Problems of Lorentz Force and Its Solution

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Abstract In the article is proven that the Lorentz force is the consequence of the dependence of the scalar potential of charge on the speed. It is shown that the dependence of Lorentz force on the speed is nonlinear, as previously supposed. It is shown also, that the forces of interaction of conductors, along which flows the current, are not symmetrical. When the direction of the motion of charges in the conductors they coincide, the force of their interaction occurs less than when directions of motion are different.

Keywords: maxwell equations, Lorentz force, Ampere law, Faraday law, laws of the induction


1. Introduction

Unfortunately, contemporary classical electrodynamics is not deprived of the contradictions, which did not up to now obtain their explanation.

The fundamental equations of contemporary classical electrodynamics are Maxwell's equation. They are written as follows for the vacuum:

\[ \text{rot } \vec{E} = -\varepsilon \frac{\partial \vec{B}}{\partial t} \quad (1.1) \]

\[ \text{rot } \vec{H} = \mu \frac{\partial \vec{D}}{\partial t} \quad (1.2) \]

\[ \text{div } \vec{D} = 0 \quad (1.3) \]

\[ \text{div } \vec{B} = 0 \quad (1.4) \]

where \( \vec{E} \) and \( \vec{H} \) are tension of electrical and magnetic field, \( \vec{D} = \varepsilon_0 \vec{E} \) and \( \vec{B} = \mu_0 \vec{H} \) are electrical and magnetic induction, \( \mu_0 \) and \( \varepsilon_0 \) is magnetic and dielectric constant of vacuum. From these equations follow wave equations for the electrical and magnetic field

\[ \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad (1.5) \]

\[ \nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} \quad (1.6) \]

these equations show that in the vacuum can be extended the plane electromagnetic waves, the velocity of propagation of which is equal to the speed of light

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad (1.7) \]

Maxwell's equations are written for the material media

\[ \text{rot } \vec{E} = -\mu_0 \varepsilon \frac{\partial \vec{H}}{\partial t} \quad (1.8) \]

\[ \text{rot } \vec{H} = n \vec{v} + \mu_0 \varepsilon \frac{\partial \vec{E}}{\partial t} = n \vec{v} + \frac{\partial \vec{D}}{\partial t} \quad (1.9) \]

\[ \text{div } \vec{D} = n e \quad (1.10) \]

\[ \text{div } \vec{B} = 0 \quad (1.11) \]

where \( \mu \) and \( \varepsilon \) are the relative magnetic and dielectric constants of the medium and \( n \) , \( \vec{v} \) and \( e \) are density, value and charge rate.

Equations (1.1 - 1.11) are written in the assigned inertial reference system (IRS), and in them there are no rules of passage of one IRS to another. The given equations also assume that the properties of charge do not depend on their speed, since in first term of the right side of Eq. (1.9) as the charge its static value is taken.

In Maksvell's equations are not contained indication that is the reason for power interaction of the current carrying systems; therefore to be introduced the experimental postulate about the force, which acts on the moving charge in the magnetic field. This postulate assumes that on the charge, which moves in the magnetic field, acts the force

\[ F_L = \mu_0 \vec{H} \times \vec{v} \quad (1.12) \]

This approach is had essential deficiency. If force acts on the moving charge, then must be known the object, from side of which acts this force. In this case the magnetic field is independent substance, comes out in the role of the mediator between the moving charges. Consequently, there is no law of direct action, which would give answer to a question, as interact the charges, which accomplish relative motion.
Relationship (1.12) causes bewilderment. In the mechanics the forces, which act on the moving body, are connected with its acceleration, with the uniform motion there exist frictional forces. With the uniform motion there exist also frictional forces. The direction of these forces coincides with the velocity vector. But the force, determined by Eq. (1.12), have another property. Rectilinear motion causes the force, which is normal to the direction motion, what is assumed none of the existing laws of mechanics. Therefore is possible to assume that this some new law, which is concerned relative motion of those only of charged tel.

Is certain, magnetic field is one of the important concepts of contemporary electrodynamics. In accordance with the Ampere law around the current, which flows along the conductor, there is a circulation of the magnetic field

\[ \oint Hdl = I \]  

(1.13)

where \( I \) is conduction current. If we to the conduction current add bias current, then we will obtain the second Maxwell's equation (1.9).

It should be noted that the introduction of the concept of magnetic field does not be founded upon any physical basis, but it is the statement of the collection of some experimental facts, which with the aid of the specific mathematical procedures in large quantities of the cases give the possibility to obtain correct answer with the solution of practical problems. But there is a number of the physical questions, to which the concept of magnetic field answer does not give. Using Eqs. (1.12) and (1.13) not difficult to show that with the unidirectional parallel motion of two like charges, or flows of charges, between them must appear the additional attraction. However, if we pass into the inertial system, which moves together with the charges, then there magnetic field is absent, and there is no additional attraction. This paradox in the electrodynamics does not have an explanation. It does not have an explanation, also, in the special theory of relativity (SR).

With power interaction of conductors, along which flows the current, forces are applied not only to the moving charges, but to the lattice. But the concept of magnetic field also to this question answer does not give, since. In Eqs. (1.1-1.13) the presence of lattice is not considered.

As the fundamental law of induction in the electrodynamics is considered Faradaylaw, consequence of whom is the first Maxwell's equation. However, here are problems. It is considered until now that the unipolar generator is an exception to the rule of flow. The existing state of affairs and those contradictions, which with this are connected, perhaps, are most are clearly formulated in the sixth volume of work [2]. We read on page 52: "the rule of flow states that the contour e.m.f. is equal to the opposite-sign rate of change in the magnetic flux through the contour when the flux varies either with the changing field or due to the motion of the contour (or to both). Two options – "the contour moves" or "the field changes" are indistinguishable within the rule. Nevertheless, we use these two completely different laws to explain the rule for the two cases: \[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \] for the "changing field". And further on:

There is hardly another case in physics when a simple and accurate general law has to be interpreted in terms of two different phenomena. Normally, such beautiful generalization should be based on a unified fundamental principle. Such principle is absent in our case".

2. Laws of the Induction

With conducting of experiments Faraday established that in the outline is induced the current, when in the adjacent outline direct current is switched on or is turned off or adjacent outline with the direct current moves relative to the first outline. Therefore in general form Faraday law is written as follows[3]:

\[ \oint E'dl' = -\frac{d\Phi_B}{dt} \]  

(2.1)

This writing of law indicates that with the determination of the circulation of \( \vec{E} \) in the moving coordinate system, near \( \vec{E} \) and \( dl' \) must stand primes and should be taken total derivative. But if circulation is determined in the fixed coordinate system, then primes near \( \vec{E} \) and \( dl' \) be absent, but in this case to the right in Eq. (2.1) must stand particular time derivative.

Complete time derivative in Eq. (2.1) indicates the independence of the eventual result of appearance e.m.f in the outline from the method of changing the flow. Flow can change as because \( \vec{B} \) it depends on time, so also because the system, in which is determined the circulation \( \oint \vec{E}'dl' \), it moves in the magnetic field, whose value depends on coordinates. The value of magnetic flux in Eq. (2.1) is determined from the equation

\[ \Phi_B = \int \vec{B} ds' \]  

(2.2)

where the magnetic induction \( \vec{B} = \mu \vec{H} \) is determined in the fixed coordinate system, and the element \( ds' \) is determined in the moving system. Taking into account Eq. (2.1), we obtain from Eq. (2.16)

\[ \oint \vec{E}'dl' = -\frac{d}{dt} \int \vec{B} ds' \].

Since \( \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \ \text{grad} \), let us write down:

\[ \oint \vec{E}'dl' = \frac{\partial \vec{B}}{\partial t} ds' - \int \left[ \frac{\partial \vec{B}}{\partial t} + \vec{v} \ \text{grad} \vec{B} ight] ds' \]  

(2.3)
In this equation integral is taken on the outline, which covers the area \( d\vec{s} \). Let us immediately note that entire following presentation will be conducted under the assumption the validity of the conversions of Galileo, i.e., \( d\vec{l} = dl \) and \( d\vec{s} = d\vec{s} \). During the motion in the magnetostatic field is fulfilled the equation

\[
\vec{E}' = \left[ \vec{v} \times \vec{B} \right]
\]

This equation is obtained not by the introduction of postulate about the Lorentz force, but directly from the Faraday law of the induction. Thus, Lorentz force is the direct consequence of the law of magnetoelectric induction.

Faraday law (2.3) indicates that how a change in the magnetic pour on, or motion in these fields, it leads to the appearance of electrical pour on; therefore it should be called the law of magnetoelectric induction. However, in the classical electrodynamics there is no law of electromagnetoelectric induction, which would show, how a change in the electrical field on, or motion in them, it leads to the appearance of magnetic field on. The development of classical electrodynamics followed along another way. Was first known the Ampere law

\[
\oint \vec{H} \cdot d\vec{l} = I
\]  
(2.4)

where \( I \) is current, which crosses the area, included by the outline of integration. In the differential form Eq. (2.4) takes the form:

\[
\text{rot} \, \vec{H} = \vec{j}_{\sigma}
\]  
(2.5)

where \( \vec{j}_{\sigma} \) is current density of conductivity.

Maxwell supplemented Eq. (2.5) with bias current

\[
\text{rot} \, \vec{H} = \vec{j}_{\sigma} + \frac{\partial \vec{D}}{\partial t}
\]  
(2.6)

However, must exist the law of electromagnetic induction, which determines magnetic fields in the changing electric field

\[
\oint \vec{H}'d\vec{l}' = \frac{d}{dt} \Phi = \Phi_{D}
\]  
(2.7)

where \( \Phi_{D} = \int \vec{D} \cdot d\vec{S}' \) is the flow of electrical induction.

\[
\oint \vec{H}'d\vec{l}' = \int \frac{\partial \vec{D}}{\partial t} d\vec{S}' + \int \left[ \vec{D} \times \vec{v} \right] d\vec{l}' + \int \vec{V} d\vec{l}' d\vec{S}'
\]  
(2.8)

In contrast to the magnetic field on, when \( \text{div} \, \vec{B} = 0 \), for the electrical field on \( \text{div} \, \vec{D} = \rho \) and last term in the right side of Eq. (2.8) it gives the conduction current \( I \) and from relationship (2.7) the Ampere law immediately follows. Thus, from Eq. (2.7) follows Ampere law. During the motion in the DC fields we obtain

\[
\vec{H} = \left[ \vec{D} \times \vec{v} \right]
\]  
(2.9)

As shown in the work [2], from Eq. (2.9) follows and Bio-Savar law, if for enumerating the magnetic fields on to take the electric fields of the moving charges. In this case the last member of the right side of Eq. (2.8) can be simply omitted, and the laws of induction acquire the completely symmetrical form

\[
\oint \vec{E}'d\vec{l}' = -\int \frac{\partial \vec{B}}{\partial t} d\vec{s} + \oint \left[ \vec{v} \times \vec{B} \right] d\vec{l}'
\]

\[
\oint \vec{H}'d\vec{l}' = \int \frac{\partial \vec{D}}{\partial t} d\vec{s} - \oint \left[ \vec{v} \times \vec{D} \right] d\vec{l}'
\]  
(2.10)

For the constants fields on transformation laws they take the following form

\[
E' = [\vec{v} \times \vec{B}], H' = -(\vec{v} \times \vec{D})
\]  
(2.11)

3. Dynamic Potentials and the Field of the Moving Charges

As already mentioned, in the classical electrodynamics be absent the rule of the conversion of electrical and magnetic fields on upon transfer of one inertial system to another. This deficiency removes SR. With the entire mathematical validity of this approach the physical essence of such conversions up to now remains unexplained.

Let us explain, what potentials and fields can generate the moving charges. The first step in this direction, demonstrated in works [4,5,6,7], was made with the aid of the introduction of the symmetrical laws of induction Eq. (2.10).

Equations (2.10, 2.11) attest to the fact that in the case of relative motion of frame of references, between the fields of and there is a cross coupling, i.e., motion in the fields of leads to the appearance fields on and vice versa. Motion in the fields \( \vec{H} \) leads to the appearance fields on \( \vec{E} \) and vice versa. This leads to the additional consequences, which were for the first time examined in work [4]. Electric field beyond the limits of the long charged rod is determined from the equation

\[
E = \frac{g}{2\pi\varepsilon r}
\]

where \( g \) is linear charge.

If we in parallel to the axis of rod in the field of begin to move with the speed of another IRS, then in it will appear the additional magnetic field. If we now with respect to already moving IRS begin to move third IRS with the speed \( \Delta v \), then already due to the motion in the field \( \Delta H \) will appear additive to the electric field of \( \Delta E = \mu_0 E (\Delta v)^2 \). This process can be continued and further, as a result of which can be obtained the number, which gives the value of the electric field of \( E_{ch} (r) \) in moving IRS with reaching of the speed of \( v = n\Delta v \), when \( \Delta v \to 0 \), and \( n \to \infty \). In the final analysis in moving IRS the value of dynamic electric field will prove to be more than in the initial and to be determined by the equation:

\[
E' (r,v_{ch}) = \frac{gch \frac{v_{ch}}{c}}{2\pi\varepsilon r} = Ech \frac{v_{ch}}{c}
\]  
(3.1)

The electric field of the single charge will be determined by the equation:
where \( v_\perp \) is normal component of charge rate to the vector, which connects the moving charge and observation point.

The equation for the scalar potential, created by the moving charge, for this case will be written down as follows [4.5.6.7]:

\[
\varphi'(r, v_\perp) = \frac{e c h v_\perp}{4\pi r} = \varphi(r) c h v_\perp \tag{3.2}
\]

where \( \varphi(r) \) is scalar potential of fixed charge. The potential \( \varphi'(r, v_\perp) \) can be named the scalar-vector potential, since it depends not only on the absolute value of charge, but also on speed and direction of its motion with respect to the observation point. Maximum value this potential has in the direction normal to the motion of charge itself.

During the motion in the magnetic field, using the already examined method, we obtain:

\[
H'(v_\perp) = H c h v_\perp \tag{3.3}
\]

where \( v_\perp \) is the speed normal to the direction of the magnetic field.

If we apply the obtained results to the electromagnetic wave and to designate components pour on parallel of speed as \( E_\parallel, H_\parallel, \) and \( E_\perp, E_\perp \) as components normal to it, then conversions fields on they will be written down:

\[
\begin{align*}
E_\parallel' &= E_\parallel E_\perp = \bar{\bar{E}}_\perp = \bar{\bar{E}}_\perp \frac{v}{c} + Z_0 \left( \bar{\bar{v}} \times \bar{\bar{E}}_\perp \right) \frac{v}{c} \\
H_\parallel' &= H_\parallel H_\perp = \bar{\bar{H}}_\perp = \bar{\bar{H}}_\perp \frac{v}{c} - \frac{1}{\mu_0 c} \left( \bar{\bar{v}} \times \bar{\bar{H}}_\perp \right) \frac{v}{c},
\end{align*}
\]

(3.3)

where \( Z_0 = \frac{\mu_0}{\varepsilon_0} \) is the impedance of free space, \( c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} \) is the speed of light.

Conversions fields Eq. (3.3) they were for the first time obtained in the work [4].

4. Power Interaction of Parallel Conductors

The concept of magnetic field arose to a considerable degree because of the observations of power interaction of the current carrying and magnetized systems. Experience with the iron shavings, which are erected near the magnet poles or around the annular turn with the current into the clear geometric figures, is especially significant. These figures served as occasion for the introduction of this concept as the lines of force of magnetic field. In accordance with third Newton's law with any power interaction there is always equality of effective forces and opposition, and also always there are those elements of the system, to which these forces are applied. A large drawback in the concept of magnetic field is the fact that it does not give answer to that, counteracting forces are concretely applied to what, since magnetic field comes out as the independent substance, with which occurs interaction of the moving charges.

Is experimentally known that the forces of interaction in the current-carrying systems are applied to those conductors, whose moving charges create magnetic field. However, in the existing concept of power interaction of the current carrying systems, based on the concepts of magnetic field and Lorentz force, the positively charged lattice, which is the frame of conductor and to which are applied the forces, it does not participate in the formation of the forces of interaction.

Let us examine this question on the basis of the concept of scalar-vector potential. We will consider that the scalar-vector potential of single charge is determined by relationship (3.2), and that the electric fields, created by this potential, act on all surrounding charges, including to the charges positively charged lattices.

Let us examine from these positions power interaction between two parallel conductors (Figure 1), along which flow the currents. We will consider that \( g_1^+, g_2^+ \) and \( g_1^-, g_2^- \) present positive and negative linear charges in the upper and lower conductors.

**Figure 1.** Schematic of power interaction of the current-carrying wires of two-wire circuit taking into account the positively charged lattice

We will also consider that both conductors prior to the start of charges are electrically neutral, i.e., in the conductors there are two systems of the mutually inserted opposite charges with the linear charges \( g^+ \) and \( g^- \), which electrically neutralize each other. In Figure 1 these systems are moved apart along the axis \( z \). Subsystems with the negative charge (electrons) can move with the speeds \( v_1, v_2 \). The force of interaction between the lower and upper conductors we will search for as the sum of four forces, whose designation is understandable from the figure. The repulsive forces \( F_1 \) and \( F_2 \) we will take with the minus sign, while the attracting force \( F_3 \) and \( F_4 \) we will take with the plus sign.

For the single section of the two-wire circuit of force, acting between the separate subsystems, will be written down

\[
\begin{align*}
F_1 &= -\frac{g_1^+ g_2^+}{2\pi r}, \quad F_2 = -\frac{g_1^- g_2^-}{2\pi r} \frac{v_1 - v_2}{c}, \\
F_3 &= +\frac{g_1^- g_2^+}{2\pi r} \frac{v_1}{c}, \quad F_4 = +\frac{g_1^+ g_2^-}{2\pi r} \frac{v_2}{c}.
\end{align*}
\]

(4.1)
Adding forces, we will obtain the amount of the composite force

\[ F_\Sigma = \frac{g_1 g_2}{2 \pi \varepsilon r} \left( \frac{v_1 c}{c} + \frac{v_2 c}{c} - \frac{v_1 c - v_2 c}{c} - 1 \right) \tag{4.2} \]

where \( g_1 \) and \( g_2 \) are undertaken the absolute values of linear changes, and \( v_1, v_2 \) take with its signs.

Where \( v \ll c \) let us take only two first members of expansion in the series \( \frac{v}{c} \), i.e., we will consider that

\[ \frac{v}{c} \approx 1 + \frac{v^2}{2c^2} \].

From Eq. (4.2) we obtain

\[ F_{\Sigma 1} = \frac{g_1 v_1 g_2 v_2}{2 \pi \varepsilon c^2 r} = \frac{I_1 I_2}{2 \pi \varepsilon c^2 r} \tag{4.3} \]

Since the magnetic field of straight wire, along which flows the current \( I \), we determine by the equation

\[ H = \frac{I}{2 \pi r} \]

from Eq. (4.3) we obtain

\[ F_{\Sigma 1} = \frac{I_1 I_2}{2 \pi \varepsilon c^2 r} = \frac{H_1 I_2}{\varepsilon c^2} = I_2 \mu H_1 \]

where \( H_1 \) is the magnetic field, created by lower conductor in the location of upper conductor.

It is analogous

\[ F_{\Sigma 1} = I_1 \mu H_2 \]

where \( H_2 \) is the magnetic field, created by upper conductor.

The results, obtained in the model of scalar-vector potential, completely coincide with the results, obtained on the basis of the concept of magnetic field.

The equation (4.3) represents the known rule of power interaction of the current carrying systems, but is obtained it not by the phenomenological way on the basis of the introduction of phenomenological magnetic field, but on the basis of completely intelligible physical procedures, under the assumption that that the scalar potential of charge depends on speed. In the formation of the forces of interaction in this case the lattice takes direct part, which is not in the model of magnetic field. In the model examined are well visible the places of application of force. These equations completely coincide with the results, obtained on the basis of the concept of magnetic field. In this case is undertaken only first member of expansion in the series \( \frac{v}{c} \). For the speeds \( v \sim c \) should be taken all terms of expansion. If we consider this circumstance, then the connection between the forces of interaction and the charge rates proves to be nonlinear. This, in particular it leads to the fact that the law of power interaction of the current carrying systems is asymmetric. With the identical values of currents, but with their different directions, the attracting forces and repulsion become unequal. Repulsive forces prove to be greater than attracting force. This difference is small and is determined by the equation

\[ \Delta F = \frac{I_1 I_2}{2c^2 \mu \varepsilon c^2} \frac{v_2^2 - v_1^2}{2c^2} \]

but with the speeds of the charge carriers of close ones to the speed of light it can prove to be completely perceptible.

Let us remove the lattice of upper conductor and after leaving only free electronic flux. In this case will disappear the forces \( F_1, F_2 \). In this case will disappear the forces of and, and this will indicate interaction of lower conductor with the flow of the free electrons, which move with the speed of on the spot of the arrangement of upper conductor. In this case the value of the force of interaction is defined as:

\[ F_{\Sigma} = \frac{g_1 g_2}{2 \pi \varepsilon r} \left( \frac{v_2 c}{c} - \frac{v_1 c - v_2 c}{c} \right) \tag{4.4} \]

Lorentz force assumes linear dependence between the force, which acts on the charge, which moves in the magnetic field, and his speed. However, in the obtained relationship the dependence of the amount of force from the speed of electronic flux will be nonlinear.

Let us note still one interesting result. Taking into account relationship (4.3), the force of interaction of electronic flux with the rectilinear conductor can be determined from the equation

\[ F_{\Sigma} = \frac{g_1 g_2}{2 \pi \varepsilon r} \left( \frac{v_2 c}{c^2} - \frac{v_1 c - v_2 c}{c^2} \right) \tag{4.5} \]

From Eq. (4.5) follows that with the identical direction the electron motions in the conductor and in the electronic flux with the fulfillment of conditions \( v_1 = \frac{1}{2} v_2 \) the force of interaction are equal to zero.

Since the speed of the electronic flux usually much higher than speed of current carriers in the conductor, the second term in the brackets in relationship (4.5) can be disregarded. Since

\[ H_1 = \frac{g_1 v_1}{2 \pi \varepsilon c^2 r} \]

we will obtain the magnetic field, created by lower conductor in the place of the motion of electronic flux:

\[ F_{\Sigma} = g_2 \mu v_2 H = g_2 \mu v_2 H \]

In this case, the obtained value of force exactly coincides with the value of Lorentz force.

Taking into account that

\[ F_{\Sigma} = g_2 E = g_2 \mu v_2 H \]

it is possible to consider that on the charge, which moves in the magnetic field, acts the electric field \( E \), directed normal to the direction of the motion of charge. This result also with an accuracy to of the quadratic terms of \( \frac{v^2}{c^2} \) completely coincides with the results of the concept of magnetic field and is determined Lorentz force.

As was already said, one of the important contradictions to the concept of magnetic field is the fact that two parallel beams of the like charges, which are moved with the identical speed in one direction, must be attracted. In this model there is no this contradiction.
already. If we consider that the charge rates in the upper and lower wire will be equal, and lattice is absent, i.e., to leave only electronic fluxes, then will remain only the repulsive force $F_2$.

Thus, the moving electronic flux interacts simultaneously both with the moving electrons in the lower wire and with its lattice, and the sum of these forces of interaction it is called Lorentz force. The sum of the forces of such an interaction is Lorentz force.

Regularly does appear a question, and does create magnetic field most moving electron stream of in the absence compensating charges of lattice or positive ions in the plasma? The diagram examined shows that the effect of power interaction between the current carrying systems requires in the required order of the presence of the positively charged lattice. Therefore most moving electronic flux cannot create that effect, which is created during its motion in the positively charged lattice.

5. Conclusion

Is shown that the dependence of the scalar potential of charge on the speed is the physical cause of Lorentz force. And this result should be considered new law. These results are obtained under the assumption the validity of the conversions of Galileo. It is shown that the dependence of Lorentz force on the speed is nonlinear, as previously supposed. When the direction of the motion of charges in the conductors they coincide, the force of their interaction occurs less than when directions of motion are different.

References