Non-Relativistic Weibel Instability Due to Inverse Bremsstrahlung Absorption in Laser Fusion Plasma

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Abstract The present work has the aim to investigate the Weibel instability induced by inverse bremsstrahlung in laser fusion plasma, where the effect of the self-generated magnetic field is induced. Specifically, we have considered the generated static magnetic field effect on the inverse bremsstrahlung absorption. For this, we consider a homogeneous under dense plasma in interaction with a high-frequency and low-magnitude laser electric field. In our investigation, the unperturbed anisotropic distribution function which takes into account the plasma heating due to inverse bremsstrahlung absorption of laser energy is calculated from the Fokker-Planck equation. By using this distribution function, we have established the dispersion relation of the Weibel modes taking into account the coupling term of quasi-static magnetic fields with high frequency fields of the laser wave. The main obtained result is that the inclusion of the self-generated magnetic field in the inverse bremsstrahlung absorption causes a decrease of the spectral range of the Weibel unstable modes and a reduction of the growth rate of the instability by two orders. This result shows clearly that the previous analyses of the Weibel instability due to inverse bremsstrahlung have an over estimated for the values of the generated static magnetic fields. Therefore, inverse bremsstrahlung overestimated the values of the generated static magnetic fields should not affect the experiences of inertial confinement fusion.

Keywords: Weibel instability, laser fusion plasma, static magnetic field, stabilization

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1. Introduction

The Weibel instability [1] is a micro-convective instability. It corresponds to the excitation of electromagnetic modes in plasma characterized by anisotropy in temperature which corresponds to a plasma described by an anisotropic velocity distribution function. This anisotropy can be generated by several mechanisms, namely: the heat transport, the expansion of the plasma and the inverse bremsstrahlung absorption [2,3,4,5,6].

The present work aims to the investigation of the Weibel instability induced by inverse bremsstrahlung absorption in the laser fusion plasma. This requires the investigation of the dispersion relation for low-frequency electromagnetic modes in plasma heated by a laser pulse.

The spatio-temporal dependence of the high-frequency laser pulse is supposed to be a normal mode. It results highlight new terms in the dispersion relation which can contribute to the Weibel modes instability. We will keep all terms and study their influence on the growth rate of the Weibel instability.

The present paper is organized as follows: in the section 1, we present the used theoretical model where the basic

equation is the Fokker-Planck equation. We consider homogeneous plasma in the presence of a high frequency and low magnitude laser electric field. In order to calculate the distribution function from the Fokker-Planck equation, we use the method of time scales separation and the iterative method. In section 2, we solve the linear part of the Fokker-Planck equation associated with the disruption of the distribution function and establish the dispersion relation of the Weibel modes. Solving the dispersion relation leads to the calculation of rates of instabilities. Finally, a brief conclusion and a summarization of the main obtained results are presented.

2. Basic Equation

To describe the laser fusion plasma corona which is fully ionized plasma where the interactions between particles are dominated by the Coulomb interaction, by Braginski notation the suitable kinetic equation is that of Fokker-Planck equation [7]. For electrons, the Fokker-Planck equation is written in the laboratory frame as:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} - \frac{e}{m_e} \left(\vec{E} + \vec{v} \times \vec{B} \right) \cdot \frac{\partial f}{\partial \vec{v}} = C_{ei}(f, f) + C_{ee}(f) \quad (1)$$

Where f is the distribution functions of electrons, m_e is the electron mass, e is the elementary charge, $\vec{E}$ and $\vec{B}$ are respectively the electric and the magnetic fields present in the plasma, $C_{ee}(f)$ and $C_{ei}(f, f)$ are respectively of electron-electron collision and electron-ion operators [8].

$\vec{E} = \vec{E}_h + \vec{E}_s$ and $\vec{B} = \vec{B}_h + \vec{B}_s$, where $\vec{E}_h$ and $\vec{B}_h$ represent the high-frequency fields associated to the laser wave, $\vec{E}_s$ and $\vec{B}_s$ mean low frequency fields associated to the disturbance in the plasma.

We point out that $\vec{B}_h$ has a small magnitude compared to

that of $\vec{E}_h$: $\frac{\vec{E}_h}{\vec{B}_h} \sim c$. In addition, The spatio-temporal evolution of the laser wave is of the form:

$$\vec{E}_h = \vec{E}_0 \text{Re}[\exp(i\omega t)] \quad (2)$$

where $\vec{E}_0$ and ω are respectively the wave electric field magnitude and the wave frequency. In order to solve the equation (1), we consider two time scales, a low-frequency hydrodynamic timescale and a high-frequency (laser field) one. Therefore, the electronic distribution function f can be written as the sum of a quasi-static distribution function f_s , which varies slowly in time and a high-frequency distribution function f_h , following the temporal variation of high frequency laser electric field $\vec{E}_h$, so:

$$f(\vec{r}, \vec{v}, t) = f_s(\vec{r}, \vec{v}, t) + \text{Re} \left[f_h(\vec{v}) \exp(i\omega t) \right] \quad (3)$$

The rating on the indices «s» and «h» refers to secular time scales low frequency and high frequency respectively and will be used throughout this work.

The separation of time scales in equation (1) leads to the following high-frequency and low-frequency kinetic equations:

$$\frac{\partial f_h}{\partial t} - \frac{e}{m_e} \vec{E}_h \cdot \frac{\partial f_s}{\partial \vec{v}} = \frac{e}{m_e} \left(\vec{E}_s + \vec{v} \times \vec{B}_s \right) \cdot \frac{\partial f_h}{\partial \vec{v}} + C_{ei}(f_h) \quad (4)$$

$$\begin{aligned} \frac{\partial f_s}{\partial t} + \vec{v} \cdot \frac{\partial f_s}{\partial \vec{r}} - \frac{e}{m_e} \left(\vec{E}_s + \vec{v} \times \vec{B}_s \right) \cdot \frac{\partial f_s}{\partial \vec{v}} - C_{ei}(f_s) \\ = \frac{e}{m_e} \vec{E}_h \cdot \frac{\partial f_h}{\partial \vec{v}} \end{aligned} \quad (5)$$

The symbol $\langle \rangle$ denotes the average over the laser wave cycle time, $T = 2\pi / \omega$. These equations make a system of two coupled equations. Note here that the terms in the

electric field $\vec{E}_s$ and magnetic field $\vec{B}_s$ reflect the inclusion of the low frequency electromagnetic field effect in our study. In particular, the first term on the right hand side of the equation (4) reflects the coupling of quasi-static fields with the laser field. Let us recall here that this

field present in the plasma is generated by the mechanism of the Weibel instability. These terms do not appear in the work reported in reference [3].

2.1. Resolution of the High-Frequency Fokker- Planck Equation

We consider the high frequency approximation, where the laser wave frequency ω_l is greater than the collisions frequency ν_{ei} .

This approximation is largely justified in the laser fusion plasma experiments. For example of typical Parameters: (electron temperature $T_e = 1 \text{ KeV}$; overage free scaling of particles $\lambda_{ei} = 1 \mu\text{m}$ and laser wave length $\lambda_l = 1.06 \mu\text{m}$ respectively) we have $\omega_l / \nu_{ei} \gg 1$.

In this case, the high frequency electronic distribution function is calculated as follow:

$$\begin{aligned} f_h(\vec{v}) = \frac{e}{m_e} \frac{i}{\omega_0^2} E_{hj} \cdot \frac{\partial f_s}{\partial v_j} \frac{e}{m_e} \frac{\nu_{ei}(v)}{\omega_0^2} E_{hj} \\ \cdot \frac{\partial f_s}{\partial v_j} \frac{e^2}{m_e^2} \left(\vec{E}_s + \vec{v} \times \vec{B}_s \right)_i \frac{\partial}{\partial v_i} \left[E_{hj} \cdot \frac{\partial f_s}{\partial v_j} \right] \end{aligned} \quad (6)$$

where the used Einstein notation means that the repeated indices represent the summation over the indices. The expression (6) represents the high-frequency component of the electronic distribution function which depends on its low-frequency component f_s .

2.2. Resolution of the Low Frequency Fokker-Planck Equation

We will now solve the low frequency Fokker-Planck equation (5) using the expression of the distribution function of high frequency f_h established in the previous paragraph. Substituting (6) in (5), we obtain then after some algebra:

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \frac{\partial f_s}{\partial \vec{r}} - \frac{e}{m_e} \left(\vec{E}_s + \vec{v} \times \vec{B}_s \right) \cdot \frac{\partial f_s}{\partial \vec{v}} + \nu_{ei}(v) [f_s] = S_{BI} \quad (7)$$

where

$$\begin{aligned} S_{BI} = -\frac{1}{2} \frac{e^2}{m^2 \omega_0^2} E_{0l} E_{0j} \frac{\partial}{\partial v_l} \left[\nu_{ei}(v) \frac{\partial f_s}{\partial v_j} \right] \\ - \frac{1}{2} \frac{e}{m} \frac{e^2}{m^2 \omega_0^2} E_{0l} E_{0j} E_{si} \frac{\partial}{\partial v_l} \left[\frac{\partial^2 f_s}{\partial v_i \partial v_j} \right] \\ - \frac{1}{2} \frac{e}{m} \frac{e^2}{m^2 \omega_0^2} E_{0l} E_{0j} \frac{\partial}{\partial v_l} \left[\left(\vec{v} \times \vec{B}_s \right)_i \frac{\partial^2 f_s}{\partial v_i \partial v_j} \right] \end{aligned} \quad (8)$$

The next step of our calculation is to linearize the equation (7) by setting

$$f_s = f_s^{(0)} + f_s^{(1)} \quad (9)$$

We calculated the second anisotropic equation using the same approximation as applied by projection of the equation (7) on Legendre polynomial $p_2(v)$ [9].

1. The terms proportional to the second anisotropic distribution function $f_{s2}^{(0)}$, are ignored. This is justified by

the fact that: $\frac{f_{s2}^{(0)}}{f_{s0}^{(0)}} \sim \frac{V_0^2}{V_t^2} \ll 1$ which correspond to the low

magnitude laser wave approximation largely fulfilled in the laser-plasma interaction experiments [6].

2. We have neglected the terms proportional to $f_{s4}^{(0)}$

and those proportional to $\frac{V_0^2}{V_t^2}$.

3. The expression of the second anisotropic distribution function is obtained in the stationary approximation is $\frac{\partial f_{s2}^{(0)}}{\partial t} \approx 0$.

The evolution of the second anisotropy equation $f_{s2}^{(0)}$ is obtained by:

$$f_{s2}^{(0)} = \frac{1}{3\sqrt{5}} \left(\frac{v_0}{v_T} \right)^2 \left(3 + \frac{v^2}{v_T^2} \right) f_M \quad (10)$$

This result shows that the second anisotropy is positive $f_{s2}^{(0)} > 0$, which corresponds to a temperature anisotropy $T_x > T_\perp$, where T_x denotes the temperature in the direction ox and $T_\perp$ in the direction perpendicular to ox . Indeed, the plasma is heated preferentially along the laser wave field direction: ox .

3. Analysis of the Weibel Instability

This paragraph is devoted to the analysis of the Weibel instability. We determine the dispersion relation of the Weibel modes and deduce the growth rate of Weibel instability. For this it is necessary to calculate the perturbed distribution function due to an electromagnetic disturbance.

3.1. Calculation of the Perturbed Distribution Function

The evolution equation of the perturbed is obtained from the low-frequency Fokker-Planck equation by considering the first term order, so:

$$\begin{aligned} \frac{\partial f_s^{(1)}}{\partial t} + \vec{v} \cdot \frac{\partial f_s^{(1)}}{\partial \vec{r}} - \frac{e}{m_e} \left(\vec{E}_s + \vec{v} \times \vec{B}_s \right) \cdot \frac{\partial f_s^{(0)}}{\partial \vec{v}} \\ + v_{ei}(v) \left[f_s^{(1)} \right] = I_{BI} \left(f_s^{(1)} \right) + I_{BI} \left(f_s^{(0)} \right) \end{aligned} \quad (11)$$

where

$$I_{BI} \left(f_s^{(1)} \right) = \frac{1}{2} v_0^2 \frac{\partial}{\partial v_x} \left[v_{ei}(v) \frac{\partial f_s^{(1)}}{\partial v} \right]$$

After some manipulation the mathematical equation (11) becomes.

$$\begin{aligned} v_{ei}(v) f_{sl,m}^{(1)} + ikv \left[\frac{l^2 - m^2}{4l^2 - 1} \right] f_{sl-1,m}^{(1)} \\ + ikv \left[\frac{(l+1)^2 - m^2}{4(l+1)^2 - 1} \right] f_{sl+1,m}^{(1)} = S_{El,m} + S_{Bl,m} \end{aligned} \quad (12)$$

Note also, that the right-side of equation (12) is zero. The result is a recurrence relation between the components:

$$\begin{aligned} v_{ei}(v) f_{sl,m}^{(1)} = -ikv^4 \left[\frac{l^2 - m^2}{4l^2 - 1} \right] f_{sl-1,m}^{(1)} \\ - ikv^4 \left[\frac{(l+1)^2 - m^2}{4(l+1)^2 - 1} \right] f_{sl+1,m}^{(1)} \text{ pour } l \geq 3 \end{aligned} \quad (13)$$

We will now solve the system of equations (13) using mathematical techniques based on continued fractions. Solving kinetic equations with continued fractions [9] was used for the first time in [4], where the collisional propagator in the Fokker-Planck equation has been explicitly reversed on the basis of spherical harmonics. By applying these results to equation (13), and after some mathematical manipulations, we obtain:

$$f_{s3,m}^{(1)} = -ikv^4 \left[\frac{9 - m^2}{35} \right]^{\frac{1}{2}} F_{3,m} f_{s2,m}^{(1)} \quad (14)$$

where $F_{3,m}$ is the continued fraction defined the following recurrence relation:

$$F_{l,m}(k, v) = \left[v_{ei}(v) + k^2 v^8 \frac{(l+1)^2 - m^2}{4(l+1)^2 - 1} F_{l+1,m} \right]^{-1} \quad (15)$$

We must note here that equation (14) is the exact solution of the infinite hierarchy of equations (13). It gives a relationship between components $f_{s3,m}^{(1)}$ and $f_{s2,m}^{(1)}$ that includes the contribution of all components $f_{sl,m}^{(1)}$ with $l > 3$. The explicit expressions of the components $f_{sl\pm 1}^{(1)}$ are then obtained as:

$$\begin{aligned} f_{sl\pm 1}^{(1)} = \pm 4 \sqrt{\frac{2\pi}{3}} \frac{e}{m_e} E_s v_t^2 y^2 F_{l,1} f_M \\ \mp i \frac{e}{m_e} \frac{\sqrt{5}}{K v_t} \sqrt{\frac{\pi}{15}} B_s \frac{v_0^2}{v_t^2} (1 - v F_{l,1}) (3 + y) y^{\frac{1}{2}} f_M \end{aligned} \quad (16)$$

3.2. Determination of the Dispersion Relation

The calculation of the dispersion relation of electromagnetic Weibel modes in the semi-collisional approximation [2,4,5,10] can be calculated using the perturbed Fokker-Planck equation coupled with Maxwell's equations presented as follows:

$$\vec{\nabla} \times \vec{E}_s = -\frac{\partial \vec{B}_s}{\partial t} \quad (17)$$

and

$$\vec{\nabla} \times \vec{B}_s = -\mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}_s}{\partial t} \quad (18)$$

where $\vec{J}$ is the current density defined by

$$\vec{J} = -e \int \vec{v} f_s^{(1)} d\vec{v} \quad (19)$$

By considering that the spatio-temporal dependence of the field $\vec{E}_s$ and $\vec{B}_s$ as a Fourier mode $\sim \exp(i\omega t + kz)$, equations (18) and (19) can be represented as:

$$kE_s = \omega B_s \quad (20)$$

$$kB_s = -ie\mu_0 \int v_x f_s^{(1)} d\vec{v} \quad (21)$$

By developing the function $f_s^{(1)}$, on the spherical harmonics basis Y_l^m , the equation (21) reads as:

$$kB_s = -ie\mu_0 \sqrt{\frac{2\pi}{3}} \int_0^\infty v^3 (f_{s1,-1}^{(1)} - f_{s1,1}^{(1)}) dv \quad (22)$$

We deduce then the dispersion relation of the Weibel modes as:

$$\begin{aligned} \frac{k^2 c^2}{\omega_p^2} &= i \frac{8\sqrt{2}}{3\sqrt{\pi}} \omega v_t^3 \int_0^\infty y^3 F_{1,1} \exp(-y) dy \\ &+ \frac{2}{3\sqrt{\pi}} \frac{v_0^2}{v_t^2} \int_0^\infty y^{1/2} (3+y) (1 - v F_{1,1}) ep(-y) dy \end{aligned} \quad (23)$$

Here, $w = w_r + i\gamma$, is the frequency of Weibel mode and γ being the growth rate which is derived in the linear approximation from equation (23) as:

$$\begin{aligned} \gamma(k) &= -\frac{3\sqrt{\pi}}{8\sqrt{2}} \frac{k^2 c^2}{\omega_p^2} \frac{1}{v_t^3} \frac{1}{\int_0^\infty y^3 F_{1,1}(k\lambda_{ei}, y) \exp(-y) dy} \\ &+ \frac{1}{4\sqrt{2}} \frac{v_0^2}{v_t^2} \frac{1}{v_t^3} \frac{\int_0^\infty y^2 (3+y) (1 - v F_{1,1}) ep(-y) dy}{\int_0^\infty y^3 F_{1,1}(k\lambda_{ei}, y) \exp(-y) dy} \end{aligned} \quad (24)$$

4. Discussion

The first term of equation (24) is a loss term; it corresponds to a loss due to collisions between particles in the collisional limit ($k\lambda_{ei} \gg 1$) while in the non-collisional limit ($k\lambda_{ei} \ll 1$), it describes the Landau damping of electromagnetic modes. The second term, $\sim v_0^2$ corresponds to the Weibel instability source. Equation (24) gives explicitly the growth rate of the Weibel instability excited by inverse bremsstrahlung absorption in laser fusion plasma. This expression contains continuous fractions which are numerically calculated [5]. We present on Figure 1 and Figure 2, the spectra of the growth rate of Weibel unstable mode for typical parameters of plasma and laser.

We point out that the profile of the spectrum $\gamma(k)$ has present a maximum. This can be interpreted by the competition between the loss effects (collisions and Landau damping) and the inverse bremsstrahlung Weibel instability growing.

However, the values of the growth rate remain in the order of $\gamma(k) = 10^{12} s^{-1}$ compared by reference [11,12]. it

is found that this reduction is independent of the values of the electron density. We believe that this is due to the choice of the Krook collisions operator $C_{ei}(f) = \nu_{ei}(V)[f - f_M]$ where the collision frequency ν_{ei} is considered constant. This approximation is not realistic since it is known that collisions reduce the anisotropy of the plasma it letting it to a steady state described by an isotropic distribution function.

Figure 1. Growth rate of Weibel instability γ as function of the collision parameter $k\lambda_{ei}$ for typical physical parameters:

$$n_e = 10^{27} m^{-3}, T_e = 2 KeV, \frac{v_0}{v_t} = 0.3 \text{ and } Z = 4.$$

The dotted line corresponds to the case without $I_{BI}(f_{s0}^{(0)})$ term.

Figure 2. Growth rate of Weibel instability γ as function of the collision parameter $k\lambda_{ei}$ for typical physical parameters:

$$n_e = 9 \times 10^{27} m^{-3}, T_e = 2 KeV, \frac{v_0}{v_t} = 0.3 \text{ and } Z = 4.$$

The dotted line corresponds to the case without $I_{BI}(f_{s0}^{(0)})$ term.

5. Conclusion

In this work we have presented a theoretical investigation and a numerical analysis (Figure 1 and Figure 2) of the Weibel instability in the laser fusion plasma using the Fokker-Planck model. The stabilization effect due to generated static magnetic field is computed. It has been shown that by taking into account the coupling of the self-generated magnetic field by the Weibel instability with the laser wave field leads to a significant decreasing in the spectral range of unstable Weibel modes. Also this coupling undergoes a reduction in the growth rate values of Weibel instable modes. For the most unstable mode, this reduction is of two ranges.

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