Scattering Events and Heat Conductivity of Layered La$_{2-x}$Sr$_x$CuO$_4$ Superconductors

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Abstract The problem of heat conduction in layered La$_{2-x}$Sr$_x$CuO$_4$ superconductor has been investigated in a new frame work of in-plane and cross-plane concepts with the help of modified Callaway model of thermal conductivity based on relaxation time approximation. Using the many body quantum dynamical theory of the expressions for thermal conductivity in context of in-plane and cross plane have been obtained and results are found in excellent agreement with experimental observations for layered La$_{2-x}$Sr$_x$CuO$_4$ cuprate superconductors. The theory explores the possibility of device fabrication cold in one direction and hot in the other.

Keywords: relaxation times, in-plane and Cross- plane thermal conductivity, scattering process


1. Introduction

A detailed understanding of thermal transport and management in layered systems is increasingly emerging as thrust area in the modern efficient energy technology and theorists are interestingly involved to explain this phenomenon. The study of heat transport in conventional and high temperature superconductors (HTS) emerged as an important tool to understand the scenario of phonon and electron energy spectrum along with various collision events. The thermal conductivity $\kappa$ was understood in such systems contributed both by electrons and phonons in the form of electronic thermal conductivity $\kappa_e$ [1] and phonon conductivity $\kappa_{ph}$ [2] related by $\kappa = \kappa_e + \kappa_{ph}$. At fairly low temperatures the Widemann-Franz law often breaks down severely and the metallic behavior of solids which become superconducting suggests that the electronic thermal conductivity starts to disappear and one can take $\kappa_e \rightarrow 0$ negligibly small with $\kappa = \kappa_{ph}$ instead of the concept of isolated channels [3,4,5]. The layered high temperature superconductor structures are highly anisotropic in character and thus the in-plane and cross-plane thermal transport becomes more and more important. Some investigations reveal [6,7] that scattering of electrons from phonons, impurities and interfacial roughness can be used to determine in-plane electron transport and resonant tunneling effects and the in plane scattering can be used to determine cross-plane transport [8,9,10] and this is further supported by the different phonon velocities in different directions [9]. Lattice vibrations can couple to each other and strongly couple with any structural defect, surface boundaries, dislocations or point defects [2,4,11,12,13,14,15].

The thermal conductivity of layered structures based on Boltzmann transport equation approach has studied by many theorists [16,17,18] using the method of relaxation time approximation (RTA). The validity of RTA, however, suffers from many inadequacies because of its derivation for low frequency phonons at low temperatures, additivity of lifetimes of individual scattering events and its incompatibility to explain inelastic phonon scattering processes [18-24]. Adopting the quantum mechanical approach these inadequacies have been successfully removed from the phenomenological theories of thermal conductivity [18,20,21]. The discrepancies involved due to the phonon dispersion relation and violation of Matthiessen’s rule of additivity of inverse relaxation time have been removed by introducing the equivalence between relaxation times and line widths [20,21]. The electron-phonon and the resonance scattering events are found to be highly sensitive in microstructures also, which successfully explain the abnormal behavior (dip and rise) of thermal conductivity curve at and above the critical temperature $T_c$. In this region an utmost care has to be taken as the thermal transport takes place in the presence of pairons in HTS which do not contribute to thermal conductivity.

In the present work we have investigated the role of various scattering processes based on many body quantum dynamics and the thermal conductivity is resolved into in-plane and cross-plane contributions which addresses the possibility of restricting the heat flow in a particular direction and allowing it in the another which can be exploited to the development of exotic technological HTS crystals for industrial use. In the present case in-plane
thermal conductivity is observed greater than cross-plane thermal conductivity.

2. Formulation of the Problem

The anisotropic considerations suggest that the thermal conductivity of a layered crystal can be resolved as a contribution of in-plane thermal conductivity $\kappa_\parallel$ and cross-plane thermal conductivity $\kappa_\perp$ as

$$\kappa = \kappa_\parallel + \kappa_\perp$$  \hspace{1cm} (1)

where $\kappa_\parallel$ and $\kappa_\perp$ can be obtained from Callaway expression [2] using relaxation time approximation in the following form

$$\kappa_\parallel = \left( \frac{k_B}{2\pi^2 \nu} \right) \beta^2 h^2 \iint_0^\infty \tau_{pp} \omega^2 e^{\beta \hbar \omega} \omega^2 \alpha^2 d\alpha d\omega$$

$$\kappa_\perp = \left( \frac{k_B}{2\pi^2 \nu} \right) \beta^2 h^2 \iint_0^\infty \tau_{pp} \omega^2 e^{\beta \hbar \omega} \omega^2 \alpha^2 d\alpha d\omega$$  \hspace{1cm} (1a)

where $\omega_D$ is the Deby Frequency, $\beta = (k_B T)^{-1}$ and in order to get rid of the inadequacies involved due to Matthiessen’s rule the relaxation times $\tau_{pp}^{-1}(\omega)$ and $\tau_{kk}^{-1}(\omega)$ are along in-plane and cross-plane directions which can be replaced by phonon line widths $\Gamma_{kk}(\omega)$ and $\Gamma_{kk}(\omega)$ [18,20,21]. The evolution of $\Gamma_{kk}(\omega)$ and $\Gamma_{kk}(\omega)$ can be obtained with the help of quantum dynamics of phonons [20,21,25,26,27,28].

3. The Collision Processes

In order to investigate the many body quantum dynamics to explore the various scattering mechanisms, let us consider the Hamiltonian for a HTS in the form

$$H = H_p + H_A + H_e + H_{ep} + H_D$$  \hspace{1cm} (2)

where,

$$H_p = \sum_k \frac{\hbar \omega_k}{4} \left[ A_k A_k^* + B_k B_k^* \right]$$  \hspace{1cm} (2a)

$$H_A = \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} \hbar \nu_i \left( A_{k_1} A_{k_2} + A_{k_2} A_{k_1} + \cdots \right)$$  \hspace{1cm} (2b)

$$H_e = \sum_q \left( \hbar \omega_q \hat{b}_q^+ \hat{b}_q + \hbar \omega_q \hat{b}_q^* \hat{b}_q + \hbar \omega_q \hat{b}_q^* \hat{b}_q^+ + \hbar \omega_q \hat{b}_q \hat{b}_q^* \hat{b}_q \right)$$  \hspace{1cm} (2c)

$$H_{ep} = \sum_{k,q} \left( g_{kk} \hat{b}_{q}^+ \hat{b}_{q} + g_{kk} \hat{b}_{q} \hat{b}_{q}^* \hat{b}_{q} \right) B_k$$  \hspace{1cm} (2d)

$$H_D = -\hbar \sum_{k_1,k_2} \left[ C(k_1,k_2) B_{k_1} B_{k_2} \right] + \hbar \sum_{k_1,k_2} \left[ D(k_1,k_2) A_{k_1} A_{k_2} \right]$$  \hspace{1cm} (2e)

In the above equations the symbols $H_p$, $H_A$, $H_e$, $H_{ep}$ and $H_D$ respectively stand for harmonic phonon-phonon coupling- [25], anharmonic phonon- [26,27,28], electron-[29,30], electron-phonon coupling- [29,30,31] and Defect-Hamiltonian [26,28,29,30,31]. This Hamiltonian can be used to evaluate the double time temperature dependent phonon Green’s function

$$G_{kk}(t-t') = \langle \langle A_k(t); A_k^*(t') \rangle \rangle$$

$$= -i \theta(t-t')(A_k(t), A_k^*(t')).$$  \hspace{1cm} (3)

in the form

$$G_{kk}(\omega) = \frac{\omega \tilde{\nu}_{kk}}{\pi(\omega^2 - \omega_k^2 - 2\omega_k \tilde{P}(k,k',\omega))]$$  \hspace{1cm} (4)

here $\tilde{\nu}_k$ is the renormalized phonon frequency and $\tilde{P}(k,k',\omega)$ is the self-energy operator or response function

$$\tilde{P}(k,k',\omega) = \lim_{\epsilon \to 0^+} \Delta_k(\omega) - i\Gamma_k(\omega)$$  \hspace{1cm} (5)

Where $\Delta_k(\omega)$ (real part of $\tilde{P}(k,k',\omega)$ ) is shift in the phonon frequency and the imaginary part is the phonon frequency line width at the half maximum of the phonon frequency peak and can be written in the form

$$\Gamma_k(\omega) = \Gamma^A_k(\omega) + \Gamma^{\nu}_k(\omega) + \Gamma^{\nu\nu}_k(\omega) + \Gamma^{AD}_k(\omega)$$  \hspace{1cm} (6)

Here $\Gamma_k^A(\omega)$ , $\Gamma_k^{\nu}(\omega)$ , $\Gamma_k^{AD}(\omega)$ and $\Gamma^{\nu\nu}_k(\omega)$ are the individual contributions of line widths (life times) due to point defect scattering, phonon-phonon scattering, interference scattering and electron-phonon scattering, respectively, more details of every term are available in the references elsewhere [26,27,28,31] and will be discussed in the rest of the paper.

The relaxation time as per Callaway formalism is given by $\tau^{-1} = \sum_i \tau_i^{-1}$ for $i$ type of scattering processes not interacting with each other which in general is never found in a real system and addresses the wrong application of Matthiessen’s rule. The $i$ type of collision events may be described in terms of boundary scattering $\tau_{1B}^{-1}$, impurity scattering $\tau_{1I}^{-1}$, phonon-phonon collision $\tau_{1ph}^{-1}$, interference scattering $\tau_{1AB}^{-1}$, electron-phonon $\tau_{e-ph}^{-1}$, resonance scattering $\tau_{R}^{-1}(\omega)$ events, etc. The problem of use of adequate dispersion relations and the inverse additivity of relaxation times can be resolved by taking the concept of renormalized and perturbed mode frequencies [18,31] and $\tau^{-1} = \Gamma_k(\omega)$ [18,20,21,31] which in accordance with Eq. (1) can be resolved in the form

$$\tau^{-1} = \tau_{1\parallel}^{-1} + \tau_{1\perp}^{-1} = \Gamma_{kk}(\omega) + \Gamma_{kk}(\omega) = \Gamma_k(\omega)$$  \hspace{1cm} (7)
where 
\[ \tau_{k \parallel}^{−1}(\omega) = \tau_{CB}^{−1} + \Gamma_{k \parallel}^A(\omega) + \Gamma_{k \parallel}^{AD}(\omega) + \Gamma_{k \parallel}^{E}(\omega) + \tau_{k \parallel}^{−1}. \] (8a) 
\[ \tau_{k \perp}^{−1}(\omega) = \tau_{CB}^{−1} + \Gamma_{k \perp}^A(\omega) + \Gamma_{k \perp}^{AD}(\omega) + \Gamma_{k \perp}^{E}(\omega) + \tau_{k \perp}^{−1}. \] (8b)

Despite several serious objections in the numerically amenable Callaway model established wide acceptability to successfully analyze thermal conductivity by the use of the concept of the relaxation times which has convoluted dependence on frequency, temperature and various distribution functions in a large number of scattering processes involved and resists it as a very sensitive quantity. However, this model was greatly modified by a large number of authors to shape it up in a physically justifiable format [1,4,19,20,21,22,23,35]. A brief account of these events for the layered systems is described as follows:

3.1. Combined Boundary Scattering

The concept of boundary scattering phenomenon [15,16,17,18,20,21,31,32] is based on the assumption that at low temperatures the long wavelength phonons of low frequency get excited and scatters from the crystal boundaries at the relaxation rate of \( \tau_B^{-1} = v / L \) where \( L = 1.22(l_1 l_2)^{1/2} \) is the Casimir length [15] and \( l_1, l_2 \) are cross sectional area of the specimen. Some limitations of Casimir theory which enforced to use \( L \) as a parameter in most of the work on thermal conductivity and this was modified by considering the involvement of crystal micro boundaries due to micro scale fluctuations in the internal boundaries of the crystal [32] in the form of \( \tau_{CB}^{−1} = v_p / L_B \), \( L_B \) being the modified Casimir length. Here the term \( \tau_{CB}^{−1} \) can be modified for the layered systems as \( \tau_{CB}^{−1} = \tau_{CB}^{−1} + \tau_{CB}^{−1} \), where \( \tau_{CB}^{−1} = v_{p||} / L_{||} (B) \) and \( \tau_{CB}^{−1} = v_{p\perp} / L_{\perp} (B) \). These microscale fluctuations incorporated in \( L_B \) offer significant resistance for longer wavelengths and strong phonon-boundary scattering is indeed the reason for increased thermoelectric performance of nanostructures and silicon nanowires in particular [33].

3.2. Impurity Scattering

The contribution of scattering from defect events starts as the temperature starts rising and higher frequency phonons begin to excite with shorter wavelengths. Phonons get localized around the impurity sites and interact with impurities offering much thermal resistance. However, the point impurity scattering was described by Klemens [4] for mass change parameter but when one develops the same problem with the help of many body quantum dynamical theory the lifetime can be obtained in the following forms [16-21]

\[ \Gamma_k^D(\omega) = 8\pi\varepsilon(\omega)\sum_{k_1} R(-k,k_1) R^*(-k,k_1) \delta(\omega - \omega_{k_1}^2) \approx A\omega^4 + A_4\omega^2 \]

It is noteworthy here that apart from the Klemens expression there arises force constant change term depending on square of frequency which of course is highly sensitive in the description of heat capacity. This result can further be obtained for a layered crystal in the form

\[ \Gamma_k^D(\omega) = \Gamma_k^{D||}(\omega) + \Gamma_k^{D\perp}(\omega) \approx A_{m||}\omega_{||}^4 + A_{m\perp}\omega_{\perp}^4 + A_{||}\omega_{||}^2 + A_{\perp}\omega_{\perp}^2 \]

Figure 1. Behavior of \( \tau_{D}^{-1} \) vs T [ inset \( \tau_{D}^{-1} \) vs x] for in-plane and cross-plane references.

3.3. Phonon-Phonon Processes

With further rise in temperature more and more phonons with higher frequencies get excited and start interacting with the cubic and quartic phonon fields of each other giving rise to phonon-phonon scattering. In HTS the quartic phonon scattering does not take place as it is a phenomenon generally operative at high temperatures and one can restrict to \( \Gamma_k^A(\omega) = \Gamma_k^{A||}(\omega) \). In the earlier work the phonon-phonon scattering relaxation time was mostly taken to vary arbitrarily with the powers of frequency and temperature and their multiplier coefficients which couldn’t justify the physics of the problem. This problem was systematically undertaken to study quantum dynamically [18,20,21,31] and revealed the following exact frequency and temperature dependence:

\[ \Gamma_k^{3A}(\omega) = 18\pi\varepsilon(\omega)\sum_{k_1} |V_3\{k_1,k_2,-k\}|^2 \eta_{S\alpha\omega\alpha} \times \delta(\omega^2 - \omega_{\alpha\omega\alpha}^2) - S\alpha\omega\alpha\delta(\omega^2 - \omega_{\alpha\omega\alpha}^2) \]

\[ \approx B_0\omega^2T \]

\[ \approx \Gamma_{||}^{3A}(\omega) + \Gamma_{\perp}^{3A}(\omega) \approx B_0\omega^2T + B_2\omega_{\perp}^2T. \]
Where $B_j (j = \|, \perp)$ are constants for a layered system.

Figure 2. Behavior of $\tau_{3j}^{-1}$ vs $T$ [inset $\tau_{3j}^{-1}$ vs $x$] for in-plane and cross-plane references

Figure 2 depicts the variation of phonon-phonon life times in plane and cross plane references with $T$ and $x$. Apart from defects this scattering infers that the trend for in plane and cross plane cases is similar but the thermal resistance offered by cross planes is certainly higher and may enable one to technological exploitation of the idea that in cross plane direction the system is cold and in the in plane scene it is hot giving way for heat transport.

3.4. Interference Scattering

At elevated temperatures the phonons of localized fields start interacting with those of anharmonic fields giving rise to impurity-anharmonicity interaction modes and interference scattering [18, 20, 21, 31]. Taking these interactions with cubic anharmonicity only the related line width takes the form

$$\Gamma_k^{AD} (\omega) = \Gamma_k^{3D} (\omega) = \frac{44\pi e}{\hbar^2} \sum_{k_1, k_2, k_3} |V_1 (k_1, k_2, -k)|^2 R(k_1, k_2) \omega_k^{-1} \eta_{\parallel}$$

with $x = h\omega / k_BT$, $x_j = h\omega_j / k_BT$; $(j = \|, \perp)$. The details of various symbols appearing in the above equations are well described in the references elsewhere [16, 17, 18, 19, 20, 28, 30, 32] and need no reproduction. Electron-phonon interaction events are highly sensitive to the frequency variations and helps in understanding the fact that the pairons or cooper pairs do not contribute to the thermal transport and this typical behavior of electron-phonon events is shown in Figure 3.

Figure 3. Behavior of $\tau_{3j}^{-1}$ vs $T$ [inset $\tau_{3j}^{-1}$ vs $x$] for in-plane and cross-plane references

3.5. Electron-phonon Scattering

Ziman [13] successfully explored the problem of heat transport by electrons which was developed later quantum mechanically [31] and found it a highly sensitive quantity by describing the electron energy line width in the form

$$\Gamma_k^{ep} (\omega) = g^2 \omega^2 \left[-A_{1e} \left(e^{\omega_0^2/2} + 1\right)^{-1} + A_{2e} \coth \left(3\omega / 2\right)\right]$$

which gives its form for layered systems as follows:

$$\tau_R^{-1} = R\eta_{\parallel}^2 T^n \left[\left(\omega_0^2 - \omega^2\right) + \left(\Omega / \pi\right)^2 \omega_0^2 \omega^2\right]^{-1}$$

with $x = h\omega / k_BT$, $x_j = h\omega_j / k_BT$; $(j = \|, \perp)$. The details of various symbols appearing in the above equations are well described in the references elsewhere [16, 17, 18, 19, 20, 28, 30, 32] and need no reproduction. Electron-phonon interaction events are highly sensitive to the frequency variations and helps in understanding the fact that the pairons or cooper pairs do not contribute to the thermal transport and this typical behavior of electron-phonon events is shown in Figure 3.

Figure 4. Behavior of $\tau_{ep}^{-1}$ vs $T$ [inset $\tau_{ep}^{-1}$ vs $x$] for in-plane and cross-plane references

3.6. Resonance Scattering

Pohl [33] associated the dip just above the maximum peak of thermal conductivity with impurity and resonance scattering and the related relaxation rate was described by him is given by

$$\tau_R^{-1} = R\eta_{\|}^2 T^n \left[\left(\omega_0^2 - \omega^2\right) + \left(\Omega / \pi\right)^2 \omega_0^2 \omega^2\right]^{-1}$$

which gives its form for layered systems as follows:

$$\tau_R^{-1} = R\eta_{\|}^2 T^n \left[\left(\omega_0^2 - \omega^2\right) + \left(\Omega / \pi\right)^2 \omega_0^2 \omega^2\right]^{-1}$$

where $R_{\|}$, $R_{\perp}$ are the in-plane and cross-plane proportionality constants related to the impurity concentration. $\omega_{0\|}$ and $\omega_{0\perp}$ are respective resonance frequencies and $\Omega$ is the damping constant.

4. Analysis of Thermal Conductivity

In order to justify the outcome of the above findings we have taken up the numerical analysis of thermal conductivity of high temperature oxide superconductor La-Sr-Cu-O samples. The experimental results of Uher [5]
for the samples $\text{La}_2\text{SrCuO}_4$, $\text{La}_{1.8}\text{Sr}_{0.2}\text{CuO}_4$, and $\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$ have been taken for the purpose of analysis for different temperature ranges $0-70K$, $0-100K$ and $0-140K$, respectively. The values of various constants and parameters used in the analysis of in-plane and cross-plane thermal conductivity are furnished in Table 1, separately for each sample. The numerical estimation has been carried out in the light of Eqs. (1), (1a) and (1b) have been portrayed in Figure 5 through 7, which reveal excellent agreements between theory and experimental observations throughout all the temperature ranges. The phonon velocity can be replaced by the group velocity $v_g \approx v_p$ which can be further resolved via simplest dispersion relation of the form $\omega^2 = \omega_{\parallel}^2 + \omega_{\perp}^2 = v_{\parallel}^2 k_{\parallel}^2 + v_{\perp}^2 k_{\perp}^2$ with $k_{\parallel}^2 = k_x^2 + k_y^2$ and $k_{\perp} = k_z$ in the Debye approximation [18]. This concept is well supported by Holland’s two mode analysis [35]. During the numerical computation it is observed that the contribution of combined boundary and impurity scattering events is highly effective at low temperatures but their significance gradually diminishes with the rise of temperature and these processes are eventually replaced by the phonon-phonon scattering and interference processes. The findings of Kristoffel et al that the impurities play a very essential role in the cuprate superconductors [36] is well supported in the present work. The situation becomes so intense that the phonons of anharmonic phonon fields start interacting with the phonons of localized fields giving rise to the interference interactions with higher magnitude of thermal resistance. The varied frequency and temperature dependence for phonon processes by earlier worker at their choice thus gets proper justification through phonon-phonon and interference processes and work at and above the thermal conductivity maxima.

Table 1. Constants and parameters used in the analysis of thermal conductivity of high temperature superconductor $\text{La}_{2x}\text{Sr}_{x}\text{CuO}_4$ samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>$T_c$ (K)</th>
<th>$\theta$ (K)</th>
<th>$g_\parallel$</th>
<th>$g_\perp$</th>
<th>$L_{\parallel}(B)(\times10^{-3}$ cm)</th>
<th>$L_{\perp}(B)(\times10^{-3}$ cm)</th>
<th>$A_{\parallel}(10^{22}$ sK$^{-1}$)</th>
<th>$A_{\perp}(10^{22}$ sK$^{-1}$)</th>
<th>$D_{\parallel}(10^{-45}$ s$^3$K$^{-1}$)</th>
<th>$D_{\perp}(10^{-45}$ s$^3$K$^{-1}$)</th>
<th>$\nu_{\parallel}(\times10^5$ cm$^{-1}$)</th>
<th>$\nu_{\perp}(\times10^5$ cm$^{-1}$)</th>
<th>$A_{\parallel,1}(10^2$ erg$^{-2}$K$^{-2}$)</th>
<th>$A_{\parallel,2}(10^2$ erg$^{-2}$K$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{La}_2\text{SrCuO}_4$</td>
<td>37</td>
<td>410</td>
<td>0.7</td>
<td>1.6</td>
<td>0.155</td>
<td>0.75</td>
<td>17.9197</td>
<td>20.0659</td>
<td>20.8982</td>
<td>218.6982</td>
<td>6.2</td>
<td>4.0</td>
<td>9.89</td>
<td>9.89</td>
</tr>
<tr>
<td>$\text{La}<em>{1.8}\text{Sr}</em>{0.2}\text{CuO}_4$</td>
<td>37</td>
<td>400</td>
<td>0.7</td>
<td>0.6</td>
<td>0.153</td>
<td>0.145</td>
<td>83.6587</td>
<td>70.6197</td>
<td>20.8924</td>
<td>20.6972</td>
<td>6.4</td>
<td>6.1</td>
<td>9.89</td>
<td>9.89</td>
</tr>
<tr>
<td>$\text{La}<em>{1.85}\text{Sr}</em>{0.15}\text{CuO}_4$</td>
<td>37</td>
<td>410</td>
<td>0.7</td>
<td>0.6</td>
<td>0.155</td>
<td>0.145</td>
<td>53.6597</td>
<td>350.6197</td>
<td>50.8982</td>
<td>90.6982</td>
<td>7.5</td>
<td>7.5</td>
<td>9.89</td>
<td>9.89</td>
</tr>
</tbody>
</table>

The electron-phonon interactions primarily participate in the thermal transport above the conductivity maximum in the case of high temperature superconductors where conductivity curve shows a cusp like trend near the transition temperature. This cusp is more and more pronounced in case of YBaCuO superconductors [5,31,37].

The phonon-phonon interactions and higher order collision events certainly take place in the system when the temperature is continuously elevated resulting in the excitation of higher and higher frequency phonons with considerably smaller wavelength enabling the collisions at smaller distances and the thermal resistance continuously becomes more and more severe. Obviously, the...
thermal transport and may be attributed to negative thermal resistance (Figure 4 inset) in this region giving cusp like behavior.

Coming to the case of in plane and cross plane thermal transport the various parameters used in computation show that in plane values are always smaller than those of cross plane values. Before going into further details let us define the thermal conductivity functions for in-plane \( \kappa_{||} \) and cross-plane \( \kappa_{\perp} \) contributions as

\[
\kappa_{||} = \frac{k_B \beta^2 h^2 \alpha^4 e^{\beta \hbar \alpha ||}}{2\pi^2 v_{||} \Gamma_k (\alpha_{||}, T) \left(e^{\beta \hbar \alpha ||} - 1\right)^2} \quad (16a)
\]

\[
\kappa_{\perp} = \frac{k_B \beta^2 h^2 \alpha^4 e^{\beta \hbar \alpha \perp}}{2\pi^2 v_{\perp} \Gamma_k (\alpha_{\perp}, T) \left(e^{\beta \hbar \alpha \perp} - 1\right)^2} \quad (16b)
\]

The variation of conductivity function for in-plane and cross plane have been portrayed with \( \chi_{||} \), T is depicted in Figure 8 and Figure 9.

![Figure 8. Variation of in-plane thermal conductivity function \( \kappa_{||}(x_{||}, T) \) versus \( x_{||} \) and T for all collision process](image1)

For reduced frequencies between \( x_j < 1 \) the thermal conductivity function shows sharp peak and the behavior becomes more pronounced at higher temperatures and as is clear from contour lines at the frequencies \( x_j > 1 \) the contribution remains almost constant, notably the curve flattens very rapidly in case of cross plane reference.

![Figure 9. Variation of cross-plane thermal conductivity function \( \kappa_{\perp}(x_{\perp}, T) \) versus \( x_{\perp} \) and T for all collision process](image2)

The curves in Figure 5 – Figure 7 show that the nature of \( \kappa_{||} \) almost follows the nature of experimental curves but the behavior of \( \kappa_{\perp} \) is completely different and slightly rising trend at initial low temperatures but immediately becomes constant for elevated temperatures inferring that it becomes least temperature sensitive.

5. Conclusions

Present investigations successfully describe the behavior of thermal conductivity of cuprate superconductors and is applicable to the all types of high temperatures superconductors. Further, the present study supports that the thermal conductivity in both directions i.e. parallel to the layers and perpendicular to the layers is always smaller than that of bulk materials. It is also inferred that the thermal transport is quite efficient parallel to the layers and along the growth axis or in the cross-plane direction is found quite small compared to the in-plane direction of La\(_{2-x}\)Sr\(_x\)CuO\(_4\) superconductors. This idea can be technologically exploited in fabrication/design of the devices in which the system should not respond to temperature in a particular direction whereas shows maximum response to temperature in the rest of the directions. In other words, the theory concludes that it is possible to develop the devices which is hot in the in plane direction but cold in the cross plane direction.

References