Dependence between Non-Energy Commodity Sectors Using Time-Varying Extreme Value Copula Methods

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Abstract

In this work, our objective is to study the intensity of dependence between six non-energy commodity sectors in a bivariate context. Our methodology is to choose, in a first step, the appropriate copula flowing Akaike criteria. In a second step, we aim to calculate the dependence coefficients (Kendall’s tau, Spearman’s rho and tail dependence) using filtered data by the $AR(1)-GARCH(1,1)$ model to study the dependence between the extreme events. Empirical results show that dependence between non-energy commodity markets increases during volatile periods but they offer many opportunities to investors to diversify their portfolio and reduce their degree of risk aversion in bearish market periods.

Keywords: non-energy commodity, dependence structure, copula, diversification, time-varying correlations


1. Introduction

The international financial markets have become closely integrated since regulations and barriers have been gradually removed over the past years so that people in different parts of the world can invest into the markets of other countries. This provides investors an opportunity to optimize portfolios by higher returns and lower risk. But this makes the financial markets become more dependent to each other and the system more complex.

However, over the last few decades international financial markets have experienced a succession of serious crisis of different causes and origins. For example, the 2007-2010 global financial crises which originated in the United States was sparked by the subprime real estate crisis, and then turned into a world financial crisis. Most of these crises are characterized by high volatility and contagion (Forbes and Rigobon, 2002). Moreover, recent studies suggest that these crises stoked greater correlations between the world’s equity markets, in particular in periods of high and extreme volatility, and thus lowered the diversification benefit potential from investing in traditional stocks.

The highly volatility and widespread contagion have prompted investors to consider alternative investment instruments as a part of diversified portfolios in order to be used as a hedge to diversify away the increasing risk in the stock markets. Since the early 2000s, commodities have emerged as an additional asset class beside traditional ones such as stocks and bonds. Many researchers, using data from before the 2000s, have found a little negative return correlation between commodity and stock returns (Greer, 2000; Gorton and Rouwenhorst, 2006). Return correlations among commodities in different sectors have also been found to be small (Erb and Harvey, 2006). Moreover, several papers have reported decreasing movements of return correlations between commodities and stocks at least before the recent financial crisis (Chong and Miffre, 2010).

These characteristics of commodity returns implied an opportunity for diversification and, thus, have attracted investors worldwide. Therefore, various instruments based on commodity indices have attracted billions of dollars of investment from institutional investors and wealthy individuals. The increasing presence of index investors precipitated a fundamental process of financialization amongst commodities markets, through which commodity prices became more correlated with prices of financial assets and with each other.

As a result, time-varying correlations in commodity markets are becoming an important issue. These cross-commodity relationships imply that two or several commodities share an equilibrium that links prices in the long run. Examples of economic long-term relationships between commodities include production relationship where upstream commodity and downstream commodity are tied in a production process, and substitute (or complementary) relationships where two commodities serve as substitute (or complement) in either consumption or production.

The modeling of the co-movements of non-energy commodity has so far received little attention in the financial literature. Yet, it is a subject of considerable importance for the pricing, risk management, and optimization of portfolios composed of multi-commodity sectors. Indeed, the hedging strategies for multi-
commodity portfolios are based on futures contracts rather than spot transactions. As a consequence, a model describing the joint evolution of multi-commodity sectors should capture at the same time their global and local dependence structures.

Our aim in this paper is to examine the dependence structure between six non-energy commodity sectors (Metals & Minerals, Fats & Oils, Grains, Other Food Items, Beverages and Agricultural raw materials) using the concept of copula to capture correlation among non-energy commodity sectors.

The remaining part of the paper is organized as follows: Section two is a review of some previous works. The third section explains the GARCH model used to model marginal distributions. The fourth section provides a brief review of the copulas theory. The fifth section describes the data and sample. The sixth section presents empirical results. The last section delivers final remarks and conclusions.

2. Literature Review

It is a common perception that commodity prices tend to move together. This perception is especially common among commodity traders who may justify an increase in the price of one commodity because the prices of other commodities have increased. It is not surprising that commodity prices move in synchronization because many of them are influenced by common macroeconomic fundamentals such as inflation, interest rates and industrial production. Related commodities are also complements (e.g., oil and metals) and substitutes (e.g., gold and silver) in consumption, and inputs (e.g., oil, copper and silver) in the production of others.

But there is no a priori reason for believing that prices of unrelated commodities should move together, expect for macroeconomic shocks affecting commodities markets in general.

For example, in a recession commodity prices decline across the board because demand declines, and in periods of general inflation commodity prices rise, partly because commodities provide a hedge against inflation. However, after accounting for macroeconomic shocks, is co-movement among commodity prices still evident?. Since the founding paper of Pindyck and Rotemberg (1990), the relations between commodity prices through the co-movement analysis have been often studied. The authors find excess co-movement in prices of unrelated commodities among others. They explain that raw commodities may have a common trend because of direct effects (an increase in industrial production leading to an increase in industrial commodities' demand for the production process and non-industrial commodities' demand due to the increase of revenues) or indirect effects (through the expectations for commodities, affecting the storage process and then the current prices) resulting from macroeconomic changes. They include in their model macroeconomic variables (nominal interest rate, industrial production, consumer price index, etc.) in order to take into account common effects.

They conclude by explaining that excess co-movement can be due to: (i) herd behavior from the traders; (ii) liquidity constraints; (iii) frequency of data. Nevertheless, few studies have supported similar conclusions. On examination of the literature, we can discern three major areas of research on commodities including:

- Dependence between Commodities (energy or/ and non-energy) and macroeconomic variables (exchange rate, interest rate and index price)
- Dependence between crude oil and other commodities
- Dependence between non-energy commodities

2.1. Dependence between Commodities and Macroeconomic Variables

The literature has identified a set of common factors that explained the increasing of co-movement among commodity prices since the early 2000: basic macroeconomic variables include those related to demand (such as the industrial production of developed countries), variables related to the cost of supplies (such as gasoline and fertilizer prices) and variables related to the international situation (such as the effective exchange rate, the US real interest rate and the exchange rate), which may affect co-movement among commodities.

Indeed, all these fundamental factors have played a role, but there are also reasons to suspect that financial markets could have been partly responsible for the surge in co-movement among commodity prices (Table a. in annexe).

2.2. Dependence between Crude Oil and other Commodities

The research on crude oil is larger than other commodities because of their extensive use by many economic sectors. This works focused on the impact of fluctuations in the prices of strategic commodities such as crude oil on the prices of other commodity counterparts. Indeed, crude oil is an important input in production of many commodities (Table b. in annexe).

2.3. Dependence between Non-Energy Commodities

To the best of our knowledge, co-movement between non-energy commodities is rarely treated in multivariate context. In this sense, this work can perfectly contribute to this area of the literature (Table c. in annexe).

3. GARCH Model

Recent developments in financial econometrics propose the use of non-linear models. Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models, being the most widely used, and many studies have showed that GARCH models have many advantages in modelling financial returns since they can usually indicate volatility clustering. Volatility clustering is called heteroscedasticity in statistics, which means the variance changes with time and it is conditioned in the past (conditional). Traditional time series models are unable to capture the volatility clustering well, but GARCH models can use this information well.

As noted above, the tests in this study are based on the ARCH family of models developed by Engle (1982) and
generalized by Bollerslev (1986). Returns and conditional variances of financial assets are modelled to reflect the stylized facts observed on financial markets (presence of asymmetry, long memory, non-linearity and thick tails of distributions, etc.). For this reason, these models have been shown empirically to provide a good explication for many financial return series.

3.1. ARCH Model

The ARCH (q) is the fundamental model of the ARCH process proposed by Engle (1982) during a study on the variance of inflation in Great Britain. The model is based on a quadratic parameterization of the conditional variance. Engle (1982) expressed the conditional variance as a linear function of the past squared innovations. A process ARCH (q) is given by:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$$

(1)

The conditional variance will be positive if \( \alpha_0 > 0 \) and \( \alpha_i \geq 0 \) for \( i=1, 2,...q \).

3.2. GARCH Model

Bollerslev (1986) generalized the initial model of Engle by establishing GARCH (p, q) model. This extension consists of the introduction of lagged values of the variance in its equation. The GARCH model permits the conditional variance to be expressed as a linear function of lagged squared error terms and lagged conditional variance terms. Thus, it allows a more parsimonious description of the lag structure.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

(2)

Where \( \alpha_0 > 0 \) et \( \alpha_i \geq 0, \beta_j \geq 0, \) for \( i=1, 2,... p \).

Compared to the ARCH model, the GARCH model includes autoregressive (AR hereinafter) terms as well as moving average (MA hereinafter) terms. Adding (MA) terms means that the conditional variance is also a linear function of its own lags, which improves the performance of ARCH model. For GARCH model:

- The term autoregressive represents a feedback mechanism that incorporates past observations into the present.
- Conditional indicates that variance has a dependence on the past.
- Heteroscedasticity means a time-varying volatility.

In general, GARCH is a mechanism that uses past variances in the explication of future variances. More specifically, GARCH is a technique to model the serial dependence of volatility. The GARCH (p, q) model successfully captures several characteristics of financial time series, such as thick-tailed returns and volatility clustering. In the present paper, we consider the applicability of the AR(1)–GARCH (1.1) model under the hypothesis that innovations follow t-Student distribution with \( v \) degrees of freedom to estimate volatility for different non-energy commodity sectors. According to Bollerslev (1986), utilizing student-t distribution as the conditional distribution for GARCH model is more satisfactory since it shows thicker tail and larger kurtosis than normal distribution. The conditional mean and variance equations of AR-GARCH-t model can be expressed as

$$r_t = \mu + \phi r_{t-1} + \epsilon_t, \epsilon_t = \sqrt{h_t} z_t$$

with \( z_t \mid \Omega_{t-1} \sim iid \), \( h_t = w + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} \)

where \( r_t \) denotes return at time \( t \), \( \mu \) is the conditional mean at \( t \), \( h_t \) is the conditional variance at \( t \) and \( w, \alpha \) and \( \beta \) are non-negative parameters with the restriction that the sum \( \alpha + \beta \) are less than one to ensure the stationarity and the positivity of conditional variance as well. The log-likelihood function can be expressed as

$$L (r_t / \Omega) = \sum_{t=1}^{T} \ln \left( \frac{r_{t+1}/z_t}{\sqrt{\pi}} \right) - \frac{1}{2} \sum_{t=1}^{T} \ln \left[ 1 + \frac{r_{t+1}^2}{\pi} \right]$$

(3)

where \( \Omega = (\mu, \phi, \omega, \alpha, \beta, v) \) is the parameter vector of the AR-GARCH-t model.

The one-step-ahead conditional variance forecast \( \sigma_{t+1/t}^2 \) for the GARCH (p, q) model is given by

$$\sigma_{t+1/t}^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i+1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j+1}^2$$

(4)

The AR(1)–GARCH (1.1) with \( t \) innovations will be used to model marginal distributions and filtered the data before applying copula.

4. Copulas and Dependence Measures

4.1. Copula Theory

Copulas have become the attention multivariate modelling in various fields. A copula is a function that links together univariate distribution functions to from a multivariate distribution function (Patton, 2007).

Copula is simply the distribution function of random variables with uniform marginals. Given two or more marginal distributions (e.g. EVT) that model individual movement of random variables, a copula function describes how the distributions 'relate together' to determine the multivariate distribution. The ability of a copula to separate the dependence structure from the marginal behaviour in a multivariate distribution is due to Sklar’s theorem (1959) presented below.

Let \( X \) and \( Y \) be random variables with continuous marginals \( F_X = u \) and \( F_Y = v \). According to Sklar’s theorem, their joint distribution function \( F(x,y) \) can be written in terms of a unique function \( C(u,v) \) as \( F(x,y) = C(u,v) \).

The function \( C(u,v) \) is called the copula of \( F(x,y) \) and describes how \( F(x,y) \) is coupled with the marginal distribution functions \( F_X(x) \) and \( F_Y(y) \). According to this theorem, the selection of the dependence structure and the choice of the univariate distributions can be independent of each other. For instance, \( C(u,v) \) might be from Gumbel family of copulas, whereas \( F_X(x) \) might be normally distributed and \( F_Y(y) \) is a Gamma distributed variable. Selection of an appropriate model that characterizes the
dependence between X and Y can be independent of the choice of marginal distributions.

There are several classes of copula functions. In this paper, we consider Archimedean copulas in the bivariate case (Frank, BB7), meta-elliptical (normal) and extreme-value copulas (Gumbel, Galambos, Husler–Reiss and Tawn). Although Gumbel and normal copulas are widely used in finance, extreme-value copulas extend the univariate EVT technique.

Extreme-value copulas are the limits of copulas of component wise maxima in independent random samples, and provide appropriate models for the dependence structure between rare extreme events.

The Archimedean copula family has a simple closed form and can be stated directly. For \(-1 \leq \theta \leq \infty\), the Clayton copula is defined as

\[
C(u,v) = \max \left\{ -\left( u^{-\theta} + v^{-\theta} - 1 \right) \theta, 0 \right\}
\]

(6)

Note that Gumbel belongs to a family of both Archimedean and extreme-value copulas. Both Clayton and Gumbel are asymmetric copulas, and exhibit greater tail dependence. In Gumbel, the dependence is highly encountered in the positive tail rather than the negative one. On the other hand, Clayton copula shows greater dependence in the negative tail.

The bivariate Gumbel copula (so-called Gumbel–Hougaard copula) can be stated directly. For \(0 \leq \theta \leq \infty\), Galambos is an extreme-value copula and expressed for \(0 \leq \theta \leq \infty\) as

\[
C(u,v) = uv \exp \left( -\left( -\ln u \right)^\theta + \left( -\ln v \right)^\theta \right)^{-1/\theta}
\]

(7)

Let \(\varphi\) be cdf of a standard Gaussian distribution. The Husler–Reiss copula for non-negative values of \(\theta\) is obtained by

\[
C(u,v) = \exp \left[ -\vartheta \left( \ln u \right) \left( -\ln v \right) \right]
\]

(8)

where \(\vartheta = \ln(u)\) and \(\vartheta = \ln(v)\).

The elliptical copulas do not have a simple closed form. The normal (or Gaussian) copula is derived from a multivariate Gaussian distribution function \(\Phi\) with zero mean. The marginals are transformed by the inverse of \(\Phi\) as follows:

\[
C(u,v) = N_\theta \left( \phi^{-1}(u), \phi^{-1}(v) \right)
\]

(9)

There are generally two approaches for estimating parameters of a copula function. Full maximum likelihood is the most direct estimation method where all parameters are simultaneously estimated. The inference for margins (IFM) method is a two-stage approach. The marginals are estimated at the first stage. At the second stage, the estimated marginal distribution is substituted into the copula function and then the dependence parameter is estimated. An attractive feature of the IFM method is that one can obtain copulas for which the dependence structure is independent of the marginal distributions.

4.2. Dependence Measures

There exist three kinds of dependence measures, namely linear correlation, rank correlation and the coefficients of tail dependence. Linear (Pearson’s) correlation is the most frequently used practical tool for measuring dependence. Rank correlations and tail dependence coefficients are copula-based dependence measures and can thus be used in the parameterization of copulas unlike ordinary correlation.

4.2.1. Rank Correlation

A rank correlation coefficient measures the correspondence between two rankings and assesses its significance. Two well-established measures of rank correlation are Spearman’s (1904) rank correlation (Spearman’s rho) and Kendall’s (1938) rank correlation (Kendall’s tau).

Kendall’s tau (\(\tau\)) and Spearman’s rho (\(\rho\)) are used to measure concordance (dependence) between random variables. Let X and Y be random variables. Rank of a random sample of \(n\) pairs \((X_i, Y_i)\) for \(i=1,\ldots,n\) is represented by \(R_i=\text{rank}(X_i)\), and \(S_i=\text{rank}(Y_i)\). The Spearman’s rho (\(\rho_n\)) is expressed as

\[
\rho_n = \frac{12}{n(n+1)(n-1)} \sum_{i=1}^{n} R_i S_i - \frac{3n+1}{n-1}
\]

(10)

Kendall’s tau is defined in terms of the notion of concordant and discordant pairs. Two pairs \((X_i, Y_i)\) and \((X_j, Y_j)\) are said to be concordant when \((X_i, X_j)(Y_i, Y_j) > 0\) and discordant when \((X_i, X_j)(Y_i, Y_j) < 0\). Let \(P_n\) and \(Q_n\) be the number of concordant and discordant pairs, respectively. The Kendall’s tau (\(\tau_n\)) is formulated as follows

\[
\tau_n = \frac{P_n - Q_n}{n(n-1)} = \frac{4}{n(n-1)} P_n - \frac{1}{2}
\]

(11)

The rank-based measures (Kendall’s tau and Spearman’s rho) provide the best alternatives to linear correlation for measuring dependence of non-elliptical distributions. Linear correlation is only ideal for elliptical distributions while most random variables are not elliptically distributed.

Linear correlation is invariant under strictly linear transformation, but not under general or nonlinear transformations. The variances of X and Y must be finite or the linear correlation is not defined. For heavy-tailed distributions, linear correlation coefficient is not defined because of infinite second moments.

4.2.2. Tail Dependence

Tail dependence is frequently observed in financial and operational risk data. When extreme events occur, the response from the market is usually asymmetric. Tail dependence measures the amount of dependence for extreme co-movements in the lower-left quadrant tail or upper-right quadrant tail of a bivariate distribution. It is of
importance because it is relevant for the study of dependence between extreme values. Tail dependence expresses the probability of having a high (low) extreme value of a series given that a high (low) extreme value of another series has occurred. The higher probability of joint extreme observations as compared with the Gaussian (normal) copula is known as positive tail dependence. Normal and Frank copulas impose zero tail dependence whereas other copulas impose zero tail dependence in one tail. For instance, the Gumbel copula has upper left tail dependence, while the Clayton copula has lower right tail dependence. Gaussian copula on the other hand exhibits no tail dependence.

Let X and Y be random variables with distribution functions $F_1$ and $F_2$. By definition, the coefficient of (upper) tail dependence of X and Y is

$$\lambda_U = \lim_{\alpha \to 1} - P\left( Y > F_2^{-1}(\alpha) \mid X > F_2^{-1}(\alpha) \right)$$  \hspace{1cm} (12)

provided a limit $\lambda \in [0,1]$ exists. $\lambda_U$ can also be interpreted in terms of VaR with the probability level $\alpha$. If $\lambda \in (0,1]$ X and Y are said to be asymptotically dependent (in the upper tail) and if $\lambda = 0$ they are asymptotically independent; and similarly for the coefficient of lower tail dependence $\lambda_L$.

$$\lambda_L = \lim_{\alpha \to 0^+} P\left( Y < F_2^{-1}(\alpha) \mid X < F_2^{-1}(\alpha) \right)$$  \hspace{1cm} (13)

When $F_1$ and $F_2$ are continuous distributions and the limit exists, then we get simple expressions for $\lambda_U$ and $\lambda_L$ in terms of the unique copula $C$ of the bivariate distribution. Using elementary conditional probability, we have the lower tail dependence

$$\lambda_L = \lim_{\alpha \to 0^+} P\left( \frac{Y < F_2^{-1}(\alpha)}{X < F_2^{-1}(\alpha)} \right)$$  \hspace{1cm} (14)

$$\lambda_L = \lim_{\alpha \to 0^+} \frac{c(\alpha, \alpha)}{\alpha}$$

For upper tail dependence we obtain

$$\lambda_U = \lim_{\alpha \to 1^+} P\left( \frac{1 - \alpha, 1 - \alpha}{1 - \alpha} \right) = \lim_{\alpha \to 1} \frac{C(\alpha, \alpha)}{\alpha}$$  \hspace{1cm} (15)

where $\hat{C} = 1 - 2\alpha + C(\alpha, \alpha)$ is the survival copula of $C$. For radially symmetric copulas we must have $\lambda_L = \lambda_U$, since $C = \hat{C}$ for such copulas.

5. Data and Sample

The data used in this study is formed from the spot prices of a set of six non-energy commodity sectors:

<table>
<thead>
<tr>
<th>Metals &amp; Minerals</th>
<th>Agricultural Raw Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum, Copper, Iron ore, Lead, Nickel, Tin and Zinc</td>
<td>Timber and Other Raw Materials (Cotton, Rubber and Tobacco)</td>
</tr>
<tr>
<td>Fats &amp; Oils</td>
<td></td>
</tr>
<tr>
<td>Grains</td>
<td></td>
</tr>
<tr>
<td>Other Foods Items</td>
<td></td>
</tr>
<tr>
<td>Beverages</td>
<td></td>
</tr>
<tr>
<td>Agricultural Raw Materials</td>
<td></td>
</tr>
</tbody>
</table>

The analysis was conducted using the monthly time series data from the period January 31, 1960 to April 30, 2014, which yielded 652 observations for each sector. The data were taken from the Database of the World Bank.

The continuously compounded monthly returns are calculated using the following logarithmic filter:

$$r_{i,t} = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right) \times 100$$

Where $r_{i,t}$ and $P_{i,t}$ denote the monthly return in percentage and the monthly price of index $i$ on day $t$, respectively. Figure 1 depicts the monthly movements in the six non-energy commodity price indexes from January 1960 to April 2014. The figure clearly shows that the price indices vary over time. However, the past decade was characterized by large fluctuations in commodity prices. First, the figure indicates that the behaviours of indexes are affected by major events, such as the subprime mortgage crisis. The increase in commodity prices was followed by a steep decline during the financial crisis in the second semester of 2008. Volatility rose sharply during the fall in commodity prices during 2008. Since the first quarter of 2009, the prices of all of the indexes have increased:

**Metals & Minerals** returns generally began increasing sharply in late 2003 due to strong demand, earlier closures of production capacity, declining stocks, and depreciation of the U.S. dollar.

**Fats & oils returns** raised in 2004, mainly due to drought in Europe and North America, as well as strong demand in China. However prices have since receded with favorable returns, but remain somewhat elevated.

**Food** returns have spiced the past couple of years, mainly for **grains** and other food items notably sugar. Grains prices have risen in part due to higher prices for fertilizers and other inputs.

**Beverage** returns have been the most volatile in the past, but supplies are adequate at present, partly due to emergence of new producers, e.g., coffee in Vietnam. Cocoa, coffee and tea prices are all up from their recent lows on various supply problems, e.g., drought in Kenya that has contributed to higher Mombasa tea auction prices.

**Agricultural Raw materials** returns have also risen, but this has mainly the result of a sharp rise in rubber prices because of high oil prices which have raised the costs of synthetic rubber, its main competitor. The price of the other main Agricultural raw material commodity such cotton is relatively subdued due to well supplied markets.

The descriptive statistics for all of the monthly return series are reported in Table 1. These statistics including mean (Mean), standard deviations (Std. dev.), maximum (Max), minimum (Min), skewness (Skew.), and kurtosis (Kurt.). ARCH refers to the empirical statistics of the statistical test for conditional heteroscedasticity of order
six. LB is the empirical statistics of the Ljung–Box tests for autocorrelations applied to raw return series. JB are the empirical statistics of the Jarque–Bera test for normality based on skewness and excess kurtosis. We can find that the mean values of returns are much close to zero, whereas the standard deviations are much larger (expect for Agricultural Raw Materials sector). For each return series, the Jarque–Bera statistic shows the rejection of the null hypothesis of Gaussian distribution at 5% significance level, suggesting the fat-tail distributions. This finding is confirmed by positive excess kurtosis.

We employ the GARCH model proposed by Bollerslev which examines past shocks and volatility on the current conditional volatility. In the case of non-energy commodities, Table 2 shows that volatility process is strongly persistent, meaning that volatility (β) impact current conditional volatility. Moreover, their sums (α + β) are very close to unity which indicates long memory. We also examine residuals produced by using copula method; we follow a three-step procedure. *Shaded numbers indicate the rejection of the null hypothesis of associated statistic tests at the 5% level.

Table 2. Descriptive statistics

<table>
<thead>
<tr>
<th>Index</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Std.dev</th>
<th>Skew.</th>
<th>Kurt.</th>
<th>JB</th>
<th>ARCH</th>
<th>Q(12)</th>
<th>Q(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal and Mineral</td>
<td>0.29</td>
<td>-30.45</td>
<td>15.85</td>
<td>20.26</td>
<td>-0.74</td>
<td>5.42</td>
<td>840.39</td>
<td>116.8</td>
<td>85.95</td>
<td>183.23</td>
</tr>
<tr>
<td>Fats and oils</td>
<td>0.25</td>
<td>-26.06</td>
<td>26.73</td>
<td>26.62</td>
<td>0.15</td>
<td>5.75</td>
<td>882.13</td>
<td>128.62</td>
<td>107.7</td>
<td>218.27</td>
</tr>
<tr>
<td>Grains</td>
<td>0.24</td>
<td>-19.62</td>
<td>22.51</td>
<td>15.72</td>
<td>0.67</td>
<td>4.6</td>
<td>615.56</td>
<td>99.15</td>
<td>135.03</td>
<td>137.5</td>
</tr>
<tr>
<td>Others foods items</td>
<td>0.29</td>
<td>-17.19</td>
<td>24.44</td>
<td>18.35</td>
<td>0.35</td>
<td>3.2</td>
<td>284.89</td>
<td>100.62</td>
<td>36.86</td>
<td>241.2</td>
</tr>
<tr>
<td>Beverages</td>
<td>0.22</td>
<td>-15.92</td>
<td>28.7</td>
<td>20.68</td>
<td>0.93</td>
<td>4.65</td>
<td>669.19</td>
<td>49.10</td>
<td>84.83</td>
<td>67.45</td>
</tr>
<tr>
<td>Agricultural Raw Material</td>
<td>0.22</td>
<td>-10.38</td>
<td>11.54</td>
<td>6.32</td>
<td>0.46</td>
<td>3.53</td>
<td>353.97</td>
<td>85.96</td>
<td>140.31</td>
<td>150.96</td>
</tr>
</tbody>
</table>

Notes: Shaded numbers indicate the rejection of the null hypothesis of associated statistic tests at the 5% level.

Figure 1. Monthly returns on a set of non-energy commodity sectors (period from 31 January 1960 to 30 April 2014)

Unlike Metal and Minerals sector, all other sector returns exhibit positive skewness, this suggests that rewards are more likely than losses. The high values for the kurtosis statistic suggest that the returns distribution have fat tails. In fact, the Jarque–Bera test strongly rejected the normality of the unconditional distribution for all the series.

Moreover, the Ljung–Box statistic suggested the presence of serial correlation in the volatility of the returns series. The autoregressive conditional heteroskedasticity-Lagrange multiplier (ARCH-LM) statistic indicated that ARCH effects were likely to be found in all the returns series. Therefore, we can conclude that the return series present some linear dependence. The statistically significant serial correlations in the squared returns imply that there is non-linear dependence in the return series. This indicates volatility clustering and a GARCH type modeling should be considered to obtain more effective estimates for Kendall’s tau, Spearman rho and tail dependence. Conditional volatility forecasts for the period 1960-2014 obtained using univariate GARCH-t model with rolling estimation procedure.

Table 3. Parameters estimates of AR (1)-GARCH (1, 1) model with residuals that follow Gaussian distribution

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Metal and Mineral</th>
<th>Fats and oils</th>
<th>Grains</th>
<th>Others foods items</th>
<th>Beverages</th>
<th>Agricultural raw material</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>0.04194</td>
<td>0.2460</td>
<td>0.04654</td>
<td>0.34237</td>
<td>0.06761</td>
<td>0.01166</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.15982</td>
<td>0.1060</td>
<td>0.39443</td>
<td>0.21915</td>
<td>0.53156</td>
<td>0.54168</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.12013</td>
<td>0.2707</td>
<td>0.19662</td>
<td>2.22406</td>
<td>1.25621</td>
<td>0.15364</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.03341</td>
<td>2.1535</td>
<td>0.04900</td>
<td>0.18473</td>
<td>0.08690</td>
<td>0.16133</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.27897</td>
<td>0.2454</td>
<td>0.88931</td>
<td>0.68514</td>
<td>0.85044</td>
<td>0.82586</td>
</tr>
<tr>
<td>JB</td>
<td>92.6</td>
<td>75.12</td>
<td>138.5</td>
<td>34.3</td>
<td>655.6</td>
<td>169.9</td>
</tr>
<tr>
<td>Q(12)</td>
<td>24.83</td>
<td>3.572</td>
<td>18.92</td>
<td>17.29</td>
<td>12.67</td>
<td>10.46</td>
</tr>
<tr>
<td>Q²(12)</td>
<td>7.384</td>
<td>9.956</td>
<td>14.97</td>
<td>5.76</td>
<td>24.69</td>
<td>12.49</td>
</tr>
<tr>
<td>AIC</td>
<td>3566.879</td>
<td>3716.165</td>
<td>3447.004</td>
<td>3626.783</td>
<td>3712.152</td>
<td>2819.923</td>
</tr>
<tr>
<td>BIC</td>
<td>3593.75</td>
<td>3743.036</td>
<td>3475.875</td>
<td>3653.634</td>
<td>3759.023</td>
<td>2846.794</td>
</tr>
</tbody>
</table>

Note: $Q(12)$ and $Q^2(12)$ are the Ljung–Box test statistic checks for the serial correlation of squared standardized residuals. The AIC criterion measures the relative goodness of fit of the estimated model. *Shaded numbers indicate the rejection of the null hypothesis of associated statistic tests at the 5% level.

We employ the GARCH model proposed by Bollerslev which examines past shocks and volatility on the current conditional volatility. In the case of non-energy commodities, Table 2 shows that past shocks (α) and past volatility (β) impact current conditional volatility. Moreover, their sums (α + β) are very close to unity which indicates long memory. We also examine residuals generated from univariate $AR(1)-GARCH(1,1)$ model to check whether the model is correctly specified and produce reliable volatility estimates. In this regard, parameters estimated provides evidence of an accurate model as the results show no serial correlation in standardized and squared residuals and no remaining ARCH effect. Therefore, our model is efficient and appropriate.

6. Empirical Results

To obtain Kendall’s tau, Spearman’s rho and tail coefficients dependence estimates using conditional copula method; we follow a three-step procedure.

• Step 1: Fitting univariate $AR(1)-GARCH(1,1)$ models for each return series assuming that innovations follow $t$-Student distribution with $v$
degrees of freedom. We obtain parameters estimation and compute standardized residuals to check the adequacy of the GARCH modeling. They are calculated as

\[
\left( z_{t-k+1}, z_{t-k+2}, \ldots, z_t \right) = \left( \frac{r_{t-k+1} - \mu_{t-k+1}}{\sigma_{t-k+1}}, \frac{r_{t-k+2} - \mu_{t-k+2}}{\sigma_{t-k+2}}, \ldots, \frac{r_t - \mu_t}{\sigma_t} \right)
\]

- Step 2: Transform each standardized residuals series into uniform (0, 1) using the probability-integral transformation.
- Step 3: Or each pair of transformed data vectors, fit the most appropriate copula according to AIC criteria, estimate their parameter by using a two-step estimation procedure (IFM) and estimate Kendall’s tau, Spearman’s rho and tail dependence coefficients.

Table 4 presents the most appropriate copula following Akaike criteria for the fifteen pair of non-energy commodity sectors. Moreover, we obtain extreme-value copulas (Galambos, Husler and Reiss, Tawn and Gumbel), Archimedean copulas (Frank and BB7), Normal copula and Archimax copula (BB4). To validate the choice of selected copulas, we perform the goodness of fit procedures given by Genest et al (2006). The empirical results of the goodness of fit test are summarized in Table 5.

To estimate the parameters of the copula, several methods are proposed in the literature including the exact maximum likelihood method, the canonical maximum likelihood (CML) method, the inference functions for margins (IFM) method, the moment’s method and the empirical copula. Empirical results of estimated copulas parameters are reported in Table 6.

Within the context of the current financial crisis, there is an increasing interest by traders, investors, portfolio managers, physical users and producers, and policy makers to understand better the performance and the distributional characteristics of increasingly important asset classes. Such enhanced understanding should lead to better returns, greater benefits from portfolio diversification, more adequate pricing of derivatives and improvement in risk management strategies. Among these asset classes are the non-energy commodities. Table 7 summarizes the Kendall’s tau and Spearman rho coefficients issues from selected copulas.
Figure 2. Dependence structure between a set of pair of non-energy commodity sectors

Table 8. Tail dependence coefficients

<table>
<thead>
<tr>
<th></th>
<th>Metals &amp; Minerals</th>
<th>Fats &amp; Oils</th>
<th>Grains</th>
<th>Other Foods Items</th>
<th>Beverages</th>
<th>Agricultural Raw Material</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Metals &amp; Minerals</strong></td>
<td>Lower</td>
<td>0</td>
<td>0</td>
<td>0.0519257</td>
<td>0</td>
<td>0.008990474</td>
</tr>
<tr>
<td></td>
<td>Upper</td>
<td>0</td>
<td>0.0692309</td>
<td>0.03497477</td>
<td>0</td>
<td>0.07041365</td>
</tr>
<tr>
<td><strong>Fats &amp; Oils</strong></td>
<td>Lower</td>
<td>0.0323422</td>
<td>0</td>
<td>0.0334762</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Upper</td>
<td>0.2430074</td>
<td>0</td>
<td>0.05761905</td>
<td>0.09682668</td>
<td>0.09682668</td>
</tr>
<tr>
<td><strong>Grains</strong></td>
<td>Lower</td>
<td>0</td>
<td>0</td>
<td>0.09682668</td>
<td>0.08652801</td>
<td>0.08872359</td>
</tr>
<tr>
<td></td>
<td>Upper</td>
<td>0.09682668</td>
<td>0</td>
<td>0.08652801</td>
<td>0.08872359</td>
<td>0.08872359</td>
</tr>
<tr>
<td><strong>Other Foods Items</strong></td>
<td>Lower</td>
<td>0</td>
<td>0</td>
<td>0.03277979</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Upper</td>
<td>0.03277979</td>
<td>0</td>
<td>0.03277979</td>
<td>0</td>
<td>0.07046881</td>
</tr>
<tr>
<td><strong>Beverages</strong></td>
<td>Lower</td>
<td>0</td>
<td>0</td>
<td>0.07046881</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Upper</td>
<td>0.07046881</td>
<td>0</td>
<td>0.07046881</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 8. Tail dependence coefficients**

- Metal and Minerals/Agricultural Raw Materials (BB7 copula)
- Fats & Oils/Grain (BB4 copula)
- Other Food/Grain (Frank copula)
- Agricultural Raw Materials/Beverages (Galambos copula)
In general, the conditional correlation matrices indicate that there exists a rather low dependence between non-energy commodity sectors. We note that Fats & Oil sector shows the strongest positive dependence with Grains sector, while the lowest degree of dependence is obtained for Agricultural Raw Materials sector and other sectors.

Our results show that non-energy commodity markets generally don’t co-move together as the dependence parameters of all copulas models are weakly significant. This result indicates that non-energy commodities are non-complementary or non-substitutable.

To quantify the extreme dependence, we compute the tail dependence coefficient (Table 8). At the extreme levels, we find asymmetric tail dependence between most of pairs. The results suggest that non-energy commodity returns are only dependent in bullish market periods, but not so in bearish markets.

This finding implies that stronger economic growth and concurrent development demand on non-energy commodities strengthen the long-run relationship between them and make them responses to common shocks such as changes in business cycles or geopolitical risk. For example, in periods of general inflation commodity prices rise together.

The bearish relationship should encourage investment in non-energy commodities. That is to say, the non-energy markets an opportunity of diversification due to their weak degree of dependence with each other.

Tail dependence expresses the probability of having a high (low) extreme value of a series given that a high (low) extreme value of another series has occurred. The higher probability of joint extreme observations as compared with the Gaussian (normal) copula is known as positive tail dependence. Normal and Frank copulas impose zero tail dependence whereas other copulas impose zero tail dependence in one tail. For instance, the Husler and Reiss copula has upper tail dependence and BB7 copula has lower right tail dependence. BB4 and Normal copulas on the other hand exhibit no tail dependence.

Tail dependence refers to the degree of dependence in the corner of the lower-left quadrant or upper-right quadrant of a bivariate distribution.

When, \( \lambda_U > \lambda_L \), the distribution is symmetric. When \( \lambda_U > \lambda_L \), the upper tail of the contour gets thinner, meaning that there is a greater probability that the energy futures will move higher jointly. Similarly, when \( \lambda_U < \lambda_L \), the lower tail of the contour becomes thinner and the probability that the non-energy commodity prices will move lower jointly is higher.

In general, the upper tail dependence is higher than the lower tail dependence (except for the Other Food Items and Metals & Minerals pair), meaning that non-energy commodity prices are more likely to move up together than go down together. Our results confirm that the dependence structure is not symmetric. However, most of the previous studies show higher lower tail dependence, meaning that financial assets are more dependent during extreme downturns than upturns of the markets.

The averages of the difference between the upper and lower tail dependence estimates provide information about dynamic asymmetries. The differences are all positive, suggesting a higher probability of joint extreme events during bull markets than during bear markets.

7. Conclusion

This study investigates the nature of dependence between different non-energy energy commodity prices. Unlike previous studies, this study uses copula method, to establish the dependence in multivariate cases.

In recent years, copula modelling has become a popular tool in the field of economics and finance. It allows greater flexibility in estimating multivariate distribution and eliminates the need for simplifying assumptions. Copulas capture dependence more broadly than the standard multivariate normal framework.

Pearson correlation may be too restrictive a criterion to measure the dependency of multivariate marginal distribution. Correlation is not a satisfactory measure of the dependence among different non-energy commodity markets because linear correlation only measures the degree of dependence but does not clearly discover the structure of dependence. Additionally, linear correlation is not invariant under nonlinear strictly increasing transformations, implying that returns might be uncorrelated whereas prices are correlated or vice versa. Finally, most of the random variables are not elliptically distributed. Therefore, the structure of dependence of these variables could be compromised when using simple correlation methods. A more prevalent approach that overcomes the disadvantages of linear correlation is to model dependency by using copulas.

In this paper, we propose the time-varying copula dependence model with marginal distributions modeled by AR (1)-GARCH with innovations that follow Gaussian distribution in order to study the dependence structure between non-energy commodity markets over time.

Our results indicate that:
- The difference between upper tail dependence and lower tail dependence is generally positive, which implies that non-energy commodity prices are more likely to move together during bull markets than in bear markets. This result is opposite to most of the previous research on copulas which has found financial assets are more likely to move down together than move up together.
- The means of the difference for all pairs are positive, suggesting a higher probability of joint extreme events during a bull market than during a bear market. Dependence between markets is studied in bivariate context. Future research may extend this methodology in multivariate context using CD-vine copula applied to high frequency data.

References

a) Dependence between Commodities (energy or and non-energy) and macroeconomic variables (exchange rate, interest rate and index price)

<table>
<thead>
<tr>
<th>Authors</th>
<th>Objective</th>
<th>Period/Methodology</th>
<th>Main findings</th>
</tr>
</thead>
</table>
b). Dependence between crude oil and other commodities

<table>
<thead>
<tr>
<th>Authors</th>
<th>Objective</th>
<th>Period/Methodology</th>
<th>Main findings</th>
</tr>
</thead>
</table>
| Aloui et al. (2014)  | This paper investigates the dependence structure of agricultural commodity prices with respect to three market risk factors: the federal funds effective rate, the USD/EUR exchange rate, and changes in the MSCI world stock market index. | 2003-2010 Model Extreme-value copula, Wavelet, VaR, CVaR | - Changes in the USD/EUR exchange rate and the stock market index are the dominant risks for agricultural commodity markets.  
- The tail dependence on the daily returns of the three market risk factors is also scale-dependent, and frequently asymmetric. |
| Delatte & Lopez (2013) | In this paper, authors propose to identify the dependence structure that exists between returns on equity and commodity futures (metal, agriculture and energy) and its development over the past 20 years. | 1990-2012 Model Copula | - The dependence between commodity and stock markets is time-varying, symmetrical and occurs most of the time.  
- A growing co-movement between industrial metals and equity markets is identified as early as 2003; this co-movement spreads to all commodity classes and becomes unambiguously stronger with the global financial crisis after Fall 2008. |
| Fernandez (2014)     | The objective of this article is to study the relationship between four U.S. price indices and 31 commodity series. | 1957-2011 Model Linear and non-linear Granger causality | - Not only shocks on commodity demand and supply may impact aggregate price indices, but also that non-commodity shocks, embodied in aggregate price indices, may impact commodity prices linearly and nonlinearly. |
| Nazlioglu (2011)     | This paper studies the impact of the increasing co-movements between the world oil and agricultural commodity prices. | 1994 - 2010 Model Granger causality | - The oil and the agricultural commodity prices do not cause each other.  
- The recent surge in the agricultural commodity prices can be attributed the changes in the oil prices. |
| Qiang and Ying Fan (2012) | This study measures the influence of the crude oil market on non-energy commodity markets before and after the 2008 Financial crisis. | 2006-2010 Model Bivariate EGARCH | - The crude oil market has significant volatility spillover effects on non-energy commodity markets, which demonstrates its core position among commodity markets.  
- The influence of the US dollar index on commodity markets has weakened since the crisis. |
| Reboredo (2012)      | This paper studies co-movements between world oil prices and global prices for corn, soybean and wheat. | 1998-2011 Model Copula | - Empirical results showed weak oil-food dependence and no extreme market dependence between oil and food prices.  
- These results support the neutrality of agricultural commodity markets to the effects of changes in oil prices and non-contagion between the crude oil and agricultural markets.  
- Oil–corn and oil–soybean dependence increased in recent years. |
| Zhang and Chen (2013) | This paper investigated the reaction of aggregate commodity market to oil price shocks and also explored the effects of oil price shocks on China's fundamental industries: metals, petrochemicals, grains and oilfats. | 2001 - 2011 Model ARMA-GARCH and ARJI-EGARCH | - The aggregate commodity market was affected by both expected and unexpected oil price volatilities in China.  
- The metals and grains indices did not significantly respond to the expected volatility in oil prices, in contrast to the petrochemicals and oilfats indices. |
| Reboredo (2013)      | This paper assesses the role of gold as a hedge or safe haven against oil price movements. | 2000-2011 Model Copula | - There is positive and significant average dependence between gold and oil, which would indicate that gold cannot hedge against oil price movements.  
- There is tail independence between the two markets, indicating that gold can act as an effective safe haven against extreme oil price movements. |
| Li Liu (2014)        | The authors study the cross-correlations between crude oil and agricultural commodity markets. | 2006 - 2008 Model Nonlinear cross-correlation and DCCA analysis | - Return cross-correlations are significant for larger lag lengths.  
- Volatility cross-correlations are significant for each lag length.  
- Cross-correlations are weak but significant.  
- The information transmission from crude oil market to agriculture markets can complete within a certain period of time. |
| Mensi et al. (2014)  | This article deals with the dynamic return and volatility spillovers across international energy and cereal commodity markets. It also examines the impacts of three types of OPEC news announcements on the volatility spillovers and persistence in these markets | 1979–2010 Model The VAR-BEKK-GARCH and VAR-DCC-GARCH | - The results provide evidence of significant linkages between the energy and cereal markets.  
- The OPEC news announcements are found to exert influence on the oil markets as well as on the oil–cereal relationships. |
| Sadorsky (2014)      | The aim of this paper is to model volatilities and conditional correlations between emerging market stock prices, copper prices, oil prices and wheat prices. | 2000-2012 Model VARMA-AGARCH and DCC-AGARCH | - Correlations between these assets increased considerably after 2008, and have yet to return to their pre 2008 values.  
- On average, oil provides the cheapest hedge for emerging market stock prices while copper is the most expensive but given the variability in the hedge ratios, one should probably not put too much emphasis on average hedge ratios. |
This paper explores the relevance of asymmetry and long memory in modeling and forecasting the conditional volatility and market risk of four widely traded commodities (crude oil, natural gas, gold, and silver).

**Period**: 1957-2011

**Model**: GARCH models

The empirical results show that volatility of commodity returns can be better described by nonlinear volatility models accommodating the long memory and asymmetry features.

This paper studies the effects of oil shocks on agricultural commodity prices.

**Period**: From January 1980 to December 2012

**Model**: Structural VAR

- Oil shocks can explain a minor friction of agricultural commodity price variations before the food crisis in 2006-2008, whereas in post-crisis period their explanatory abilities become much higher.
- After crisis, the contributions of oil-specific factors to variations in agricultural commodity prices are greater than those of aggregate demand shocks.

### c). Literature review: Dependence between non-energy commodities

<table>
<thead>
<tr>
<th>Authors</th>
<th>Objective</th>
<th>Period/Methodology</th>
<th>Main findings</th>
</tr>
</thead>
</table>
| Ciner (2001)          | This study examines the long run trend between the prices of gold and silver futures contracts traded on the Tokyo Commodity Exchange. | **Period**: 1992 - 1998  
**Model**: The Augmented Dickey Fuller (ADF) tests and Johansen’s contagion analysis | - Statistical findings indicate that the frequently cited long-term stable relationship between the prices of gold and silver has disappeared.  
- These two markets should not be regarded as substitutes to hedge against similar types of risks. |
| Hammoudeh et al. (2011) | This paper examines volatility and correlation dynamics in price returns of gold, silver, platinum and palladium, and explores the corresponding risk management implications for market risk and hedging. | **Period**: 1995-2009  
**Model**: VaR, different GARCH models | The empirical results highlight the advantages of precious metals in improving portfolio performance. |
| Kantaporn et al. (2012) | This paper studies the tail behavior of the palm oil future markets and the dependence structure between the returns on palm oil future price in three palm oil futures markets. | **Model**: Univariate EVT Gumbel and HuslerReiss copula | - The results demonstrated that both methods have a similar outcome.  
- The returns on palm oil future price among KLSE and SGX-DT have dependence in extreme, whereas the returns on palm oil future price among KLSE and DCE, SGX-DT and DCE do not have any dependence |
| Arezki et al. (2014)  | This study examine the relationship between the South African Rand and the gold price volatility | **Period**: 1979–2010  
**Model**: VECM | - The main finding is that prior to capital account liberalization the causality runs from the South African Rand to the gold price volatility.  
- The gold price volatility plays a key role in explaining both the excessive exchange rate volatility and current disproportionate share of speculative (short-run) inflows that South Africa has been coping with since the opening up of its capital account. |
| Power et al. (2013)   | The objective of this article is to model the dynamic hedging problem for more than one commodity (corn and fed cattle) when the joint dependence structure is allowed to be possibly non-elliptical. | **Period**: 2000-2010  
**Model**: Nonparametric Copula-GARCH/ GARCH-DCC/ GARCH-BEKK | The non-parametric copula-GARCH leads to economically and statistically significant improvements according to the expected shortfall (tail risk) criterion, but not in terms of variance reduction. |
| Smiech and Papiez (2012) | The aim of the paper is to analyze causality between the prices of four different metals: gold, silver, platinum and copper. | **Period**: 2000-2012  
**Model**: Granger causality | - The price of copper was the Granger cause of the prices of the remaining metals, while in the later period the price of platinum became the Granger cause of the prices of the remaining metals.  
- Past prices of gold and silver did not improve the forecasts of prices of other metals. |