American Journal of Educational Research, 2018, Vol. 6, No. 12, 1719-1725
Available online at http://pubs.sciepub.com/education/6/12/20

DOI:10.12691/education-6-12-20

Error Patterns in Turkish Pre-Service Elementary Teachers' Arithmetic Operations in Power Numbers

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Received November 01, 2018; Revised December 09, 2018; Accepted December 22, 2018

Abstract This study investigated the pre-service elementary teachers understanding of exponent and how they solve the exponents equations and their different solving techniques and errors. Advanced mathematics automatically develops an understanding of basics of mathematics which is enough to explain and justify the reasonings of basic of arithmetic operations. The sample of the study is seventy-nine first year pre-service elementary teachers at the university of Turkey. Two items solved in detail. Content analysis of these two item solutions were analyzed for pre-service elementary teachers understanding of mathematics and their error patterns. Majority of them solved the items by well know strategies at the books but some of them made basic computation errors. They could not apply the basic exponents rules to solutions. Elementary teachers should learn the basic of rules without computation errors since they were teaching basics of operations. Elementary teaching programs should cover more mathematics courses with ambitious mathematics teaching with basic operations and logic.

Keywords: *pre-service elementary teachers, power numbers, algebraic operations, error patterns, error analysis*

Cite This Article: Sevim Sevgi, "Error Patterns in Turkish Pre-Service Elementary Teachers' Arithmetic Operations in Power Numbers." *American Journal of Educational Research*, vol. 6, no. 12 (2018): 1719-1725. doi: 10.12691/education-6-12-20.

1. Introduction

Elementary school students ask lots of questions which are generally starts with the why. They are at the concrete level of [1,2] and the questions are so normal at these ages. In the classroom, primary school teachers probably answer these questions as ".. hmm... I don't know why because that's what I've always done". Elementary mathematics subjects are basic algebra, geometry, measurement and data and statistics. These subjects are approximately same across countries elementary mathematics curriculums. Elementary teacher education programs usually cover basic mathematics courses at the first year of program. Elementary teachers should learn basics of mathematics deeply to make platform for the advanced mathematics. One of the important mathematics concepts learning and understanding of exponents are central to many college mathematics courses, including calculus, differential equations and complex analysis [3,4].

Using exponents and exponents rule is a prerequisite skill that is significant to success in all algebra [5]. Multiply Common factors, solve quadratic equations, simplify radical and rational expressions, and identify the shapes of functions are based on the exponent's usage and exponent rules. [6] pointed out that understanding exponents and the concept of very large numbers – often written in scientific notation – is critical to grasping exponential functions representing real-life situations such as growth and decay. A study conducted by [7] confirmed a lack of understanding that even older algebra students have regarding exponents. Pinchback's study found that students often did not interpret expressions such as (z − 4)2 correctly and instead translated it as (z − 4) (z + 4) (confusing it with the difference of two squares) or performed other incorrect procedures such as attempting to factor (z−4)2 to (z + 2) (z−2). While this study was conducted with older students, it reveals the deeply-rooted and long-term lack of understanding students exhibit with exponents beyond the basics of 22 = 4, 33 = 27, etc. [8].

Mathematics teachers content knowledge and pedagogical knowledge is a primary knowledge for improving mathematics achievement. Knowledge of content and nature of errors provide corrective feedbacks to teachers [9]. Teachers can determine their misconceptions and difficulties, so they can increase their content and pedagogical knowledge. In the early ages, students make systematic and consistent errors at arithmetic operations. Teachers suppose these errors as careless mistakes or reflect a lack of frequency. This case is not always right, some of the errors are misconceptions. These errors may persist thought students' academic life, and they affect the acquisition of advanced mathematics [10]. Aim of this descriptive study is to investigate the reasons of pre-service elementary teachers' errors in algebraic operations in powers. Specifically, the research questions are the following:

- What conceptual structure do pre-service elementary teachers for power numbers (within the standard algorithms)?
- How do the pre-service elementary teachers' conceptual structures influence their explanations in the context of standard algorithms?
- How does the pre-service elementary teachers' perform beyond the context of the standard algorithm and how do their conceptual structures relate to their explanations in those contexts?

2. Theoretical Framework

[3]'s theoretical framework used to analyze the pre-service teachers answers to determine their understandings of exponents. Weber used Dubinsky's APOS theory [11] to understand how students develop their understanding of exponentiation as functions. Weber's analysis of understanding exponentiation as actions and processes is very similar to [12] analysis of how students view functions in general.

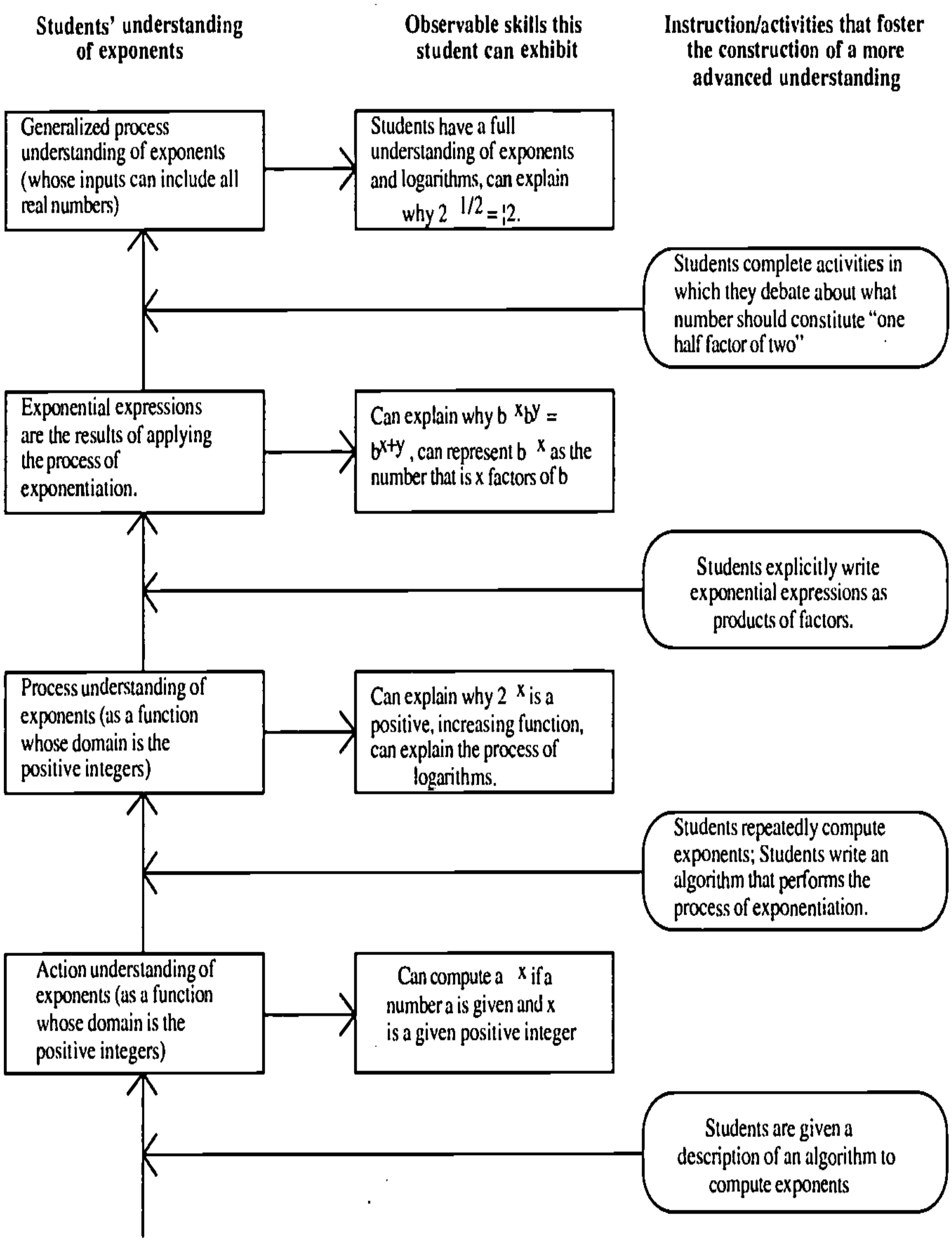

Figure 1. Weber's framework [3]

3. Research Methods

The researcher examined the error types in algebraic operations made by 79 Turkish pre-service elementary teachers from a large, urban state university. A written test was administered to students. 57 of them were female and 22 of them were male (Figure 2 Gender and Figure 3 Age Distribution) and their age range was 18 to 23. 61 of 79 students age distribution were given at below and 18 of the pre-service elementary teachers did not fill their age to their demographic form.

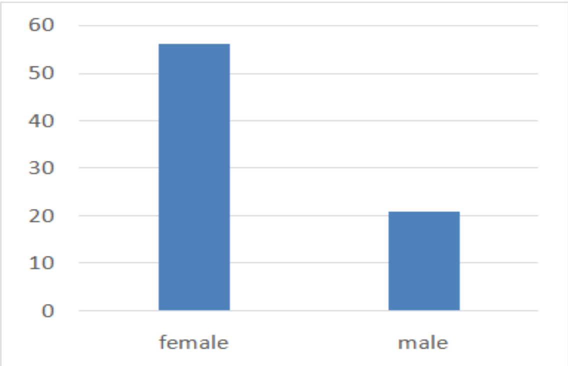

Figure 2. Gender Distribution

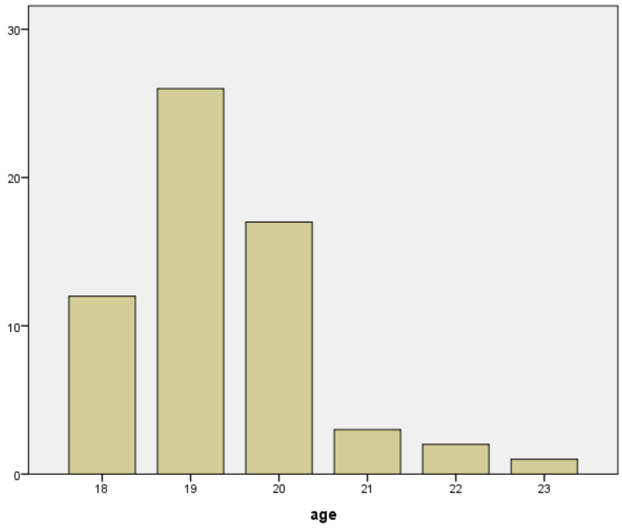

Figure 3. Age Distribution

The researcher examined the error types in algebraic operations made by 79 Turkish pre-service elementary teachers from a large, urban state university. A written test was administered to students. 57 of them were female and 22 of them were male (Figure 2 Gender and Figure 3 Age Distribution) and their age range was 18 to 23. 61 of 79 students age distribution were given at below and 18 of the pre-service elementary teachers did not fill their age to their demographic form.

In Turkish Education system, all freshman students were taken a university entrance exam and according to results of this test, they were choosing their universities and departments. The mathematics level of these students is so similar according to the university entrance exam scores since their scores are so similar and the test is standardized one. Each freshman pre-service elementary teacher completed a written mathematics achievement test. Pre-service elementary teachers were attending Basic Mathematics I course which is two hours in a week though the fall semester. Mathematics achievement test was designed as final exam of the course. A coding system was based on [13] and [14] studies was used to grade the power questions. A reliability check was performed by two experts at mathematics education department.

3.1. Items

Items were given at below as Figure 4 and solution strategies of item one were given at Table 1.

1. $\dfrac{5^{4860}+5^{4858}+5^{4856}}{5^{4856}+5^{4854}+5^{4852}}$ what is the answer of this expression?

2. $\dfrac{2^{2x+1}-2^{2x-1}+2^{2x-2}}{2^{x+4}+2^{x+2}+2^{x}}=\dfrac{1}{48}$ what is the value of x?

Figure 4. Items of power numbers

Freshman students were used 4 different solution strategies to solve the item. These solution strategies were given at Table 1.

Table 1. Solution Strategies of first item

Solution Strategy	Solution
1	$\dfrac{5^{4856}\,(5^4+5^2+1)}{5^{4852}\,(5^4+5^2+1)}=\dfrac{5^{4856}}{5^{4852}}=5^4=625$
2	$\dfrac{5^4(5^{4856}+5^{4854}+5^{4852})}{(5^{4856}+5^{4854}+5^{4852}))}=5^4=625$
3	$\dfrac{5^{4852}\,(5^8+5^6+5^4)}{5^{4852}\,(5^4+5^2+5^1)}=\dfrac{(5^8+5^6+5^4)}{(5^4+5^2+5^1)}=\dfrac{5^4(5^4+5^2+1)}{(5^4+5^2+1)}=5^4=625$
4	$\dfrac{(5^{x+8}+5^{x+6}+5^{x+4})}{(5^{x+4}+5^{x+2}+5^{x})}=\dfrac{5^x\,(5^8+5^6+5^4)}{5^x\,(5^4+5^2+5^1)}=\dfrac{(5^8+5^6+5^4)}{(5^4+5^2+5^1)}=5^4=625$

Solution strategies of second item which were applied by pre-service elementary teachers were given at Table 2.

Table 2. Solution Strategies of Second Item

Solution Strategy	Solution
1	$\frac{2^{2x+1}-2^{2x-1}+2^{2x-2}}{2^{x+4}+2^{x+2}+2^{x}}=\frac{1}{48}.\frac{2^{2x+2}\left(2^{3}-2^{1}+1\right)}{2^{x}.\left(2^{4}+2^{2}+1\right)}$ $=\frac{1}{48}.\frac{7.2^{2x-2}}{21.2^{2x}}=\frac{1}{48}.\frac{2^{x-2}}{3}=\frac{1}{48}.2^{x-2}=\frac{1}{16}=2^{-4}$ ten, $x=-2$dir.
2	$\frac{2^{2x}\left(2-2^{-1}+2^{-2}\right)}{2^{x}\left(1+2^{2}+2^{4}\right)}=\frac{1}{48}\to\frac{2^{2x}\frac{7}{4}}{2^{x}21}=\frac{1}{48}\to x=-2$
3	$\frac{4^{x}2}{4}-\frac{4^{x}}{2}+\frac{4^{x}}{4}\to\frac{84^{x}-24^{x}-4^{x}}{4}=\frac{74^{x}}{4}\frac{\frac{74^{x}}{4}}{21.2^{x}}=\frac{7.4^{x}}{4.21.2^{x}}\to\frac{2^{x}}{12}=\frac{1}{48}$ $\to 2^{x}16+2^{x}4+2^{x}=21.2^{x}\ 2^{x}=\frac{1}{4}\to x=-2$
4	By substitution 2x= a, $\frac{\left(2.a^{2}-\frac{a^{2}}{2}+\frac{a^{2}}{4}\right)}{(16a+4a+a)}=\frac{\left(\frac{7}{4}a^{2}\right)}{(21a)}=\frac{1}{48}, a=-2$

3.2. Analysis

In analyzing the data, I focused on the pre-service elementary teachers' content knowledge for teaching elementary school students that would enable those children to construct meaning. Little is known about the pre-service teachers' conceptions of power number, grounded theory with open coding is used for analysis [15].

4. Results

Results of the research quetsions are interpereted in two parts.

4.1. Results of First Item

70 of 79 students answered the question with one of the four solution strategy. They started to solve first item with exponential rule. On the other hand, 6 of the students' answers were not related with exponents and three of them write nothing as solution. Two of these were given at Figure 5 and Figure 6.

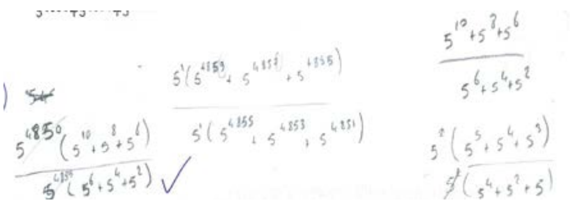

Figure 5. Unrelated solution example of first item

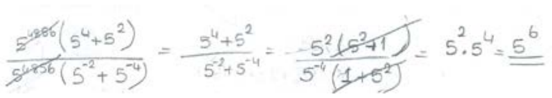

Figure 6. Unrelated solution example of first item

4.1.1. Results of First Solution Strategy of First Question

36 of freshman used first solution strategy to find the answer but six of those students were not calculated the value of 54 and two of them estimated but found a wrong number. These errors can be originated from the exam environment, exam anxiety. 17 of those right answers applied exponential the rule of ax / ay= ax-y to their solutions. Nearly, half of the students do not apply this rule to their solutions.

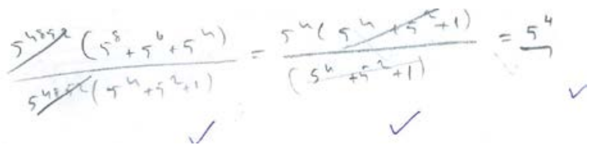

Figure 7. Example of first solution strategy of first item

4.1.2. Results of Second Solution Strategy of First Question

Only two students used this strategy to solve the expression, but they did not estimate the value of 54 at their solutions. Students think that there is no need to estimate the value of 54 at their solutions but their level cannot be understandable from this solution. They may be having misconceptions about the estimation the value of 54.

4.1.3. Results of Third Solution Strategy of First Question

19 of these students used the third strategy to solve the question but only seven of them were make all steps right. Six of them did not estimated the value of 54, one of them estimated the value of 54 as 651. This error can be a result of calculation errors instead of misconceptions. Five of them applied only first step to solve the problem but they made errors related to taking common parentheses. Two

of them applied the second step to solution but one is made a common parentheses error and second one has a misconception related to simplification.

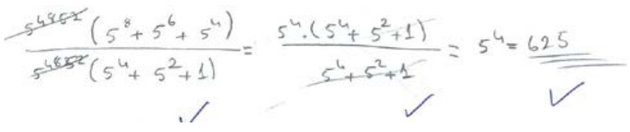

Figure 8. Example of third solution strategy of first item

At the third solution Strategy, only one student used common parentheses algorithm is right but at the last step student made an addition mistake. Only one student also used 54856 as common multipliers and all steps were right. Other Student who used 54856 as common multipliers and estimation of 54 is wrong. At the fourth Solution Strategy, some of them used substitution method or changing the unknown as 4852 = x, 5 4852 = ax or 4856= a. One of them used 4856= a substitution but common multiplier was not right. Misconception related to putting instead of number itself was made at this solution. Other one used the same substitution but at the last step he made a calculation error.

4.1.4. Results of Fourth Solution Strategy of First Question

Only five students applied this strategy but none of these students found the right answer. One is not estimated the value of 54 and the one is not substituted the value of a4. Other three were made algebraic errors and common multipliers errors at their solutions.

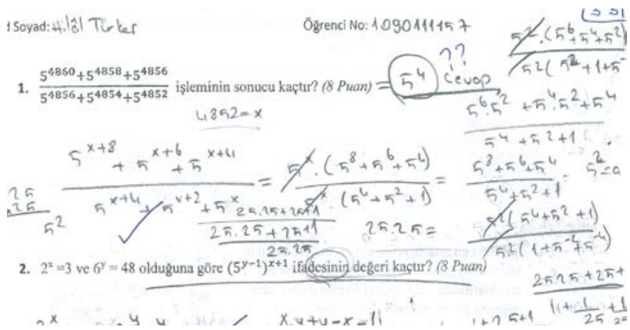

Figure 9. Example of fourth solution strategy of first item

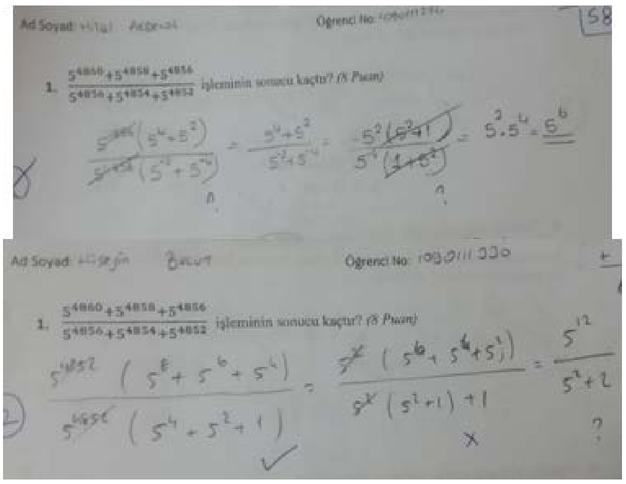

Figure 10. Common parentheses error of first item

Nine of the students have an error at the common parentheses. Example of this was given at Figure 10. [16] indicated that "One of the most frequent algebraic acts is manipulation – changing an expression into an equivalent expression that has the same value, for example, replacing 3y + 6 by 3(y + 2)" (p. 256).

4.2. Results of Second Item

68 of 79 pre-service elementary teachers solved the second item with one of the four strategies. One of them did not solve the item, there was nothing at the exam paper. Ten of the preservice teachers were applied strategies which were not related with exponents because of this reason these nine solutions were not grouped at the solution strategies. All of the solution strategies were started with the common parentheses rule.

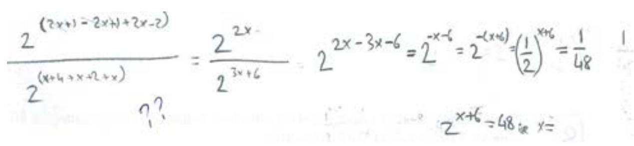

Figure 11. Ungrouped solution of second item

Some the pre-service elementary teachers have over generalization of addition with natural numbers to exponents. To illustrate Figure 12 was given:

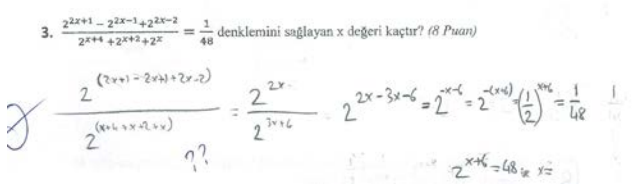

Figure 12. Example of over generalization of second item

One of them have used common parentheses in a correct form but made an error about calculation step but the source of error is not explicit.

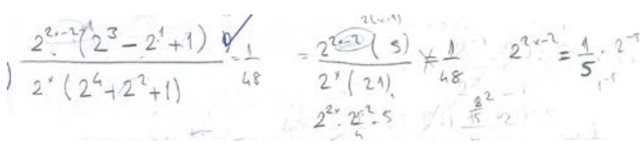

Figure 13. Ungrouped example of second item

One of them confused the 2^{2x} with the 2^{x}, this error source is exponents with the unknown. Pre-service teacher has error about the unknown quantity.

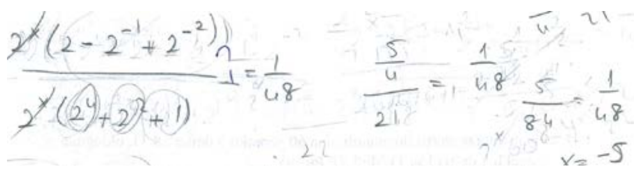

Figure 14. Example of confusion of rules

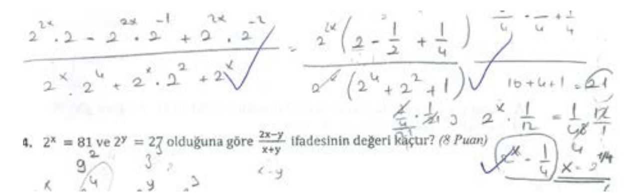

Figure 15. Problem with negative powers

One of them have problems with the negative power or writing rational numbers in terms of exponents.

One of them have problems with the $2^1 = 0$ or having common parentheses error.

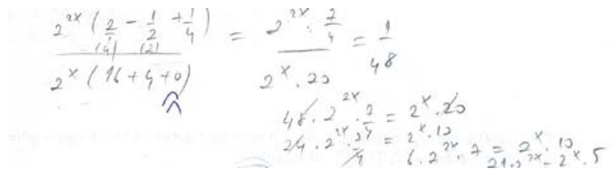

Figure 16. Parentheses error of second item

4.2.1. Results of First Solution Strategy of Second Item

Nine of the pre-service elementary teachers used the first solution strategy but only three of the solutions were right. The remaining six solutions has errors at operations level and application of exponents rule. One of them confused the 2^{2x} with 2^x. Example of operations error is at Figure 17.

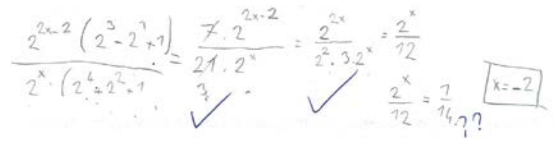

Figure 17. Example of first solution strategy of second item

4.2.2. Results of Second Solution Strategy of Second Item

56 of 68 pre-service elementary teachers were used this strategy. They prefer this common parenthesis method since this one uses without an expression one. 32 of 56 ones solved the second item by second solution strategy without errors or misconceptions. On the other hand, remaining half of the pre-service elementary teachers made an exponents error (11 pre-service teachers), operations error (18 pre-service teachers) or both operations and exponents error (3 pre-service teacher). They confused $2^x = 1/4$ with $2^x = 4$, this can be originated operations with negative numbers and also operations with the exponents.

4.2.3. Results of Third Solution Strategy of Second Item

Only two pre-service elementary teachers were used third solution strategy. One solution is right with all steps, but the other solution has problem as exponents error.

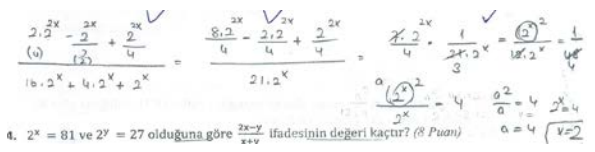

Figure 18. Example of third solution strategy of second item

4.2.4. Results of Fourth Solution Strategy of Second Item

Only one pre-service teacher tried to use the fourth solution strategy, but solution strategy has a ration numbers error.

This algebraic operation requires students to have an intuitive sense of the properties of numbers [8,17]. as cited [8] stated that "Contemplating the properties of numbers is one way of standing back from the engagement in the particular and becoming aware of the processes" (p. 70). Justification of the properties of numbers will be help students maintain their knowledge about numbers and build relational understanding [8].

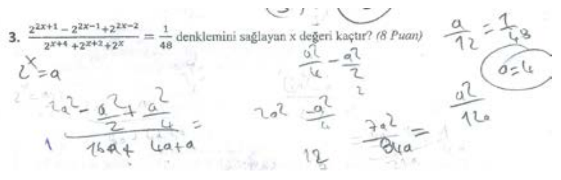

Figure 19. Example of fourth solution strategy of second item

5. Discussion and Conclusion

Data frequency analyses revealed that most common type of error was miscalculation for addition, multiplication and division of power numbers. The second most common error type was related to common parentheses of numbers. Frequencies of the error patterns of powers numbers was given.

Pre-service elementary teachers' errors suggested that their error patterns in arithmetic operations is related to conceptual and procedural knowledge and skills. Error analysis lets instructors to ensure that all students have an opportunity to succeed in mathematics. Error analysis can yield important information about pre-service elementary teachers' thinking, understanding, and misconceptions [18,19,20]. Misconceptions come from prior knowledge and errors are the result of their naïve concept [21]. One of the reasons of these misconceptions and errors is mathematics curriculums thought at schools since most mathematics curriculum at all levels frequently stress the prototype concept of exponents which involves, most of the time, the algebraic "unconscious manipulation" [4]. The problem of unconscious manipulation emerges from nearly all the research on the teaching of algebra in secondary schools [22]; a problem which is linked to the inability to translate into symbols, interpret them, and generalize.

Diagnostic assessment of student error patterns gives teachers the opportunity to deliver meaningful instruction to every student [23] and promote equality among a diverse population of students. Teachers' knowledge of common student misconceptions and errors as one of the six constructs of mathematics teacher effectiveness to foster student learning [24,25,26].

6. Further Research

The findings of research require replication, and extension to a larger and more diverse and larger samples than sample of research. Findings are tentatively compelling and serving to provide insights into pre-service elementary teachers' level of understanding of exponential concepts. The findings encourage further investigations, which could include the following:

- The use of larger samples of pre-service teachers from a wider range of attainment.

- The use of larger samples of in-service teachers from a wider range of attainment.
- The development of questionnaires involving not only fractional rational exponents but also decimals.
- The use of the exponential notation in other scientific areas.
- The extent to which experience with technologies may influence students' knowledge.

7. Limitations

First limitation of the study is convenient sampling. Future studies would be enhanced by including pre-service elementary teachers from different universities.

Second limitation is the administration of only 2 questions. More questions should be administered to pre-service elementary teachers.

Acknowledgements

Earlier version of this paper is presented as an oral presentation at the ECER 2018 Bolzano - The European Conference on Educational Research, 02-04 September 2018, Bolzano, Italy.

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