

From Pulsating to Rotating Dissipative Light Bullet

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Abstract Dissipative light bullets can be considered as (3+1) D spatiotemporal dynamics of stable optical solitons in the three spatial dimensions, in addition to localization in the temporal domain. They share many common characteristics with other multi-dimensional phenomena and their study remain an open challenge. Here, we report semi-rotating and rotating light bullets from pulsating dynamic when a suitable ansatz function is chosen. Using the complex cubic-quintic Ginzburg-Landau equation model and thanks to a variational method known as the collective variable approach, we can observe stable pulsating light bullet. The influence of the initial conditions leading to an asymmetric rotating light bullet in the transverse plane, are investigated.

Keywords: *pulsating light bullet, rotating light bullet, complex Cubic-Quintic Ginzburg-Landau Equation, dissipative soliton, collective variable approach, spatial-Temporal Pulse, semi-rotating light bullet*

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1. Introduction

In nonlinear dissipative medium, the combined interplay between gain, loss, spectral filtering, dispersion, diffraction, and nonlinearity creates a light bullet which is confined in the three spatial dimensions and localized in the temporal domain [1]. The confinement and localization result mainly from a balance between dispersive and nonlinear conservative effects on the one hand and between nonlinear gain and nonlinear loss on the other hand [2]. These double balances are strongly coupled, providing unique properties to light bullets [3,4]. Light bullet is a localized structure in dissipative systems considered in nonequilibrium thermodynamics [5].

Unveiling the dynamics of (3+1) D spatiotemporal solitons gained great interest for delivering high-capacity information and exploring multi-dimensional nonlinear dynamics that widely exist in physics, chemistry, biology and materials science [6,7].

The (3+1) D spatiotemporal dissipative solitons are not simply extensions of a spatiotemporal optical field that have been studied for a long time in Hamiltonian systems. These dissipative systems are not isolated, but are kept in contact with an external source to promote energy transfer. It assumes that there are not only energy losses but also energy supply [1].

Light bullets can exist for any sign of chromatic dispersion [8] and have extended regions of stability. The existence of stable light bullets with complicated shapes

was demonstrated using numerical simulations in a dissipative medium described by the (3+1) D complex cubicquintic Ginzburg-Landau equation (3D CGLE) [8,9,10]. Recently, theoretical models have been developed for understanding the generation of (3+1) D spatiotemporal dispersions and strong nonlinear interactions [11]. The numerical simulations and theoretical challenges give rise to allow experimental observations and potential applications of these optical structures. In fact, advances in understanding of multimode linear telecommunications systems, computing, algorithms, and modeling multidimensional control of light-light and light-matter interactions create a strong foundation to address these challenges [12]. Recent investigations have focused in nonlinear photonic crystals, in particular on the fabrication, characterization and application of (3+1) D structures with properties and functionalities that are either difficult or virtually impossible to achieve with lower dimensional structures [13]. An experimental framework for real-time observation, however, is yet to be demonstrated [14], whereas the time-averaged measurements of (3+1) D solitons have recently been demonstrated by sampling the laser beam profiles [15] and manipulating the pump power [16].

Progress towards the search for (3+1) D spatiotemporal solitons or light bullets requires specific conceptual, analytical, numerical and experimental advances which pass through an appropriate physical model and the synthesis of an adequate material for the generation and propagation of spatiotemporal solitons [17]. Advances in both theoretical and experimental directions are needed,

but in this work we tackle the first of the two challenges. We present a semianalytical results that are based on the complex cubic-quintic Ginzburg-Landau equation (3D CGLE) model, which includes both spatial diffraction and temporal dispersion. This 3D CGLE model is useful to describe spatiotemporal dynamics and a vast variety of nonlinear phenomena in a variety of nonlinear dissipative systems [18], such as interactions observed in passively mode-locked laser cavities or transmission lines [18,19]. The (3+1) D complex cubic-quintic Ginzburg-Landau equation admits stationary and pulsating solutions [1,9,20,21], although all of them have been found numerically in specific regions of the equation parameters [19]. With the semianalytical methods [21-24], the domains of existence of stationary and pulsating solutions have also been studied extensively. Furthermore, there are no exact analytical soliton of 3D CGLE in our knowledge. Consequently, we can expect that the only way of finding light bullets is an entirely numerical procedure or semi-analytical methods.

That being said, numerical simulations of the 3D CGLE for a given set of parameters and a given initial condition is tedious and time consuming with the standard computers [8]. As the 3D CGLE has different type of solutions (unstable, stationary, pulsating,...), the task becomes more tedious when we explore the regions of existence for light bullets as well as the unveiling of their complex dynamics. Like that, the study of the light bullet remains a very hard problem to tackle in its generality and requires enormous computer power. To overcome this complication, semi-analytical methods based on various physical backgrounds can be useful to perceive 3D CGLE solutions more efficiently and envisage their parameters domains of interest. Recently, thanks to a variational method known as the collective variable approach, we highlighted a rich variety of solutions of (3+1) D spatiotemporal solitons that include stationary, pulsating [21,22] and a new type of breathing solutions[25]. One of the precious advantages of this collective variable method is that it makes it easy to make more complex dynamics evident.

However, only full computations can assert or cancel the dynamics found in such a way. In the present study, using the collective variable approach and a suitable trial function, we find a rich variety of pulsating light bullet. The present work is planned as follows. Section 2 presents the model of our study and semianalytical methods known as the collective variable approach. The section 3 relates to the propagation of pulsating light bullet. The influence of initial condition on the dynamic of rotating light bullet is discussed in Section 4. And finally the section 5 is devoted to some concluding remarks.

2. Model of Study

In our semi-analytical study, we consider the propagation of light bullets in a system described by an extended complex cubic-quintic Ginzburg-Landau equation model. This model has been used successfully in a number of studies to provide a wide range of physical phenomena in dissipative systems observed in passively mode-locked laser cavities or transmission lines. It includes cubic and quintic nonlinearities of dispersive and dissipative types,

and transverse diffraction effects, to take into account theoretical and numerical grounds for the development of “dissipative light bullet” experiments. The propagation equation reads [2,5]:

$$\begin{aligned} \psi_z - i \frac{D}{2} \psi_{tt} - \frac{i}{2} \psi_{xx} - \gamma \frac{i}{2} \psi_{yy} - i |\psi|^2 \psi - i \nu |\psi|^4 \psi \\ = \delta \psi + \beta \psi_{tt} + \varepsilon |\psi|^2 \psi + \mu |\psi|^4 \psi \end{aligned} \quad (1)$$

Equation (1) is written in normalized form. The coefficients D , γ , ν , δ , β , ε and μ are real constants. The physical meaning of each term depends on the real problem that must be examined. ψ represents the optical envelope and is a complex function of four real variables $\psi = \psi(x, y, t, z)$, where t is the retarded time in the frame moving with the pulse, z is the propagation distance, and x and y are the two transverse coordinates. The left-hand-side of equation (1) contains the conservative terms, namely D denotes the dispersion propagation regime, being anomalous when $D > 0$ and normal $D < 0$, γ represents the Kerr nonlinearity coefficient and ν which represents, if negative, the saturation coefficient of the Kerr nonlinearity. Dissipative terms are written on the right-hand-side of equation (1). The meaning of the corresponding parameters is the following: δ is the coefficients for linear loss (if negative), ε represents the nonlinear gain (if positive), β denotes the spectral filtering (if positive) and μ is the saturation of the nonlinear gain (if negative).

By observing, equation (1) is nothing but a natural extension of the one-dimensional CGLE, and admits a variety of localized solutions, from stationary to pulsating, as well as period-doubling bifurcation [25]. As the 3D CGLE has several parameters that define the existence of stable solutions, the task seems extremely difficult for a given set of parameters and an initial condition.

One of the possibilities to overcome this complexity, is to resort to a semi-analytical approach.

The aim of this method is to reduce an infinite-dimensional to an ordinary differential equation (ODE), which can be solved numerically with relative ease. The semianalytical approach is based on a trial function theory with a finite number of parameters, as a result, the resulting dynamical system controls the evolution of parameters such as the light bullet amplitude, width, and chirp. In practice, the principle of the semi-analytical approach technique consists to associate collective variables [26] with the pulse's parameters of interest for which equations of motion may be derived. The objective being to simplify the characterization of a light bullet by using a lowdimensional equivalent mechanical system based on a finite number of degrees of freedom. Each degree of freedom can then be represented by a coordinate called collective variable (CV). It's an effective tool for a significant reduction in the number of used variables for the description of the soliton dynamics. In [27] using the same technique we carried out the dynamical behavior of pulsating solitons in the two-dimensional Complex Swift-Hohenberg equation. The basic principle consists in decomposing the optical field $\psi(x, y, t, z)$ in the following way [26,28]:

$$\psi(x, y, t, z) = f(X_1, \dots, X_N, x, y, t) + q(x, y, t, z), \quad (2)$$

where f , the trial function is a function of the CVs. The trial function is a simple analytic function that has amplitude and phase profile, is chosen according to the propagation equation and, at best, the configuration of the optical pulse. The component $q(x, y, t, z)$ is a residual field that represents all other excitations in the system (noise, radiation, dressing field, etc.). The option of precise form of the trial function which introduces the collective variables in the theory is crucial for obtaining solutions with the desired properties, especially when approximations are made.

Here according to our previous study [21], and in order to describe rotating dynamics, we choose the following Gaussian ansatz function:

$$f = A \exp \left(-\frac{t^2}{w_t^2} - \frac{x^2}{w_x^2} - \frac{y^2}{w_y^2} - s_1 xy + \frac{i}{2} c_t t^2 + \frac{i}{2} c_x x^2 + \frac{i}{2} c_y y^2 + \frac{i}{2} s_2 xy + if \right) \quad (3)$$

where A , w_t , w_x , w_y , s_1 , s_2 , c_t , c_x , c_y and f represent the collective variables: A stands for the light bullet amplitude, w_t , w_x and w_y are related to the temporal and the transverse widths along x and y axis respectively. c_t , c_x , and c_y are respectively the temporal and the transverse chirp parameters, and f is the global phase that evolves along with propagation.

$$\begin{aligned} \dot{A} &= A\delta + \frac{7}{16}\sqrt{2}A^3\varepsilon - \frac{2\beta A}{w_t^2} + \frac{2}{9}\mu\sqrt{3}A^5 \\ &\quad - \frac{1}{2}Ac_tD - \frac{1}{2}Ac_x - \frac{1}{2}Ac_y \\ \dot{w}_t &= w_t c_t D - \frac{1}{8}\sqrt{2}w_t A^2\varepsilon - \frac{2}{27}\mu\sqrt{3}A^4w_t \\ &\quad + \left(4 - w_t^4 c_t^2\right) \frac{\beta}{2w_t} \\ \dot{w}_x &= w_x c_x - \frac{1}{8}\sqrt{2}w_x A^2\varepsilon - \frac{2}{27}\mu\sqrt{3}A^4w_x + \frac{1}{4}s_1 s_2 w_x^3 \\ \dot{w}_y &= w_y c_y - \frac{1}{8}\sqrt{2}w_y A^2\varepsilon - \frac{2}{27}\mu\sqrt{3}A^4w_y + \frac{1}{4}s_1 s_2 w_y^3 \\ \dot{s}_1 &= -s_1 c_x - s_1 c_y + \frac{1}{4}\sqrt{2}\varepsilon A^2 s_1 \\ &\quad + \frac{4}{27}\sqrt{3}\mu A^4 s_1 - \frac{s_2}{w_y^2} - \frac{s_2}{w_x^2} \\ \dot{c}_t &= -Dc_t^2 - \frac{\sqrt{2}A^2}{2w_t^2} + \frac{4D}{w_t^4} - \frac{8\sqrt{3}\nu A^4}{27w_t^2} - \frac{8\beta c_t}{w_t^2} \\ \dot{c}_x &= s_1^2 - c_x^2 - \frac{s_2^2}{4} - \frac{\sqrt{2}A^2}{2w_x^2} + \frac{4}{w_x^4} - \frac{8\sqrt{3}\nu A^4}{27w_x^2} \\ \dot{c}_y &= s_1^2 - c_y^2 - \frac{s_2^2}{4} - \frac{\sqrt{2}A^2}{2w_y^2} + \frac{4}{w_y^4} - \frac{8\sqrt{3}\nu A^4}{27w_y^2} \\ \dot{s}_2 &= -s_2 c_x - s_2 c_y - \frac{1}{2}\sqrt{2}A^2 s_1 \\ &\quad - \frac{8}{27}\sqrt{3}\mu A^4 s_1 + \frac{4s_1}{w_y^2} + \frac{4s_1}{w_x^2} \\ \dot{f} &= \beta c_t + \frac{7\sqrt{2}A^2}{16} - \frac{D}{w_t^2} - \frac{1}{w_x^2} - \frac{1}{w_y^2} + \frac{2\sqrt{3}\nu A^4}{9} \end{aligned} \quad (4)$$

The parameters s_1 and s_2 , which we shall call *rotating parameters*, are introduced in the trial function (Eq. 3) to tie the transverse widths in order to have rotating dynamics. The choice of trial function is the first step for the collective variable approach. The second step of characterization would be to carry out a variational analysis neglecting the residual field $q(x, y, t, z)$, this approximation is called the bare approximation [26]. As is the case in most practical studies, one can consider the fact that the pulse propagation can be completely characterized by the ansatz function ($\psi(x, y, t, z) = f$) by neglecting the residual field.

Using the bare approximation to the 3D CGLE, as in our previous studies (see all the details in [21,22,26,27,29]) we get the ten collective variables that evolve according to the following set of ten coupled ordinary differential equations. This variational equations give us the first idea on the dynamic of the light bullet without having to know the exact pulse field $\psi(x, y, t, z)$.

The equations (Eq. 4) are usually functions of time that evolve subject to the constraints of the system and finally converge to a fixed point or a limit cycle. One of the major benefit of the semi-analytical approach lies in the fact that the pulse propagation can be completely characterized without having to solve the exact equation (Eq. 1) 3D CGLE. For example, in [30,31] applying this method we investigated the phenomenon of dissipative soliton resonance found earlier using numerical simulations and confirmed by the method of moments. The second major benefit of the semianalytical analysis is that it gives a detailed qualitative picture of the role and mode of action of each term of the 3D CGLE. It helps to see clearly the influence of each equation (Eq. 1) parameters on the various physical parameters of the light bullet. We notice that the evolution of the amplitude is dominated by the linear loss (δ), the nonlinear gain (ε) and its saturation (μ), as well as that the terms of spectral filtering (β) and dispersion term (D). It appears distinctly that the Kerr nonlinearity coefficient (γ) does not act explicitly on the amplitude. The progression of temporal parameters namely the temporal width (w_t) and chirp (c_t) are deeply affected by the dispersion term and the spectral filtering. Nonetheless, the nonlinear gain and its saturation, and the saturation coefficient of the Kerr act differently on these temporal parameters. The temporal widths are impacted only by the nonlinear gain and its saturation, and the transverse chirp parameters by the saturation coefficient of the Kerr. We point out that *rotating parameters* s_1 and s_2 act only on the spatial magnitude (w_x , w_y , c_x and c_y) and none of the soliton's parameters are influenced by f , the global phase. The *rotating parameters* s_1 and s_2 , absolutely necessary in this present study are influenced by the nonlinear gain (ε) and its saturation (μ).

3. Pulsating Light Bullet

The 3D CGLE admits stationary and pulsating solutions in the parameter space of the (3+1) D complex cubic-quintic Ginzburg-Landau equation. In [21] we predicted approximately the domains of existence of stable light bullets using Gaussian trial function which admits asymmetric pulse shapes in the transverse plane (x, y). A major goal of our study is to provide a quick pulsating

light bullet solution in the parameter space of the (3+1) D CGLE using the trial function with rotating parameters (Eq. 3). The stable solutions correspond to the stable fixed points of the system, obtained from the ordinary differential equations (Eq. 4). The unstable fixed points can be divided in two categories corresponding to the limit-cycle attractor and the unstable solutions. The dynamic of the pulse in the limit-cycle attractor area matches to the pulsating solutions of the (3+1) D complex cubic-quintic Ginzburg-Landau equation. The pulsating solutions possess inherent stability the same way as stationary stable solutions do. In our quest to find pulsating light bullets of the (3+1) D CGLE, we fixed the values of the following parameters $\beta=0.1$, $\gamma=1$, $\mu=-0.1$, $\delta=-0.4$, $\nu=-0.0$ and varied the two remaining parameters (D and ε), according to our previous study [21]. To carry out our simulations, we chose the following initial condition: $A=2.8697$, $w_t=2.8117$, $w_x=1.1473$, $w_y=1.1473$, $s_1=0$, $c_t=-0.0061$, $c_x=-0.0011$, $c_y=-0.0011$ and $s_2=0$ corresponding to a fixed point and affected by a slight perturbation corresponding to the photon noise in the system. Thus, for each pair of (D , ε) values, the use of the Newton-Raphson method helps to know if it corresponds to a limit-cycle. An illustration of the evolution of pulsating light bullet is presented in Figure 1 for the following values of the parameters $\varepsilon=0.59$ and $D=6$. The behaviour of pulsating solutions are general feature of most nonlinear dissipative systems and can be attributed of limit-cycle attractor.

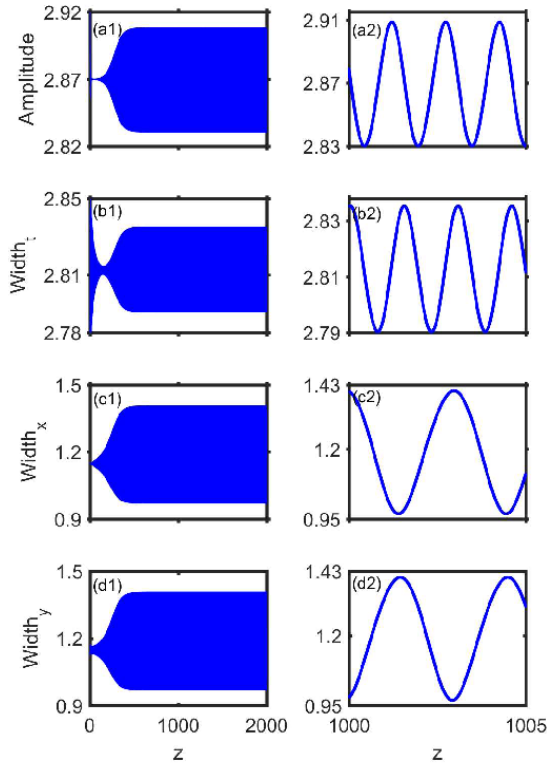


Figure 1. Radially asymmetric pulsations of pulsating light bullet obtained with the use of the semi-analytical method and the trial function. The evolution of the light bullet's parameters with respect to z is shown: (a1) the amplitude A , (b1) the temporal width w_t , (c1) the radial width w_x along x and (d1) the radial width w_y along y . The onset of stable harmonic pulsations appears clearly on enlarged views at right (a2)-(d2)

The dynamic of this pulsating light bullet starts with small oscillations right after the initial condition. These oscillations in the z direction gradually develop in amplitude (a1), temporal width (b1), and spatial widths (c1) and (d1), and become stationary at $z > 400$. The enlarged views of oscillations (a2)-(d2) show a nearly permanent harmonic evolution of the parameters that run between two fixed values. One can clearly notice that the x and y oscillations are out of phase and have the same amplitude. Likewise, the temporal width quickly evolves with respect to the spatial width. The light bullet periodically plays out of phase consecutive contractions in the x and y directions, while keeping almost constant its peak amplitude. The temporal and spatial chirps oscillations can be seen from the evolution plot in the Figure 2. They qualitatively describe the same dynamics as the temporal and spatial widths, characteristic of the pulsating light bullet.

This pulsating act has comparatively the same dynamics that we pointed out numerically in our previous studies [25,27]. Likewise, in [32,33] the authors experimentally investigated the dynamical diversity of pulsating solitons from single-soliton to multi-soliton in a fiber laser.

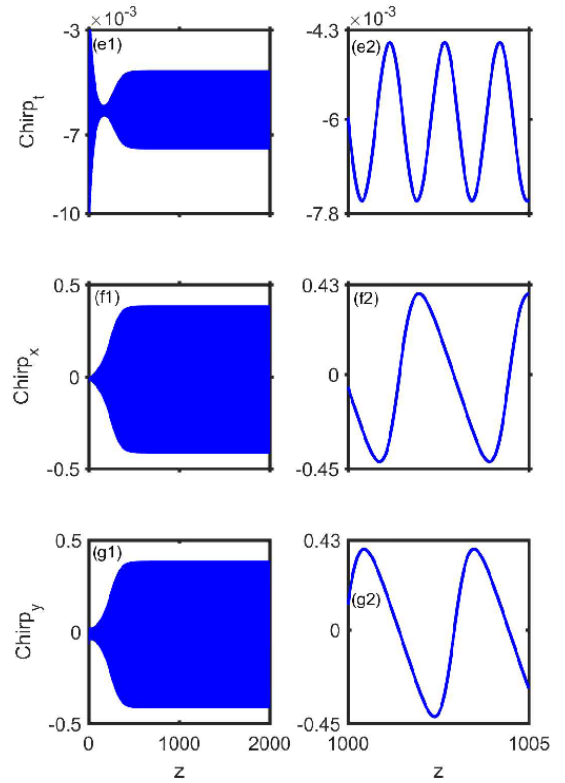


Figure 2. Radially asymmetric pulsations of pulsating light bullet obtained with the use of the semi-analytical method and the trial function. The evolution of the light bullet's parameters with respect to z is shown: (e1) the temporal chirp c_t , (f1) the radial chirp c_x along x and (g1) the radial chirp c_y along y . The onset of stable harmonic pulsations appears clearly on enlarged views at right (e2)-(g2)

4. Rotating Light Bullet

One of the useful parameters which can help to control the dynamic of the pulse in dissipative systems is the total

energy Q given by the three dimensional integral over $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{t}$:

$$Q(z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |\psi(x, y, t, z)|^2 dx dy dt \quad (5)$$

For dissipative systems, this quantity evolves according to the state of solutions; it can be stationary, pulsating or tend towards infinity.

Thereby, the total energy Q quantitatively and qualitatively provides the main information about the dynamic of the light bullet. Moreover an obvious advantage of the collective variable approach also lies in the fact that it helps to express the total energy with respect to the light bullet's parameters. In this way, we can easily reveal the total energy as a function of light bullet's parameters, like this:

$$Q(z) = \pi\sqrt{2\pi} \frac{A^2 w_t w_x w_y}{2\sqrt{4 - w_t^2 w_x^2 w_y^2}} \quad (6)$$

The above expression shows that the light bullet energy Q strongly depends on its amplitude A , temporal w_t and spatial widths w_x, w_y . The choice of the trial function is therefore very essential as it controls quantitatively and qualitatively the behaviour of the light bullet.

4.1. Semi-rotating Light Bullet

The *rotating parameters* S_1 and S_2 evolution (d), the total energy Q (a), temporal width $Width_t$ (b), and radially asymmetric pulsations $Width_x$ and $Width_y$ (c) obtained with the use of the collective variable approach and the trial function for the following values of the parameters $\beta = 0.1$, $\gamma = 1$, $\mu = -0.1$, $\delta = -0.4$, $\nu = -0.08$, $\varepsilon = 0.59$, and $D = 6$ are presented in Figure 3.

Small oscillations appear right after starting the evolution and become stationary at $z > 400$. The (a2)–(d2) represent a close-up view of the oscillations. We note the total energy Q oscillates between two fixed values, with the same number of periods as the temporal width w_t . However, the *rotating parameters* S_1 and S_2 remain constant (fixed at zero) throughout the propagation.

We can remark that the radial width ($Width_x$ and $Width_y$) oscillations are out of phase and have the same amplitude. It appears that the radial widths oscillate slowly while the amplitude of the light bullet oscillates quickly. This points out periodic out of phase consecutive contractions of the light bullet in the x and y directions as shown in the Figure 4. It follows an oscillating and semi-rotating motion of the light bullet - antisymmetric mode inflation-deflation. The transition from position I corresponding at the distance 923.9323 to the position IV corresponding at the distance 949.9141 is done according to three distinct dynamics: the light bullet plays pulsating motion, stretches along the x axis (in position II) and along the y axis (in position III) and a semi-rotating motion. The semi-rotating dynamic takes place between positions I and IV, without the light bullet making a complete rotation, because the radial widths ($Width_x$ and $Width_y$) are not linked, while $S_1 = S_2 = 0$.

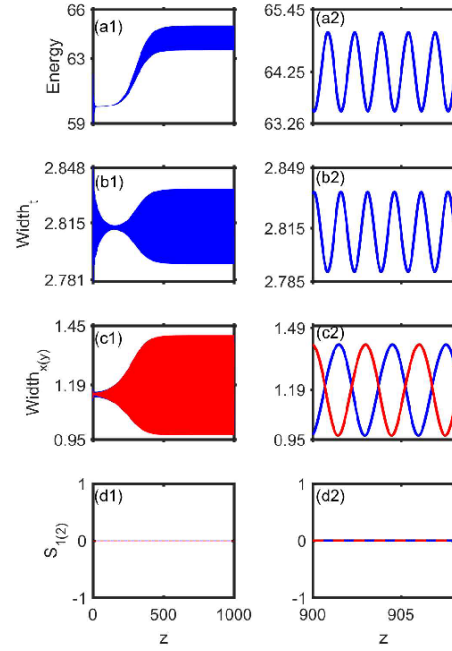


Figure 3. Sample asymmetric pulsating dissipative light bullet. (a1) Evolution of the total pulse energy with respect to z , showing the onset of pulsations on enlarged views at right (a2). (b1) Evolution of the temporal width w_t and (c1) the radial widths $w_{x(y)}$ along x and y . The enlarged views of those pulsations are plotted in (a2)–(d2). The pulsations along the x and y axes are out of phase and share the same amplitude. (d1) Evolution of the parameters $S_{1(2)}$. $S_{1(2)} = 0$ and remain at 0 throughout the propagation

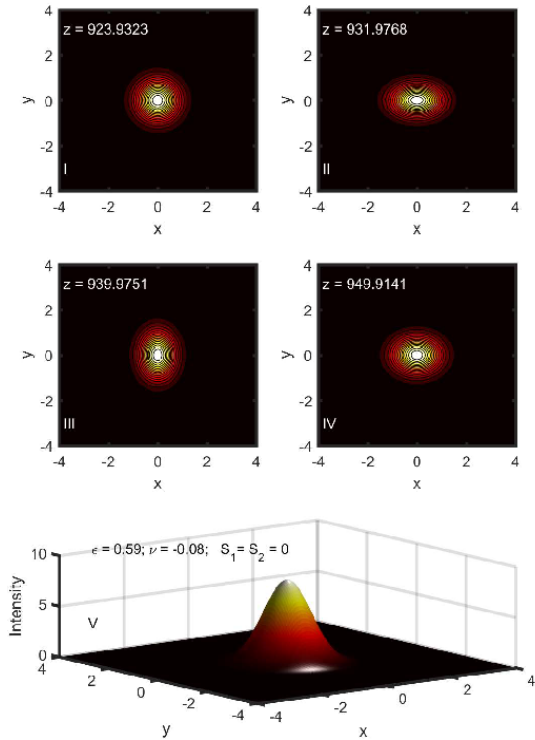


Figure 4. Semi-rotating pulsating dissipative light bullet in different position in (x, y) plane during its motion. V Spatial profile of semi-rotating light bullet reconstructed from the collective variable approach. Corresponding for the following values of the parameters $\beta = 0.1$, $\gamma = 1$, $\mu = -0.1$, $\delta = -0.4$, $\nu = -0.08$, $\varepsilon = 0.59$, and $D = 6$. And the following initial condition $A = 2.8697$, $w_t = 2.8117$, $w_x = 1.1473$, $w_y = 1.1473$, $s_1 = 0$, $c_t = -0.0011$, $c_x = -0.0011$, $c_y = -0.0011$, and $s_2 = 0$

4.1. Rotating Light Bullet

When keeping all the parameters corresponding to the Figure 3 constant and varying slightly S_1 and S_2 ($S_1 = S_2 \neq 0$) but $S_1 = S_2 \approx 0$, the overall dynamic of the light bullet changes.

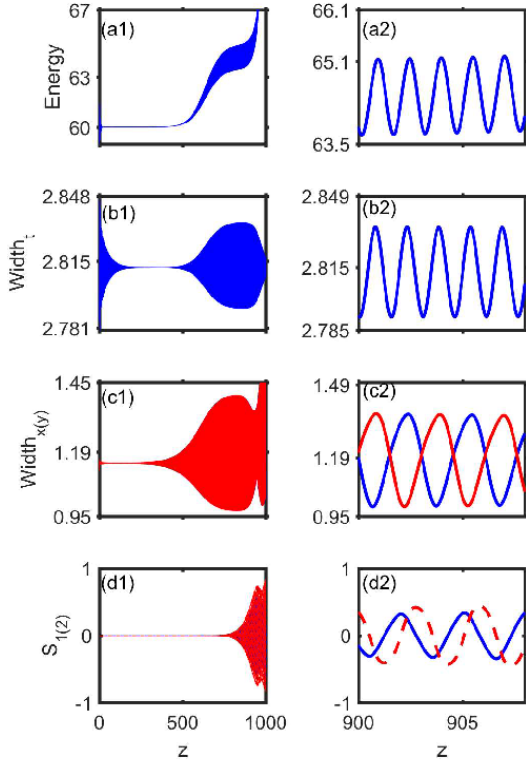


Figure 5. Sample rotating pulsating dissipative light bullet. (a1) Evolution of the total pulse energy with respect to z , showing the onset of pulsations on enlarged views at right (a2). (b1) Evolution of the temporal width w_t and (c1) the radial widths $w_{x(y)}$ along x and y . The enlarged views of those pulsations are plotted in (a2)-(d2). The pulsations along the x and y axes are out of phase and share the same amplitude. (d1) Evolution of the parameters $S_{1(2)}$. $S_{1(2)}$ starts at 0 and oscillates when the light bullet undergoes a rotation

The stable pulsating light bullet becomes unstable and quickly undergoes a collapse. Before the collapse we witness a transitory phase during which the light bullet oscillates while turning on itself, that we call *pulsating rotating light bullet*. As shown in Figure 5, the (a2)-(d2) represent a close-up view of oscillations. As illustrated in Figure 5, which represents the same situation as in Figure 3, except the parameters S_1 and S_2 which have been modified for $S_1 = S_2 = 3 \cdot 10^{-7}$. The dynamic of the light bullet begins with a stationary phase up to a distance $z \approx 500$. Subsequently, the total energy increases and the light bullet switches dynamics with quasi-stable oscillations and finally collapses. Prior to the collapse, the parameters S_1 and S_2 also begin to oscillate, and hence bind the radial widths, which drives the impulse into a quasi-stable rotating motion over a short distance. The pulsating rotating dynamic comes from the change of the parameters S_1 and S_2 . Note that the collapse is characterized by a crumbling of the temporal width (b1) while the radial widths increase (c1) as well as the total energy (a1). During the pulsating phase, S_1 and S_2 remain fixed at 0 and the radial widths are out of phase with periods oscillation less than that of the temporal width. To

clearly highlight the rotating motion, we are interested in the dynamics in the (x, y) plane, this situation is summarized in Figure 6. One can notice that the light bullet occupies different positions at specific distances.

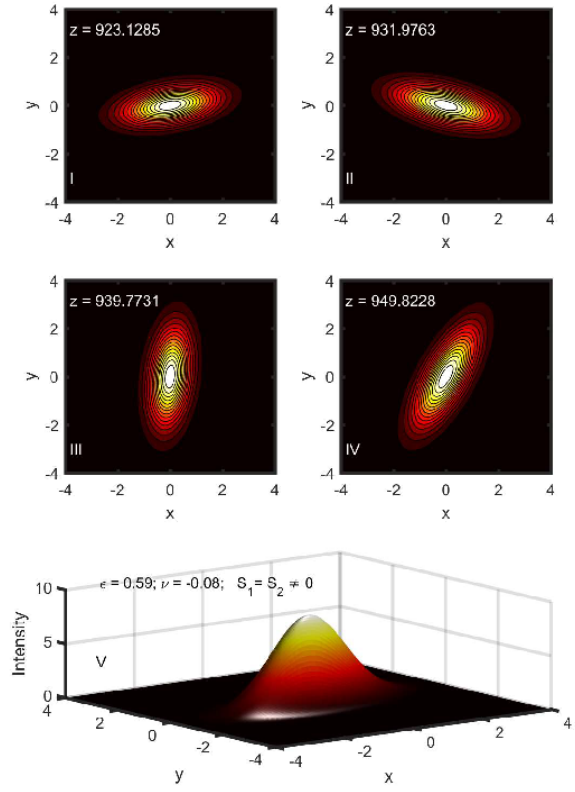


Figure 6. Rotating pulsating dissipative light bullet in different position in (x, y) plane during its motion. V Spatial profile of rotating light bullet reconstructed from the collective variable approach. Corresponding for the following values of the parameters $\beta = 0.1$, $\gamma = 1$, $\mu = -0.1$, $\delta = -0.4$, $\nu = -0.08$, $\varepsilon = 0.59$ and $D = 6$. And the following initial condition $A = 2.8697$, $w_t = 2.8117$, $w_x = 1.1473$, $w_y = 1.1473$, $s_1 = 0$, $c_t = -0.0061$, $c_x = -0.0011$, $c_y = -0.0011$ and $s_2 = 3 \cdot 10^{-7}$

From position I to position IV, it performs a rotation in the (x, y) plane. During the rotating, it also plays pulsating, we are therefore seeing a *pulsating rotating light bullet*.

5. Conclusion

We have demonstrated efficient and quick approximate solutions in a dissipative medium described by the (3+1) D complex cubic-quintic Ginzburg-Landau equation. Using a semianalytical method known as the collective variable approach, we predict semi-rotating and rotating pulsating light bullet when a suitable trial function is chosen. The validity of this study is based on the choice of the trial function and comparison with limited number of direct calculations is required. Nevertheless, the collective variable approach is incomparably quicker than direct numerical computations. As well, this approach helps to reveal elementary types of pulsating light bullet: asymmetric *inflating-deflating* and *pulsation-rotation*. The simplicity of the method is related to arbitrary choice of ansatz function which helps to find specific solutions. Of course, these must be at the final stage confirmed, complemented or invalidated by numerical studies.

Pulsating rotating light bullet needs to be confirmed experimentally, and no doubt they will have potential applications to parallel pre-processing and dynamical routing of optical data, and also in various fields in physics, biology and chemistry where there are also pulsating.

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