scale galaxy clustering. These difficulties sharpened with the discovery of CMB anisotropy by COBE in 1992, and several alternatives including ΛCDM (Lambda Cold Dark Matter) and mixed cold + hot dark matter came under active consideration. The ΛCDM model could be extended by adding cosmological inflation, quintessence or other elements that are current areas of speculation and research in cosmology. If the Hubble constant is too high, the CMB alone requires an almost flat universe with $\Omega_m + \Omega_{\Lambda} + \Omega_k \approx 1$.

The organization of structure appears to follow as a hierarchical model with organization up to the scale of superclusters and filaments. Larger than this, there seems to be no continued structure, a phenomenon that has been referred to as the End of Greatness [50]. The End of Greatness is an observational scale discovered at roughly 100 Mpc (roughly 300 million lightyears) where the lumpiness seen in the large-scale structure of the universe is homogenized and isotropized in accordance with the cosmological principle. It was not until the redshift surveys of the 1990s were completed that this scale could accurately be observed. The multistage phase transitions could explain the phenomena of the end of greatness, which seem to approach the frozen boundaries of the cosmos. An opposite effect works on the galaxies already within a cluster: the galaxies have some random motion around the cluster center, and when these random motions are converted to redshifts, the cluster appears elongated. The vortex-field theory contributes a unified framework based on absolute space and time, with using mathematical computational convenience. As we mentioned, we have only one second timespan leaving for computational details need to study further, and hopefully we will have more supporting evidences via vortex optics and related studies. Also, we keep order (r) as an extendable component and remain various coupling constants for deep discussions and explorations in future.

7. Discussion

Methodologically, there is a basic equivalence between the scalar-vector-spinor (SVS) framework and the scalar-vector-tensor (SVT) one, where there is a difference with the purely formal one of replacing each tensor index by a pair of spinor indices. Essentially, the TeVeS is the tensor version of SVS in cosmology, while the SVS covers both micro- and macro-physics. However, spinors provide better theoretical perfection, while tensors contribute computational convenience. As we mentioned, the spinors are more fundamental than tensors in the description of space-time structure and spins.

The vortex-field theory contributes a unified framework to understand both micro- and macro-physics with using same mathematical language. It is a quantized system, and it also includes Newton physics and Einstein physics.

In Newton physics, the mathematical language focused on scalars and vectors and the physical concepts set around force $F$, so the balance of mathematics and physics resembles as

$$M(\phi(s,t), A(s,t)) = \kappa P(F(s,t), p(s,t))$$

In Eq.(3a), Newton focused on force and momentum based on absolute space and time, with using mathematical scalars and vectors. In the Newton system, the concept “force” is important. However, “force” is not necessary in SVS model.

In Einstein physics, the mathematical language focused on scalars and tensors, and the physical concepts set around energy-momentum tensor $T$, so the balance of mathematics and physics resembles as

$$M(\phi(s,t), A(s,t), G(s,t)) = \kappa P(E(s,t), p(s,t), T(s,t))$$

In Eq.(3b), Einstein focused on the relation of space-time curvature tensor $G$ and energy-momentum tensor $T$, leading to general relativity, where the scalar function $\phi$, vector $A$ and Einstein tensor $G$ are independent without relations.

Now the SVS model as a unified framework is concise, which provides mathematical methodology for describing both micro-particles and macro-galaxies. Action principle (P1) fits the universe, while duality principle (P2) acts mainly on micro-world and equivalence principle (P3) works mainly for global system. Certainly the computational details need to study further, and hopefully we will have more supporting evidences via vortex optics and related studies. Also, we keep order (r) as an extendable component and remain various coupling constants for deep discussions and explorations in future.

The quantum SVS model matches standard model, since standard model as the simplest successful model that provides a reasonably good account of the following observational evidences of the universe, including 1). the existence of the cosmic microwave background and the large-scale structural distribution of galaxies; 2). the abundances of hydrogen (including deuterium), helium, and lithium, as famous BBN hypothesis of elements synthesis; and 3). the accelerating expansion of the universe observed in the light from distant galaxies and supernovas [51].

Totally, the cosmological constant problem is naturally solved and three kinds of Higgs are expected. A special case may be interesting in further consideration, i.e. if two scalar functions $\phi$ and wave function $\psi$ combine into one as $\phi + i\psi$, does the SVS model keep same function and similar results?

Meanwhile, we have strong evidences to deny supersymmetric particles and dark sectors, because we have only one second timespan leaving for supersymmetric and dark particles in the known universe. This means that there are little space (inflation size) and time (<1s) for the existence of supersymmetric particles and dark sectors, according to present physical knowledge. However, the phenomena of both galaxies’ rotation curves and gravitational lenses indicate the stability of so called ‘dark sectors’. Therefore, new theory beyond standard models should focus on the modified theories based on general relativity and quantum field theory, rather than supersymmetric partners and dark sectors.

Another issue concerns that there is an observational paradox in the accelerating cosmological model as there is no obvious gradually rare distribution of stars in the sky. If the universe is accelerating expansion, there will be a gradually rare distribution of stars in astronomical observations as the accelerating expansion will be quicker than the generating galaxies. However, it never show so. The stars in the sky resembles stable since human
observed the universe (at least, our Milk Galaxy is stable), which means that any galaxy is always integrated system without dispersion. Perhaps X-ray and supernova astronomy might provide more evidences [52,53].

Then limitations also exit. We still keep some unknown things, such as the large-scale structure of the Universe, which may observe if one only uses redshift to measure distances to galaxies. For example, galaxies behind a galaxy cluster are attracted to it, and so fall towards it, and so are slightly blueshifted. On the near world, things are slightly redshifted. Thus, the environment of the cluster resembles a bit squashed if using redshifts to measure distance.

The quantum SVS model keeps both simplicity and elegance, for approaching physical unification or quantum cosmology without dark sectors, while three physical principles maintain its backbone.

8. Conclusion

Along the way of particle standard model and cosmological standard model, we find quantum SVS model as vortex-field theory contributes an improved understanding for approaching unified physics, characterized by

1. Mathematically and physically, one cliff means one vortex. When scalar, vector and spinor indicate respective features, the unified cliff denotes its total state. The quantum SVS model might be an effective model for describing the Universe without dark sectors, where inner local interactions describe electroweak and strong quantum fields while outer global relations clarify gravity and repulsion naturally. Three principles would contribute the principled construction of physics around the standard models, where action principle, duality principle and equivalence principle interact each other to construct the foundations of physics.

2. Locally, particle standard model is naturally deduced as gauge fields. It is estimated that there are heavier Higgs around 17.58TeV and lighter Higgs around 233.7MeV.

3. Globally, mathematical cliffs equal to physical measurements with spinor form as extended global equations including both gravity and repulsion as

\[
\begin{align*}
M_{KL'PQ'} &= \hat{G}_{KL'PQ'} \\
 &= G_{KL'PQ'} - g_{KL'PQ'} \Lambda + H_{KL'PQ'} \\
 &= \kappa \hat{T} \cdot KL'PQ' = \kappa P_{AB'CD}'.
\end{align*}
\]

Locally, electromagnetic, electroweak and strong fields will be linked together when space-time is embedded into mathematical cliffs and physical measures, while the Einstein-Friedmann equations apply to describe the universe globally.

Conclusively, the quantum SVS model as vortex-field theory keep all features of two standard models, without dark sectors, for interpreting and understanding the universe from micro-particles to macro-galaxies. The SVS model contributes a unified framework within uniform mathematical language, which provides a harmonic structure and essence in both mathematics and physics. The vortex-field theory could combine two standard models well, leading to a mathematically and physically concise unification, which might stimulate further studies.

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