About the Peculiar Aspects of Relativity and Beyond: A Pedagogical Perspective

Paolo Di Sia *

Faculty of Education, Free University of Bolzano-Bozen, Italy

*Corresponding author: paolo.disia@gmail.com

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Abstract In this paper a pedagogical, but rigorous introduction to the key concepts of relativity is presented. Starting by the notions of space and time, of fundamental importance for the evolution for the global knowledge of children already in the primary school, we arrive to indications about the teaching of relativity in high school and university. Interesting considerations related to fascinating topics, like the evolution of the concept of space-time and the unusual amazing peculiarities emerging by the extensions of relativity, will be done.

Keywords: *pedagogy, primary school, secondary school, university, space, time, relativity, knowledge, unified theories*

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1. Introduction

The Einstein's theory of general relativity (1915) is a theory concerning space, time and gravity. Before 1905, the causal structure of space and time identified a notion of simultaneity: for a given event A, i.e. a point in space at an instant of time, it is possible to define the future of A, all events that, in principle, can be achieved by a particle which starts from A [1,2,3]. The same matter can be said for the past of A. What is not in the past or in the future of A is the set of simultaneous events to A. This implied the possibility of separating the study of space-time in separate studies of space and time. This is the pre-relativistic notion of the structure of space-time. The theory of relativity has aroused a great philosophical interest for its deep implications on crucial concepts, such as the concepts of space, time, matter, absolute, relative, and for the assignment of physical meaning to non-Euclidean geometries, all issues having their roots and inheritance to the essence of man, since childhood.

Identifying space with the gravitational field, physics becomes a real geometry, a doctrine of the possible forms in the space. Even this close link to the geometry leads to the early experiences of child, relative to her/his orientation in space. Hans Reichenbach spoke of a theory based on a critique of the apodictic character of each "a-priori" concept, by means of the method of successive approximation of the principles to the experience, i.e. the main modality with which child compares her/himself to the external world [4,5]. Therefore the Einstein's theory becomes an important and precious contribution, described with high mathematics elegance, which is based on the fundamental concepts of space and time and has a large cross on the first child's activities, related to the knowledge and understanding of the world [6].

2. Notions of Space and Time in Primary School

The theme of space and time leads to interesting proposals for cross activities as early as in the primary school. It is possible to consider:

- 1) the use of space as exploration of different environments and organization of spatial relationships;
- 2) the acquisition of the spatial orientation and the intuition of the first simple concepts of geometry and measurement (logic-mathematical area) [7];
- 3) to know how to manage the basis motor skills in different situations, in an increasingly and autonomous modality, through:
 - the self-knowledge,
 - the knowledge of the other (psycho-motor area),
 - the knowledge of the objects,
 - the knowledge of space-time [8].

The physical activity, especially for children, tends to turn on conscious and unconscious mental processes. This stimulates the verbalization of experience too, for translating the gestural code in verbal communication, enhancing not only the language skills, but also the exercise of listening, attention, observation and reflection skills [9]. When asked on what the word "space" brings to mind, the children's responses are varied, but all oriented to an extension [10], which can be:

- a) finite, local or global (space is my home, is a square, a pine forest, a farm, a forest, a big city, the earth, the world);
- b) it can tend to infinity (it is a thing, where you run.....and never ends, another land, the sky with many planets, a thing that never ends.....infinite, the sea that never ends) [11].

About the concept of time, it is very important and normally is better understood during the growth. As children, it is difficult to understand, but with the help of teachers and the continuing work, following a process and a graduated path, starting by the primary school, children acquire very important skills for life and for living together with others [12].

For each person it is important to respect times and to know an orientation in time, to be able to live and work with others, for example:

- realizing the flow of time,
- to give importance at activities, which take place in time,
- living in the present,
- retracing the past,
- making advances on the future,
- dealing with the different kinds of time,
- becoming aware of the difference between cyclical time and time of the temporal sequence,
- expliciting the relativity of historical documents (a class chooses what to remember, cannot remember everything).

The concept of “elastic” time, that can be deduced by relativity, definitely helps children, which become adolescents in their personal growth [13,14].

3. Notions of Space and Time in lower and upper Secondary School

Growing up in age, adolescents mature, think and approach also disciplines such as philosophy. The concept of space has been studied with considerable interest by philosophy, in particular in relation to its nature and its subjectivity/objectivity. Space was intended as the place of bodies or as containing them; in the first case the space is full, in the second it is necessary to admit the existence of vacuum.

Aristotle supported the concept of space as a place; the space of a body is the limit in which it is contained. The space is for him finished and its maximum extension coincides with that of the celestial spheres. These concepts, starting from the Aristotelian tradition, formed a thought line that, passing through Descartes and Spinoza, reached Kant [6]. The notion of space as a container, partially filled and partially empty, systematically formulated for the first time by the Greek atomists, became one of the foundations of classical physics through the work of Newton. He introduced the concept of “absolute space”, the “sensorium” of God, the reality through which God communicates with Nature.

With contemporary philosophy, it has opened the question of jurisdiction in dealing with problems inherent the nature of space, considering the new picture of the situation with the new changes starting from the theory of relativity, and considering that the physical universe, for a properly description, requires the use of other geometries, different by the Euclidean geometry [15]-[17]. All these ideas put the growing boys in front to new concepts, so as in front to a serious reflection, discussion and interdisciplinarity.

About the concept of time, philosophically it has been considered as a homogeneous flow, in which the changing things are “immersed”. The Greek thought considered this flow as circular (the symbol was the wheel).

Plato, in the work “Timeo”, studied the problem of time in a complete philosophical significance, defining it as “the moving image of eternity”, which “proceeds according to the number” [18]. Aristotle explicited the link between time and thought, which remained a constant of philosophy. In the work “Physics” [19], he defined time as “the number of movement according to “before” and “then””.

By the Renaissance, it appeared the scientific conception of time, which considers time as a set of homogeneous instants, until the Newtonian concept of “absolute time”. Newton spoke of time as an attribute of God, “absolute time” that can be measured such as “relative time” [20]. Modern physics has challenged the classical notion of time; relativity requires the acceptance of different temporal sequences, depending by the speed of the various observers [15]. The problem of the subjectivity and objectivity of time is an issue of great emotional and intellectual impact for adolescents, who are walking toward adulthood, in particular in relation to the personal available amount of time, compared to the finite length of the human life [21].

4. Special Relativity

Special relativity changed the assumptions on the causal structure of space-time, leading to define the notion of “light cone” of an event, but not a notion of simultaneity. About the invariant structures of space-time, if in the pre-relativistic physics we had the time interval between each pair of events and the spatial interval between simultaneous events, in special relativity we find the “space-time interval”, defined as $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$, where c is the speed of light in vacuum and (x, y, z) the coordinates of a generic point P in space. The special relativity is a theory of Lorentzian flat geometry, with pseudo-euclidean metric.

5. General Relativity

General relativity is born by the attempts to formulate a theory of gravity, which is compatible with the basis ideas of special relativity and the “equivalence principle”. This principle suggests that the “free-fall motion” in a gravitational field can be considered as analogous to the inertial motion of the pre-relativistic physics and special relativity [1,2,3,22]. Gravity can be interpreted as a change in the structure of space-time, which allows to an inertial observer an acceleration relatively to another. Einstein discovered that this idea could be implemented simply by generalizing the flat Lorentzian geometry in a “curve” geometry. General relativity is therefore a theory of space and time, taking into account of the physical effects of gravity in terms of curved geometry of space-time [23]. The geometry of space-time is not fixed “in advance” (as in special relativity), but “dynamically evolving”.

6. Peculiar Involved Basis Notions at Academic Level

Introduced the notion of tangent vector for describing an infinitesimal displacement from a point P_1 to P_2 , the set

of all tangent vectors in P_1 has the natural structure of vector space, but in a curved geometry a tangent vector at P_1 cannot be naturally identified with a tangent vector at a generic point Q different from P_1 . Therefore the more general notion of “tensor” in P_1 must be used, which generalizes the notion of “four-vector”. An illuminating example of a tensor field, i.e. of a tensor defined on all the points P_i of a space, is a “metric”, which is an inner product on the tangent vectors [24].

An elegant and mathematically clear way for defining a tangent vector is a “derivation”, i.e. a directional derivative operator acting on functions. The most intuitive way is to consider a curve, locally described by the coordinates $x^\mu(t)$ of the point on the curve as a function of the parameter of the curve “ t ”, and to identify the tangent to the curve at the point $x^\mu(t)$ with the set of n numbers $(dx^1/dt, \dots, dx^n/dt)$ to the point on the curve, labeled by the parameter “ t ”. About the need of having a mathematically precise notion of “set of points” constituting space-time, the appropriate notion is that of “manifold”, which is locally seen like $\mathbb{R}^n$. Defined the tensors, it is possible to define a tensor field of type (k, l) and to use the notion of “parallel transport” along a curve. The notion of curvature completes the essential mathematical material needed for the formulation of general relativity [13,25].

7. About the Teaching of General Relativity at Academic Level

Anticipating special relativity to general relativity, also at qualitative level in secondary school, the obstacles of conceptual nature decrease. It is important the knowledge of classical mechanics, as well as notions of electromagnetism; the even qualitative knowledge of electromagnetic waves helps for learning the concept of gravitational wave.

Approaching special relativity, it is interesting to possibly give the appropriate emphasis at the geometric point of view. Even at qualitative level, are significant:

- 1) a clear explanation that space-time in general relativity does not have the structure of vector space;
- 2) the introduction of the notion of tangent vector to a curve;
- 3) the introduction of the notion of a space-time metric and its use for determining the “proper time” spent along a curve;
- 4) the introduction of the notion of “geodesic curve” as curve that “extremes” this time.

For a course at academic level, it is believed that the known key thoughts are:

- a) how to calculate the elapsed time along arbitrary “time-type” curves.
- b) How to determine time-type geodesics (which represent the possible paths of “free-fall” particles) and null-geodesics (representing possible paths of light rays) in space-time [13,25]. If students do not possess the necessary tools for understanding the Einstein’s equation, it will be not possible the derivation of the solutions, so they will have to “accept” that space-time “births” as solution of the Einstein’s equation.

c) After that, it is possible to discuss the Schwarzschild solution, representing the external gravitational field of a spherical body, and the solutions of FLRW (Friedmann-

Lemaitre-Robertson-Walker), representing homogeneous and isotropic cosmologies.

d) In relation to the Schwarzschild solution, it is possible to solve the equations of the “null-” and “time-type” geodesics, and then to derive predictions for the motion of planets and the “bending” of light, so as the nature of “black-hole” of the (extended) solution of Schwarzschild.

e) Other key issues are the gravitational radiation and the nature of black-hole, all fascinating topics for students minds and that open the way to unified theories [26].

8. Charming Topics of Unified Theories

The current theories about the unification of fundamental forces include new dimensions, that our senses do not see or feel, hidden extra-dimensions [15,16,17]. Students can think about simple everyday situations, such as the picture of a point on a sheet; it is actually a three-dimensional object, with three dimensions, because of the ink of the pen. It is the “representation of a point”, not “a point”. For the same reason, the sheet of paper on which we have drawn the point, however subtle may to be, has still a thickness [6,27]. We identify normally a hair with a line, but on fact it has a thickness; we identify normally the tip of a pin with a point, but on fact it is a sphere, and so on. This enables the students to understand that in reality there are always insignificant sizes, if compared to the others, relatively to the context in which we are.

The simplification in mathematical and physical modelling research led to classical physics, relative to the real world, easily observable with the available tools, but if we try to understand more about the “ultra-small” and “ultra-big world”, we do not have many available tools, nor the nature provides obvious evidence.

Superstring theories are one of the attempts to explain all fundamental forces of Nature and known particles in a unique comprehensive framework, considering the vibrations of tiny supersymmetric (special symmetry of physics) strings (one-dimensional objects) [28,29]. Strings are the basis constituents of such theories, one-dimensional objects instead of entities with null length (mathematical points). At each point of visible space it would exist therefore six (or seven) new dimensions, strongly wrapped to form geometric objects.

Quantum geometry is a recent field of mathematics that introduces a new concept of space, unifying methods of classical differential geometry with non-commutative algebras and functional analysis; it incorporates in geometry many ideas of quantum physics, works with quantum spaces and includes the classical concept of space as a special case [30,31,32,33].

Differently by the classical geometry, quantum spaces cannot be interpreted as a collection of points; there are non-trivial quantum fluctuations of geometry at all scales [30]. A very interesting application of quantum geometry in physics regards the mathematically consistent description of physical space-time at every scale, in particular at ultra-microscopic distances, i.e. of order of the Planck length (about $1,6 \cdot 10^{-35} m$).

All these innovations open the way to new amazing scenarios, such as time travels, the multiverse, the discrete

nature of space-time, quantum computing, which until a few years ago were subjects of science fiction. These scenarios attracted always children, youngs and adults; this should be taken into account since primary school [6,27].

9. Summary

The study of relativity, one of the two pillars of modern physics, and the evolution of this theory toward an unified picture of the universe, one of the objects of study and research for current theoretical high energy physics and advanced mathematics, can surely be used to help children in the process of evolution, growth, self-knowledge and love for knowledge, which is one of the most important aspects of human life.

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