

The Scheme of Movement as an Organizer of Action in Classical and Relativistic Mechanics

Gabriel Dias de Carvalho Junior^{1,2,*}

¹Physics Department, University of Namur, Namur, Belgium

²Faculty of Science, Catholic University of Louvain, Louvain-la-Neuve, Belgium

*Corresponding author: gabriel.carvalho@unamur.be

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Abstract This paper reports on the use of the concept of scheme as one of the references for analyzing the process of making out the meaning of relative time. It is part of the current perspective that discusses the inclusion of Modern Physics topics in high school education; the paper investigates the conditions for such inclusion to occur. To this end, a didactic sequence was produced based on the transition between the key concepts of Classical Mechanics and the Special Theory of Relativity, where one of the central points was the discussion of the influence of the reference frame in the study of motion. The research activities lasted 16 classroom hours in a third-grade year high school and were quite diverse, with lectures, group discussions, open problem-solving, and thought experiments. We analyzed, in this paper, episodes of verbal interaction and written activities of the students related to the concept of reference frame and its influence on the notion of relative time. The theoretical framework used for the analysis of the students' productions was the Theory of Conceptual Fields, which was supported by the content analysis technique. Different operational invariants were identified in the students' schema of motion and time. We concluded our research by indicating that there must be a reciprocal assimilation between the schema of time and motion for the students to be able to propose correct answers to problems in Mechanics.

Keywords: *scheme, operational invariants, subject in action, motion, time*

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1. Introduction

This paper presents the use of the concept of scheme in the analysis of interaction episodes between teacher, researcher, and students. These episodes occurred throughout the application of a teaching sequence in a third-grade class of a Federal Institute in Brazil. The teaching sequence, conducted during 16 class hours, was about the transition between Classical Mechanics (CM) and the Theory of Special Relativity (TSR).

We aimed to investigate students' meaning making process on relative time [1], and the general goal was to precisely follow students' cognitive trajectories when they studied the transition between classical and relativistic models for Mechanics.

In this sense, we surveyed works that, in some way, dealt with the senses of time and frame of reference. The first one we considered was performed by [2], which presents a conclusion linked to the choice of references by the subjects to study movements. In this work, the authors indicate that the Earth works, for the subjects, as a kind of privileged reference, in relation to which the states of movement and rest are defined.

In a work focused on the construction of a conceptual profile, [3] addresses the displacement of the profile of high school students when studying Classical Mechanics and relativity. The author constructs the different zones of the conceptual profile that evolve from the beginning of an egocentric view to the relativistic view, in addition to identifying the points that may be possible obstacles to understanding the scientific concept of reference.

Reference [4,5] present results of conducting a contextualized teaching sequence based on TSR. For the authors, the meanings of time and space are essential in this study and should be themed to avoid the erroneous attribution of their meanings within the TSR. The general conclusion is that it is possible to teach TSR in high school, which enables a deeper understanding for students in relation not only to the theory itself, but also to the way with which scientific knowledge is produced and validated. However, this type of approach may not be available to all teachers because of some problems in their initial training.

A possible response to this need began to be drawn, in his doctoral thesis, by [6]. The author explored the understanding of TSR in undergraduate physics students. In this context, he studies the essential elements— theoretical and methodological—for the formation of teachers capable of presenting TSR in high school as a

body of knowledge relevant to the dialogue with respect to the ways of validating scientific knowledge.

Based on a theoretical framework that intends to re-signify the movement of conceptual change, the work of [7] seeks to discuss the appropriation of the concept of time itself by adolescents. For this, instead of investing in the classic model of conceptual change, the authors use the coordination class theory to classify the forms of thought and the levels of appropriation of the concept from didactic intervention activities.

A more general approach on time as a concept was made by [8], who researched how the same is structured from considerations related to Bachelard's notion of epistemological profiles. In this work, the author discusses the various zones of the time profile—from those most linked to a personal notion of time—with a strong egocentric connotation in order to derive formulations linked to the modern notion of time, whether in TSR or quantum physics.

Also exploring the concept of conceptual profile, [9] study the conditions for implementing the study on TSR in a Brazilian high school. The authors seek, even if in an initial manner, a dialogue between Piaget's theory of balancing and some elements of Vygotski's sociocultural psychology to propose that it would be interesting, from the didactic point of view, if the FMC be presented over the three years of high school so that there would be a more complete understanding of key concepts of physics.

Reference [10] also seeks to investigate the possibilities of didactic intervention linked to TSR. However, the path chosen was that of a discussion linked to the nature of science and the historical aspect linked to the emergence of TSR. In this sense, the authors report an interesting experience pertaining to insertion of this conceptual field in a terminal class of a high school in Santa Catarina from a didactic module written especially for this purpose.

Still, in this didactic bias, [11] reports a strategy of presentation and discussion of the phenomena of temporal dilation and spatial contraction based on diagrams. The author defends the idea that these are difficult to understand from the students' point of view and that the use of diagrams could help them in the construction of representations, thus facilitating learning.

Reference [12] presented a didactic sequence focusing on the transition between both Classical and Relativistic Mechanics. The authors used the Theory of Conceptual Fields to conceive the role of activities and analyze students' productions. The main conclusion points to the fact that students tend to take into account absolute frames of reference to interpret problems in mechanics.

These studies, despite presenting relevant discussions for the area, did not examine how a given student perceives the changes in the epistemological status of some key ideas of physics on the border between two distinct conceptual fields. In this context, this paper presents a part of the research carried out for this purpose.

We aimed to discuss whether CM works, for the learners, as a necessary level for understanding TSR, without which an individual does not recognize the relativistic model as a possible source of destabilization and, therefore, does not learn it.

TSR is a conceptual field that breaks several assumptions of CM, such as the absolute character of space, time, and mass. The research conducted at the border between these two fields can, therefore, be fruitful by revealing ways of thinking and acting of students in situations that require a high level of abstraction.

Because it is a complex and multifaceted theory, it is necessary to choose the study subjects in which it is possible to investigate the aspect related to the elaboration of relative time. This is possible for several reasons, among which two stand out. First, the construction of relative time within the TSR is a theoretical prediction conducted from the postulates and, as such, has the same epistemological status as other effects of spatial contraction. Secondly, as [13] indicates, the notion of absolute time has a stronger ontological status than the other notions treated in TSR, since this notion is considered absolute in two ways: as a physical quantity (1) that does not interact with any other and (2) whose measure does not depend on the frame of reference we used.

Therefore, the transition between the notions of physical time in the two conceptual fields provides more elements of analysis for this research and requires an articulation between time, space, and speed. By fixing the focus of the analysis on the notion of time, one can construct a research instrument that uses the key concepts of CM linked to movement (reference, space, speed) for investigating their role in the construction of the conceptual relations of TSR.

One of the important steps of this research, then, was the discussion about the movement as a relative notion to create the very conditions for the discussion about relative time. That is why our attention was focused, at the beginning of the research activities, on the discussion about the concept of frame of reference, which will be the focus of this work.

The organization of the didactic sequence used was based on a discussion about the influence of the choice of a frame of reference in the study of movement and how this choice is arbitrary. Then, the teacher presented Newton's first law, which deals with frames of reference without acceleration as the basis for the entire construction of Newtonian mechanics. After showing that the laws of mechanics do not change in inertial frames of reference, the teacher presented some paradoxes between Classical Mechanics and Electromagnetism. These contradictions were the starting point for the study of the theory of relativity and its consequences. At this moment, the teacher presented the concept of relative time and the relativity of simultaneity. After this, the following subjects were worked on: relative space, energy, and relativistic mass, without, however, analyzing the students' productions from the point of view of the research.

The analysis presented here is focused on the validity of using the notion of time as a scheme and investigating the reciprocal assimilation between the schemes of time and movement, which is one of the findings of our research. For this, we will present the concept of scheme adopted by us and make the articulation between this concept and the research episodes to support our analyses and conclusions.

2. Concept of Scheme

2.1. Scheme in Piaget

The concept of scheme that will be used for the development of this work is associated with that presented by [14]. The choice of this concept is due to the fact that it brings together all the complexity and operability of the subject in situation, our study focus.

This concept is presented as follows:

We will call action schemes what, in an action, is transportable, generalizable or differentiable from one situation to the next, that is, what is common to the various repetitions or applications of the same action.

In this definition presented by Piaget, three main characteristics for the schemes are perceived.

In the first place, a scheme is oriented to a class of similar situations because only in this way will it be possible to generalize some action. There would be no scheme without the existence of situations. To put it another way, the phylogenetic and ontogenetic processes of the constitution of schemes presuppose subject-object interaction, or scheme-situation, as [15] argue.

Second, the schemes are linked to what is repeated in the application of the action. Before referring to an automatism, this statement is linked to the idea that the exercise of assimilation (and, therefore, the very way in which a subject knows objects and interprets them) is only possible from the existence of regularities in action. Reference [14] said, in this sense, that "schemes are like a summary of actions likely to be actively repeated". The idea of repetition is linked to the way in which the subject prepares the action, not to the action itself.

Finally, and because of the two other characteristics already mentioned, schemes are processes that have an internal organization specific to each of them. This process must have, then, a certain sequence of steps that characterizes each type of scheme, as well as a content that gives the context for the unfolding of the action.

Actions are generalized in the form of "schemes" whose organization crystallizes in relatively constant form, and this relative constancy of schemes is translated by the construction of invariants of the real, such as the scheme of the permanent object which postulates the existence of substances under the perceptual frames [16].

The fact that there is an organized process that characterizes the scheme should not be indicative of a reduction of this to a mere procedure or an algorithm. The scheme can adapt to the contingencies of situations from the double dialectic of assimilation and accommodation and can, as [17] points out, be oriented towards the construction of inferences.

2.2. Scheme in Vergnaud

In Vergnaud, scheme is seen as an invariant organization of activity for a given class of situations. Thus, it must be understood, in the Theory of Conceptual Fields (TCC), as a dynamic and functional totality in which its constituent elements always work in a solidary way. This definition results in an important relationship: the construction and development of schemes is a process profoundly dependent on situations.

It is possible to perceive Vergnaud's affiliation to Piaget's ideas since the concept of scheme continues to present the characteristics of a situation-oriented process that has a specific internal organization. However, in Piaget, the conceptual dimension was not thematized despite being essential. In this aspect, it can be understood that Vergnaud advances by proposing that there is an implicit conceptual basis—the Operative Invariants—in every intelligent action of the subject.

More precisely, <the scheme> is an invariant organization of activity for a defined class of situations. Invariance characterizes organization, not activity; the scheme is not a stereotype; it allows, on the contrary, to treat contingencies and novelties, which would not be the case if it were a stereotype. To be addressing a class of situations, is a universal. To study the activity of individuals it is then necessary to identify the different categories of situations with which they are confronted. [17]

Vergnaud takes up an important Piagetian thesis according to which knowledge (or intelligence) is adaptation. This important idea, however, needs further refinement in order to make it operational. In this sense, there are two central questions that must be answered: What adapts? And adapts to what?

The most immediate answer could be "the subject adapts to the object and the object to it", as Piaget said. But there is also the need to know what aspect of the subject and the object adapts. Vergnaud indicates, in this context, that schemes adapt to situations.

As a scheme is oriented towards a class of situations, it is possible that, when addressing a given problem, the subject must adapt itself to the characteristics encountered. This causes not only a structural change in the scheme, but also modifications in the assimilated object. Therefore, the scheme-situation interaction provides the basis for multiple influences between the subject and the object.

In Vergnaud's Theory of Conceptual Fields (TCF), the schemes, as the basis of the cognitive functioning of the psychological subject, must encompass the operational invariants. Only in this way can the cycle of the subject's actions behave a conceptual dimension that will allow him/her to recognize the pertinent elements and operate with them in action. In addition, schemes' conceptual helps us explain the fact that the more it is advanced in the process of cognitive development, the greater is the diversity between the schemes found and the ways in which a given subject acts in a situation.

Because TCF is a theory that focus the subject on action, it seeks to understand the ways that allow one to approach and satisfactorily manage situations.

The central thesis here is that each situation incites, in the learner, the use of its own set of schemes that, therefore, enables the development of new schemes, skills, and concepts that are contained in each conceptual field. In this sense, it is necessary that the subjects recognize something that can be managed in the situations and that they can establish "bridges" or build relationships with the concepts provisionally stabilized.

Therefore, subjects must share something with the social sphere of concepts in a given conceptual field in order to be initiated in their practices. These personal constructions are used in action by the subject and must

also be available in the cultural domain as a series of propositions, relations, and meanings. They are considered as the link between the subjective and cultural domains because the subject is immersed in the cultural domain and culture is "embodied" in the subjects. In CFT, these formulations are called Operational Invariants (OI) and have two categories: concepts-in-action (CIA) and theorems-in-action (TIA).

These two categories of OI are only well differentiated in relation to the role they play in the conceptualization process from the analysis of two instances: pertinence and veracity. First, it is important to emphasize that, in CBT, the conceptualization process is activated by the interaction with the elements of the situations and the subjects are active in their recognition and operation.

In this process, there are two instances that combine in the action of the subjects. The first belongs to the relational domain and can be considered as true or false based on comparison with some theoretical framework; the second can never be considered true or false since it does not use propositions or relations about the physical world, thus having the status of pertinent or non-pertinent with respect to the situation being proposed. The concepts considered pertinent by the subject in action are called concepts-in-action and, therefore, do not have the status of "pertinent" or "not pertinent" instead of "false" or "true". The propositions constructed with the concepts, and which are held to be true by the subject in action, are called theorems-in-action. These two categories vary between subjects and, for the same subject, vary from one action to another depending on what is taken as pertinent or true.

It can be seen, therefore, that the way in which Vergnaud presents and uses the concept of scheme bears similarities with Piaget's conception. First, in both authors, one perceives the idea that there is something in the cognitive functioning of the subject that is responsible for the organization of the activity. This means that, at all levels of development, there is a regulation of the action of the subjects.

Another feature in common between the two authors concerns the fact that the schemes have a specific orientation to a class of situations, even though the term "class of situations" may have a sufficient elasticity to suit the description of the schemes that each author wants. This orientation to situations is revealed, mainly, by the repetition of a certain sequence of actions adapted to them.

However, Piaget became more interested in the construction of a development model for the schemes, especially regarding the first years of life of the subjects. At this moment, the conceptual dimension is not thematized by Piaget, nor is the ability to generate inferences despite knowing that the author does not exclude these possibilities. For example, in the permanent object scheme, one perceives the inference about the existence of objects even if they are outside the perceptual field of the child.

Vergnaud's scheme is a centerpiece of the subject's action. The characteristics of the organization of action and of being a cycle of transformation are clearly derived from the Piagetian notion. On the other hand, the greater detailing of the dimensions that make up a scheme and the attention to the specific conceptual domains represent a fundamental difference between the two authors.

In this sense, the domain of operational invariants, used by Vergnaud in his analysis of the schemes, reveals, in our evaluation, the greatest difference in relation to Piaget without, however, breaking with the Piagetian tradition. It is possible to say that the description of the operative invariants and the establishment of their mechanism of action represents what Vergnaud advances the most in relation to the more specific analysis of the subject's activity in specific domains of knowledge.

3. Important Works on the Formation of the Notion of Time

3.1. Time as a Scheme

The first of the theoretical positions is related to the Geneva school's production of the notion of time. There are several works that explore this notion and study several aspects. Two works, in particular, present important elements for the interest in focus here.

The first work was the research carried out by Piaget; it was based on a suggestion of Einstein which is synthesized in "Le développement de la notion de temps chez l'enfant" [18]. The second work represents the consolidation of a set of researches on time under numbers XX and XXI of the yearbook of studies on genetic epistemology, which received, respectively, the names of "L'épistémologie du temps" [19] and "Perception et notion de temps" [16].

The central question of the first work is the investigation of whether "the subjective intuition of time is primitive or derived and, from the outset, sympathetic or not to that of speed" [18]. Thus, Piaget intends to investigate whether the notion of time is its own entity, independent of the concepts of space and speed and, in a certain way, shielded from cultural influences or, if on the contrary, this notion is constructed in coordination with the notions of space and/or speed.

In this sense, Piaget already presents, since the beginning of the work, his position on this question, stating that

Time is the coordination of movements: whether it be classical displacements or movements in space, or internal movements that are actions simply sketched, anticipated or reconstituted by memory, but to which the result is also spatial, time performs on its part the same function as space with respect to immovable objects. [18]

For Piaget, temporal operations derive from preoperative conducts, and time is nothing other than the organization of this set of operations. Time, however, is not reduced to a simple relationship. It shows a multiplicity of progressively coordinated functions and encompasses, little by little, velocity as an inverse function, in the sense that the faster a movement, the shorter the time required for it to occur [19].

The most significant conclusions of these studies point to the fact that one cannot talk about the notion of time as dissociated from situations and other concepts, especially from the concept of speed. Piaget claims that there is an operative time which is associated with time intervals and the notion of succession. This time develops in a manner

analogous to logical operations and is in close connection with the notion of speed. About this relationship between time and speed, the author also states that

The construction of time begins when different speeds are compared between them, speeds of human activities as material movements, and such construction ends with the coordination of these speeds: the notion of time and speed are then correlated. [18]

In affirming this, Piaget leads his argument to conclude that time is a scheme which is formed and differentiated throughout the development of the subject due to the confrontation of the various situations to which a subject is exposed. This is a position that gives time an operative nature of great relevance because it houses on it a set of concepts, relations, and modes of action for the class of situations that involve temporal notions.

In this way, Piaget concludes his research by saying that time does not belong to the category of "a priori forms of sensibility" as Kant thought. On the contrary, it is necessary, and, as occurring in the constitution of every scheme, the structures of reversibility must be developed so that it can act in fullness.

Finally, Piaget briefly discusses the possibility of broadening his conclusions to the relativistic domain. In this sense, he states that

Relativistic time is only the extension at great speeds of a principle valid from the primitive states of the formation of physical and psychological time, from the genesis of time in young children. [18]

The current criticisms of this work of Piaget are placed in several points and were very well summarized by [20]. It is possible to perceive, in these criticisms, both positions contrary to the experimental method used by Piaget and questions about the generality of the conclusions. In none of the works cited by the authors, however, was there the questioning of the notion of time as a scheme.

3.2. Time as a Concept

In his work on the constitution of physical time, [8] considered time as a concept and aimed to understand how elementary and high school students construct it. The context of Martins' research is more restricted than that of the Geneva school, since he investigated only the construction of the concept of physical time.

The theoretical reference of the work was Bachelard's Epistemology, where the author deepens the notion of an Epistemological Profile for time. The author established a chronology of conceptions about time, from the Greeks to the studies on the theory of relativity and quantum physics, in order to identify the zones of the epistemological profile for the concept considered.

Martins proposed to establish such an epistemological profile and he did so by relying on four different zones, which are, roughly, both the categories that can access the subjects when solving problems as well as categories indicative of a paradigm of scientific thought in various eras of the history of sciences. These categories are naïve realism (subjective notion of time and marked by egocentrism), empiricism (time as a quantity that can be measured in a univocal way), traditional rationalism (time

as an abstract mathematical parameter) and surrealistism (the modern view on time).

One of the points that Martins' work adds to the discussion about time is the possibility of establishing parallels between the historical and personal development of conceptions about time [8]. In this sense, the author works with the concept of time in a cultural perspective that Piaget and the Geneva school had not done.

Some of the conclusions of this research are close to those obtained by Piaget in the works cited above. That is, there is a genetic route for the construction of the notion of time that is made by the coordination between the other important notions of space and speed. In addition, the categories presented by Martins reveal a progressive growth in the level of abstraction, in which each category, in a certain way, encompasses the previous one(s) in a recursive process of signification.

We consider that the relevance of this research lies in addressing, from the point of view of the construction of concepts, the forms of understanding about time. The study conducted by Martins with several different age groups allows not only the identification of the zones of the epistemological profile of each student, but also the analysis of an ontogenetic and phylogenetic development of the notion of time.

We defend the idea that the two conceptions can be compatible with the concept of scheme in Vergnaud. The existence of a time scheme, associated with a set of situations (evaluation of speeds, distances, durations, for example) and concepts (frequency, durations of events, instant, speed, for example), arises from the necessary organization of actions in this area.

This time scheme has different sets of epistemic contents in each subject that can be used from what each one perceives as pertinent in the situation faced. These sets of epistemic contents—the operational invariants—are the personal basis for what Martins identified as the zones of the epistemological profile of time.

In this sense, the use of time as a scheme that organizes the actions of the subject and that works in solidarity with the movement scheme seemed to us to be more adequate for the analysis of the subject in the given situation and, also for this reason, our affiliation to the TCF occurred as a theoretical reference for the research.

4. Materials and Methods

4.1. Place and Research Subjects

The research was conducted in third-grade students (aged 16–18 years) from a federal public school in the state of Minas Gerais. Because it is a unique federal public school in the region, there is great competition in the selection exams. At the beginning of high school, classes are always full, with students exceeding the capacity of the classrooms (40 students). Because of distaste and dropouts, the third-grade classes are empty. The student group followed in this research had 16 students.

The students who participated in this research when they were in the first grade of high school studied Mechanics from an unconventional didactics sequence.

The didactical approach was oriented to the discussion of the central concepts, to the detriment of an exhaustive study of the concepts considered more "peripheral", and their mathematical formulations.

Although kinematics was not taught separately, as is common in Brazilian schools, its central elements were taught throughout the discussion on Classical Mechanics' model. For example, the concept of acceleration was only discussed within the context of Newton's second law. Despite it all, it was ensured that the key elements of Mechanics, such as the concepts of force, energy, or speed, were discussed together and in full agreement with what Vergnaud says about the need for various concepts to address a situation and the relational character of the concepts.

At that moment, the study of mechanics was initiated with a discussion of Galilean relativity and the need to take into account the frame of reference for the study of movements and for the measurement of velocities. In this line of reasoning, the principle of Inertia and Newton's first law were discussed as the cornerstones of the whole logic of CM. This study served as a starting point for the discussion about the limits of validity of a given scientific theory and about the role of the explanatory models. Then, based on the limitations of CM, Einstein's TSR postulates were presented as a possible way to resolve these limitations. As a theoretical exercise, incursions were made on some consequences of the postulates of relativity on the relative concepts of time and space.

The students who were part of the research, therefore, had already studied, briefly, some topics of TSR during the study of Mechanics. In addition to the postulates of restricted relativity, the dilation of time, the contraction of space (in the didactic sequence indicated in the previous paragraph), and the mass-energy relationship (when studying the concepts of work and energy) were taught. This study had occurred about two and a half years before the application of this research. The following year, they had already studied some concepts of Electromagnetism and Optics. Thus, it was expected that the situations and concepts presented throughout the research were not understood in the same way as before.

This recursive didactic approach never represents as a simple repetition for an individual who learns, since his cognitive repertoire, schemes, and knowledge-in-action are different from what they were in the past. On the contrary, this is a way to use different situations for increasingly complex and rich activities. Therefore, there exists a belief that students could, even if unconsciously, use the formulations of the TSR when solving problem situations.

In conclusion, the choice of these students for the application of the research was due, in large scale, to the possibility of investigating, after a long time, what were the changes in their ways of thinking and conceiving a physical theory.

4.2. Planning activities

The research methodology was planned together with the teacher and the researcher. This methodology respects the teacher's style, interests, and way of approach to

physics, and supports the construction of the data needed for the research.

The identification of schemes and OI requires that several records of the subjects' activities be checked, since, because they are not explicit, they must be inferred. In this sense, any research activity that proposes to achieve this objective should allow the subjects to be confronted with situations for which they do not have consolidated answers yet. Therefore, our design for the research was organized based on moments when the students (1) solved written conceptual problems, (2) discussed open problems in small groups, (3) interacted with the teacher in conceptual discussions, and (4) watched videos and animations about the concepts that were being studied.

We organized the research activities from four axes, starting from the concepts related to CM until the study of the consequences of the postulates of the TRR. We discussed the next three moments.

I) Presentation of the fundamentals of Mechanics: The research conducted by [2] indicates a central problem in the use of the concept of frame of reference. The discussion about the relativity of time naturally involves the use of such a concept. It was essential for us to resume this concept and place it in a central perspective for the study of movements, and hence, we needed to start the didactic sequence from this concept. Despite this, we wanted to rescue some notions, such as movement, rest, trajectory, speed, acceleration, and force, to present the principles of Galileo's relativity and inertia. Given this, it would still be possible to justify, for the students, the emergence of a new conceptual model such as TSR.

II) Identification of the tension points between the conceptual fields of Classical Mechanics and Electromagnetism: So that TRR could be fruitful as a coherent model for students, we understood that we could try to explore the limits of the classical model and identify its weaknesses. This approach has been identified in the literature as one of the strategies in the organization of teaching narratives [21]. Thus, we organized the discussion based on two important situations (inconsistency of equivalence between inertial frames of reference in Electromagnetism and the application of magnetic forces by a conductive wire traversed by electric current) to show that the results predicted by MC are inconsistent with Electromagnetism. It was at this time that we could discuss the pertinence of a new conceptual model.

III) Presentation of the TSR postulates as a possible resolution of the problems discussed above: In response to the problem of incompatibility between MC and Electromagnetism, the postulates of relativity were presented and the meaning of each one was explained. The central point we wanted to achieve was to show that the conceptual model proposed by the TRR would be a way to solve the problems raised earlier. However, resolving the inconsistencies presented required a change in the epistemological statutes of key quantities of physics, such as time. Following [17], this approach allowed us to create a context in which students could use their schemes to build new meanings for time.

4.3. Data Construction and Analysis Methodology

The problems proposed to the students were collected for initial evaluation of the possible models they used. The first time, cognitive trajectories were constructed for each of the students through their written records. In these trajectories, the answers they presented, and some possible interpretations made by the researcher, were recorded.

Written activities were summarized, and each student received a distinct code for each question. The analysis of the answers was transcribed to a table where only the codes were included. After performing the analyses, the students' names were then revealed for the construction of a first version of their cognitive trajectories together with the episodes of interaction between both teacher and colleagues. In this sense, we sought new actions that could corroborate the analyses performed or refute them in the activities performed in groups and in interactions with the teacher.

Finally, we tried to group the students based on the similarity with which they approached and solved the problems in order to verify the consistency of the operational invariants and the schemes we had inferred. This process allowed us to verify how students progressed as the teaching activities were developed by the teacher.

With this first version of cognitive trajectories in hand, we made a submission pertaining to the analysis of other researchers into our research group. This step was essential to evaluate the existence of a logical line between the proposals of the activities and the students' responses.

This first version of cognitive trajectories was presented and discussed with other researchers into our research group. This step was essential to evaluate the existence of a logical line between the proposals of the activities and the students' responses. After this internal discussion, all the results were presented to researchers in Gérard Vergnaud's research group. Interactions with them were essential to refine the interpretation of the data in this research and allowed for the mitigation of the subjective aspects of personal interpretations inherent in qualitative research.

5. Results

We will describe the first five lessons of the research activity, which were exactly those that dealt with the study on the concept of frame of reference and the relative characteristic of the movement. In each of them, we presented what we expected to verify, thus justifying the reason for choosing the tasks placed before the students. We also explained the episodes and productions of the students who were most important to us for the establishment of their cognitive trajectories. We also present the tenth lesson that represented an activity of individual problem resolution (theoretical problems) by the students.

Our complete analysis of cognitive trajectories took into account many more elements than those that were explained here. This article presents just a part of the

whole research, which focused the notion of frame of reference and the fundamentals of Classical Mechanics.

1st lesson:

This first lesson was dedicated to the presentation of the concept of frame of reference and its influence on the study of body movements. This concept is essential for the whole study, both of CM and for TSR. Therefore, the pedagogical interest of the teacher is in starting the work from a key concept that is present in the two conceptual fields.

The initial interaction between teacher and students was conducted in order to rescue situations that would have already been faced so that the next concept discussed could find echo. This teacher's attitude was recurrent throughout the classes in which the subjects were already known to the students. In the classes in which the concepts treated were more abstract and represented and, in a way, novelties for the students, the classes had a lecture character on the part of the teacher.

Teacher: (...) What do you mean by frame of reference? You << to a student in the classroom >>, what do you mean by frame of reference? For you, when you hear the words "frame or referential", what comes into your head at that first moment, that ... "start"?

Mariana: Something we take to analyze ... to reference ... something... when we analyze a movement, we take something to know ... if certain thing that we take... if the movement is in a way ... something like that.

Teacher: Antonio, when you see those words "frame of reference", what is the first idea that comes into your head; that initial idea that comes into your head?

Antonio: << uses a lot of gestures >> Oh, it's what you take as the basis for you to analyze something else. Because you don't usually analyze "that," you analyze "from that." So, you take it as a basis for "that," you understand? For you to analyze the movement ... otherwise, you take "that" as a base. Like the movement of a car... you consider another to analyze it. If it goes straight... its speed. Like this, like this.

Teacher: <<agreeing>>. Let's see one more here. Cassiano, what do you say about the frame of reference? What's in your head?

Cassiano: It's a point ... initial... that you will draw conclusions about ... it...

Teacher: A starting point, from there you draw conclusions about it. Lara!

Lara: Oh! I think... I think in the same way as Cassiano. It will be a point from which you ... will... from that you will make a comparison with some other object or place to is having ... to have a comparison even, from that point you will make a comparison.

2nd lesson:

The first written activity occurred in the class following the teacher's initial explanation about the relativity of movement and the need to choose a frame of reference for the study of movements. The text and the proposed problem are discussed below.

01) Read the following humorous text, written by Millôr Fernandes, which was published in "Jornal do Brasil" on 01/26/1990.

VERY STRONG PREJUDICE

Every time I see, in newspapers, magazines, television, and on the street, people increasingly "free" of prejudices and ... And yet they are all convinced that the Earth revolves around the sun. Why is that? Ask them and they will answer, "See, Galileo 'proved out' this a long time ago." But for whom? He may have proven out for scientists. The common man, and even we, the pejoratively called intellectuals, accept. Without thinking. Prejudicially. As before Galileo we believed that the sun revolved around the Earth. But between Galileo - whose 'evidence' we never know or know what they are - and reality, which we can see, we must believe in our eyes. Our eyes see, with absolute certainty, that the Sun rises there (to the east) and sets on the other side (to the west), spinning around an absolutely standing Earth (earthquakes apart), on which we walk without feeling the slightest movement. For me, the sun revolves around the Earth!

What can we say about the text shown above? Is it the Sun or the Earth that moves? What does it mean to say one or the other thing?

02) ANALYZE the following statement and INDICATE whether it is true or false. Justify your indication. "If object A is moving relative to another object B, and object B is moving relative to C, then A is moving relative to C."

For the analysis of the answers, it was identified how the students apply the concept of frame of reference to address situations involving the movement and/or rest of bodies. The focus was not on investigating who presented the answer considered scientifically correct, but instead on how the student dealt with the problems involving the relativity of movement.

The two problems presented to the students had different characteristics according to the intended objectives. The first one was constructed from an ironic text by Millôr Fernandes, in which he disagrees with Galileo's statement about the Earth's movement around the Sun. For the author of the text, we are prejudiced in accepting the movement of the Earth without knowing its evidence, as were Galileo's contemporaries in defending the immobility of the Earth as dogma. It should also be noted that there is an ambiguity in the text—the author mentions the movement of the Earth around the Sun but presents evidence that mentions the daytime movement of the Sun in the celestial vault, which is related to the movement of rotation of the Earth. However, the theme addressed in the activity concerned the movement of translation of the Earth around the Sun. The question addressed to the students asked them to pose themselves for or against Millôr's assertion that the Earth does not move around the Sun.

This first question is an intermediary perspective between the most immediate evocation of the concept frame of reference (as was taught by the teacher at the beginning of the first lesson) and the formalization of this concept with extrapolation in new situations. This concept is at the core of a well-known situation of students and is already studied throughout their school trajectories.

However, each student should perform an internal activity of interpreting the arguments of the author and relate them to the concept of frame of reference, in addition to recognizing that it is an ironic text.

This problem-solving approach was important mainly because it allowed students to familiarize themselves with new concepts and work with them in relatively known situations. The conscious use of knowledge in its most immediate form helps in the construction of new relationships since it allows individuals to test the limits of their knowledge in domains that are not totally new. Thus, it is possible to do a progressive enlargement of the possibilities of action of the movement schemes of each subject.

In this activity, it was possible to notice that eight students made an explicit mention of the relativity of the movement between the Sun and the Earth. Within this category of answers, some were complex while others were quite direct. This fact reinforces the idea that students are able to reproduce school discourses with reasonable precision, especially in a situation that bears resemblance to what is discussed and presented in the classroom, which is in line with the fact that the operationalization of a given scheme can lead to more "automated" conduct if the situation so requires.

For example, Cassiano says that Millôr Fernandes's position "can be accepted as long as he has classified the Earth as a point of reference", and later says that "Galileo is also right because he considered the Sun as a reference". In this same line of reasoning, Antônio stated that "The text wants to tell us in a way that makes us think about reference. And for this question I answer it depends on the frame of reference. It means the choice you will do to analyze the movement, which you will take for yourself as a reference."

Still following this notion of relativity of movement, Thales states that

"The text above demonstrates a point of view on the movement of the Sun with respect to the Earth and vice versa, point of view that is not wrong, but it is also not the only one that is correct since there are two different references (...) This means that the movement is relative to the referential."

Only Nayara denied the relative characteristic of the movement. In her response, the student says that "we see the sun rising and coming from the other side, concluding that the movement is specifically of the Sun", which reveals a rudimentary use, in action, of the concept of reference. But then she states that:

"There are those who believe that the Earth moves around the Sun, because when we see around, we see a much larger body. Since we do not see the sun larger than Earth, some believe that it makes the move. But it's the Earth who really moves."

Nayara's scheme of movement no longer holds a fully egocentric characteristic in the same sense as attributed by Piaget to this concept [22]. This means that the student does not interpret the world from herself because, for example, she can analyze the movements from a certain reference external to her. But something curious occurs that will also be perceived in the OI of other students. If the frame of reference may not be the subject himself, Nayara always seems to look for another object to be her

absolute reference system. Between egocentrism and total decentralization, there seems to be a step common to almost all the students surveyed, which is linked to the establishment of this absolute reference external to the subject, i.e., the greater object presents in the analysis.

In this sense, Nayara seems to use a TIA linked to the movement scheme: "the larger body is stationary, and the smallest one moves". From it, the student concludes that we have the impression that it is the Sun that moves around the Earth because we see it as smaller than Earth. But since the Sun is, in fact, larger than Earth, it remains stationary. Although the relativity of movement does not exist for Nayara, and even if it is possible to have some impression of the solar movement, it is, as per the conception of the student, immobile.

In general, the research conducted in this area shows that, in fact, the Earth is the privileged reference for all everyday movements [1,2,3,6-14,16,18-27]. What is being added to this discussion is the reason why the Earth is intuitively chosen as a privileged reference. This is due to the fact that it—the Earth—is the largest body present in the situations of our life. When the situation is magnified to an astronomical scale, students tend to choose the Sun as an absolute frame of reference.

This trend can be verified in several other situations in which the notion of movement is present and will be perceived in several episodes narrated here. If the reference to be used is the body of large dimensions observable, the choice of the reference is not thematized. On the other hand, when the analysis requires comparison between two bodies of comparable size, the chosen reference is formally cited.

Still regarding this activity, Osvaldo presented an approach whose essence interacts with the absolute and relative views of the movement. He states, at the beginning of his response, that "it is proven that it is the Earth that moves around the Sun." He then states that the choice of the frame of reference defines which body is seen in motion. For this student, there are two distinct situations. The first one is anchored in the field of science. The other dimension, linked to everyday phenomena, does not allow us to affirm which body is in motion because the impression of movement is linked to the adoption of a frame of reference.

The second question, placed in a theoretical domain and which is more distant from the situations discussed in the classroom by the teacher, required the students to judge an affirmative that did not mention a specific domain of daily life. To solve this situation, it would be necessary for the students to go beyond an evocation or the use of a predicative knowledge. Despite it, the approach to this situation required the concept of frame of reference in a situation that had not yet been explored by the teacher and which, therefore, explores the use of part of the knowledge that was not yet consolidated. This characteristic of the proposed activity differs from the previous one by focusing on the operational form of knowledge.

The concept of frame of reference should be mobilized to handle for the presented problem. But there should be an articulation with the definitions of movement and rest. Thus, the activity analysis was conducted in order to evaluate whether, during the students' responses, the

words "movement" and "rest" were alone or if they were accompanied by the indication of a reference system.

Only four students were able to present the concepts of "movement" and "rest" with the indication of a frame of reference. Among them, only Osvaldo had presented, in the first activity (Millôr Fernandes' text), an explanation that seemed, to us, to be mixed between the relativity of movement and the adoption of an absolute movement with respect to Earth.

This is what is perceived, for example, in Thales' answer. The student indicates that:

"The above statement is false, since A is in motion regarding to C, when the frame is placed in C or B, since if the reference is placed in A, the other objects will go backwards while object A remains at rest. So, in this case C is moving relative to A and not the other way around."

Osvaldo's answer was that

"The statement is false. Because objects A and C could be at rest relative to each other, while object B is moving in relation to A and C. Thus, with the reference in B, A would be moving in relation to B and, with the reference in C, B would be moving in relation to C. But A and C would be at rest in relation to each other".

We can see in two others answers a partial mention of the relativity of movement, that is, the concepts of "movement" and "rest" linked to systems of reference. But when the word "speed" was mentioned, it was always disconnected from the frame of reference. In this category of answers, there are only students who had made the distinction between the references for the movement of the Earth and the Sun.

This is what can be seen in Maria's answer, who stated that "It depends. Because if object A is at the same speed as object C, then A is at rest relative to C, if object C have a greater speed than the speed of object A, then A is moving relative to C."

In these first two categories, we have a super category, with six students who were able to present, in whole or in part, the notion of relative movement in their answers. All of them had also presented this form of reasoning in the first question.

The third and last category includes three of the students and is marked by the mention of "movement" and "rest" without the relationship with the adoption of frames of reference. This means that such answers are written absolutely. Within this category is Nayara, who reiterates his previous position by denying the relativity of movement when choosing as an absolute reference—the car (which is greater than the driver).

In addition, in this last category, there are also students who, in the previous situation, had established the relativity of the movement between the Sun and the Earth. This fact can be explained by the difference between the types of activities and reveals that the notion of relative movement is not yet consolidated even though some students are already able to enunciate the principle of relativity of movement.

3rd lesson:

After watching a set of videos that dealt with the relativity of movement and trajectory, the students

responded to the second written activity, which featured a Garfield cartoon that made mention of the relativity of movement. In this cartoon, Jon says: "You could move a little bit!" So, Garfield answers, "Jon is not considering Earth translation."

Garfield's activity brought good elements for the continuity of the analysis of conceptions about the influence of the reference on the movement. In this activity, it was possible to identify the same three categories presented in the previous activity.

This fact is interesting for two main reasons. Firstly, it reinforces the existence of certain forms of action on the part of students. This trend can indicate the existence of schemes mobilized to operate within the class of situations involving displacements.

In this sense, we consider it valid and very useful, from the point of view of our analyses, to conceive the movement as a scheme that articulates the concepts of frame of reference, displacement, durations and speed. In this sense, this scheme is oriented towards seeking a reference system from which the possible displacements will be analyzed based on the absolute character of space and time. In this sense, the three categories listed from Piaget's works are present when considering the scheme of movement.

The most "primitive" scheme of movement is the one that organizes actions from one's own point of view and is, therefore, very closely linked to an egocentric view of the world. The most "evolved" is what coordinates the various points of view as possible for the establishment of displacements and the analysis of velocities. Between the two, there is an internship—perceived, for example, in Nayara—that is no longer totally self-centered, but that organizes actions from the search for the greatest body present in the situation and attributes to it the epistemological status of privileged reference. In this sense, it seems to be an attribution of a pseudo relativity to the notion of movement and rest.

The second reason, related to the first, concerns the trend of stabilization of new knowledge. When the first activity was conducted for the first time, the students could have only repeated information they had just heard from the teacher. There are marks of this form of action even in Garfield's activity—when students write that the movement depends on the frame of reference and then deny this idea. This form of response, however, tends to decrease over time.

4th lesson:

This is a 100-minutes lesson, and it was marked by work with Newton's laws, especially the law of Inertia. To this end, the teacher resumed the discussions already done during the introduction to the study of CM.

To show the application of this law, the teacher proposes a situation in which two different references need to explain the movement of a ball that is inside a bus. In this context, the following episode occurs.

Teacher: (...) Suppose you're here in a bus. (...) Okay, Vívian? You're standing here, in a bus ... and in front of you, Vívian, there's a ball. The bus is moving here, in uniform rectilinear motion to the right with a certain speed there. And there's an observer here on the street

watching Vívian pass by. Vívian, for you, is this boy at rest or he moves?

Vívian: He moves.

Teacher: He's on the move, isn't he? And according to him, are you on the move or at rest?

Vívian: On the move.

Teacher: On the move. For you, is the ball at rest or on the move?

Vívian: At rest.

Teacher: It is at rest. And for him, is the ball at rest or on the move?

Vívian: On the move.

Teacher: (...) Let's imagine that the Earth is a reference at rest. And you are in uniform rectilinear movement ... in the situation, you are also in a moving frame. Physical situations are equivalent for our observer who is at rest as well for your uniform rectilinear motion, do you agree with me? (...) Now I will play with you, folks. This bus, it has a certain speed. In a certain instant, the driver stops the bus. The ball in the bus ... what's going on with it? When the driver stops the bus, what happens to the ball?

Vívian: It starts rolling.

Teacher: Vívian, explain it to me, why did the ball roll forward?

Vívian: Because every moving body tends to get moving according to the law of inertia.

Teacher: All right! But there's an important detail here: for you the ball was moving?

Vívian: No ... it was at rest.

Teacher: It was at rest! So how do you explain it to me by the law of inertia if the ball was at rest? By the law of inertia, it had to continue what?

The situation presented by the teacher is very different from what the students had already studied. It is important to note that, at this point, Vívian's explanation was no longer justified, since, for this student, the ball was at rest.

This is a situation that goes beyond the validity domain of what is being studied, since Vívian's reference is not inertial in relation to Earth and, therefore, the terms of "inertial forces" or "fictitious forces" should be used. The teacher's intention, however, was only to "provoke" the students to develop the idea that there are limits of validity for each theoretical system. This was the final discussion of this lesson and the starting point for the next one.

The analysis of the footage made with the camera that was aimed at the students allows us to realize their discomfort. There was even an exchange of glances between Osvaldo and Lara, as if one was asking the other what the correct answer would be. The students, in general, made a typical expression of what they were thinking, looking up and frowning.

At this moment, it is interesting to realize that if the concept-in-action of Inertia is not relevant to explain the situation presented, it is necessary to take another concept. The context in which the problem was presented is that of Mechanics and, therefore, Lara used another concept-in-action to solve the problem: the concept of force.

However, as we can see in the continuation of the episode, his scheme of movement used the conceptual content: "there can only be a movement when there is the application of a force". As Lara still uses the separation between real and imaginary movements, in a sense similar

to what was discussed in the lesson 2, her explanation is similar to reality, i.e., it refers to the movement that the bus made in relation to the ground (which is still consistent with the pseudo-relativity of the movement already detected). The other movement, which does not have the same epistemological status as the first, is an impression that, therefore, does not require a scientific approach. The sequence shown below illustrates this analysis.

Lara: Unless a force act on it... every action, a reaction.

Teacher: Nobody kicked the ball.

Lara: No, but from the moment he stopped <the bus> ... to every action there is a reaction. From the moment ... you were in ... the bus was on the move. You don't notice the movement of the bus because you're inside the bus and you're on the movement with it. From the moment the bus stops ... I mean ... ceases the movement, you continue ... tends to remain moving. The bus stops, but you keep going ahead. That's the way it is for me.

Teacher: But you see. Isn't your reference the bus?

Lara: Yes.

Teacher: Weren't you at rest in relation to the bus?

Lara: Yes.

Professor: The bus was at rest in relation to you. So, you do not ... for the bus reference, has no such notion of movement. Is that clear? (...) He... the bus did not apply to the ball any force. But we just got to a weird point, didn't we? Because, by inertia, the ball should continue what? Stop. If you ask this guy out here why the ball kept running, what can he say?

The teacher explains, then, that there is a limitation in this situation. Newton's laws are valid in inertial frames of references. When the bus brakes, there is an acceleration and then other considerations (he does not say which) should be considered.

5th lesson:

Throughout this class, there was a debate between the teacher, the researcher and the students.

The researcher mentions the activities already carried out and seeks to make a synthesis of what has already been studied. When he discusses the theoretical exercise on three bodies that may or may not move one in relation to the others, Lara is the only student to claim that she has positioned herself in favor of the statement, saying that

"I began to think that A... I started to create a lot of theory. A was a car, B was me and ... No! A was a car, B a person, C is ... dunno... a dog. First, I wonder A were ... that the car was moving relative to me. Then, in this case, I'd be out of the car. Then I imagined myself in the car. Then A would be on the move in relation to C, because the car would be moving. Me, in the car and the dog would be at rest."

Even though it seems that the student did not understand the problem and separated it into two parts, this positioning reveals the way Lara uses the concept of frame of reference in action. The Earth is, in this case, an absolute reference in relation to which other ones should be established. When the student is out of the car, it is moving relative to her because she lies on Earth. When she's in the car, she's moving relative to the dog because

it's on Earth. In this explanation, the Earth is the immovable reference according to which all other relations of movement and rest are established and, as we already presented and discussed, the largest body present in the situation.

After this moment, the researcher resumes the example developed by the teacher about the ball inside the bus and proposes a mental experiment to the students. The bus will travel straight for a long time and with constant speed relative to the ground (the road is straight, without imperfections). In addition, the bus windows are capped, which prevents the people inside from looking out. The question is: imagine an experiment that can be done inside the bus to know if it is at rest or on the movement.

Antonio tries a formulation, resuming the situation proposed by the teacher, saying that

"The ball can be an example, that's right? ... Or, if you ask the driver to stop! If you have a bump, like that. This will prove that the bus was moving. Because you left ... You were inside the bus, stopped relative to any other object inside the bus. If you keep moving, it proves that the whole ... the ... bus was on the movement too."

On this statement by Antônio, the researcher presents a new situation. The bump that Antonio said could be caused by a collision between a car and the bus, even if it was parked on the ground. Thales intervenes at this moment and states that, in this case, passengers should see the windows breaking, etc. This fact may indicate an attempt to deny external disturbance, considering some details that are not significant for the crucial aspect issue, which was the indistinguishability between rest and URM or, more simply, the students' lack of understanding of the problem presented by the researcher.

The researcher then returns to this question and requests an experiment that can be done while the bus could be considered as an inertial frame of reference. Antonio presents another situation, saying that

"I think I know. When... as ... you throw up the chalk <<he makes mention of a situation already presented by the teacher earlier>> ... then, when you're at rest, the chalk drops... straight. And when you are moving, it falls into a parable bow. That could explain it, right? Because if the bus is moving, the chalk will fall into a parabola arc. Could it be?"

From this speech, and without wanting to present ready answers, the researcher asks the students what they think about it. Osvaldo, opposing Antonio, states that "Oh, I don't think so because if an outside person who saw it, he would see it falling into a parable bow." This position is shared by Cassiano, who agrees with Osvaldo's formulation.

Antonio, however, convinced of his positions, counter-argues that "But you could ask someone who was on the outside, like that."

Lara presents the argument that these two observers appointed by Antonio belong to the same system. This idea is completed by Osvaldo, who says:

"But these two persons don't understand why they're at the same speed. Same thing with Earth, then. If you throw a chalk up, you'll get back the chalk. But, what ... isn't the Earth moving? It would not be valid this experiment ... would not be valid!"

These arguments make Antonio agree with Osvaldo. It is curious to note that, in Osvaldo's speech, there is no explicit indication of the references to affirm that observers or the Earth are in motion. However, this non-explicitness of references may be due to the greater "informality" of the speech between two students, since the context of the discussion already had delimited references.

After a time of reflection on the part of the students, Cassiano states that there is no way to distinguish the rest of the Uniform Rectilinear Movement from the situations described. The students seem to agree with this position. Thales' speech summarizes up this position.

"I think that... How can you affirm that if you do not have ... an ... information needed for you to draw a conclusion. That's the same thing ... you're inside the bus, closed ... it's the same thing ... if I locked you in a room all dark, locked ... on the first day of ... April ... and I got you back on the 22nd... If I ask you the date, if you can't measure it, you're going to lose your notion of time. From the moment you don't have the information you need to draw a conclusion, you can't say anything! I think so."

To end this debate, the teacher takes up some of the most important points already presented and stresses that the indistinguishability between rest and URM is the conceptual basis of Newton's mechanics. At this point, Cassiano asks: "maybe it is because we are always moving?", a statement that resumes the idea of absolute movement.

The other classes followed as planned, with the introduction and discussion of the TSR postulates and their more immediate consequences, such as the dilation of time.

10th lesson:

This lesson was wholly dedicated to individual resolution of the problems presented below.

01 Robots that have already been sent to explore Mars need to be automatons, meaning they cannot be controlled from Earth in real time. How do the already studied aspects of relativity explain this fact?
02) Imagine a planet at a distance of 10,000 light-years from Earth. Is it possible for an Earth space traveler to reach this planet still alive?

As can be seen, this activity presented three very different situations to investigate how the concepts of TSR could be recognized and used in the resolution of theoretical problems. These questions are related to Einstein's second postulate. However, each of them was conceived to evaluate different aspects.

Question 01 is based on the fact that the speed of light is not infinite. In this way, there will always be a delay between the emission of a light signal on Earth and its arrival at the robot placed on Mars. In this case, time dilation is not pertinent to arrive at the correct answer. Even so, this recognition is not trivial.

Osvaldo, for example, used the second postulate as an argument, saying that this is due to a

"Mismatch of time between Earth and Mars, so it is necessary that the robot itself make its own decisions of what to do, otherwise the time difference could result in the loss of it, due to a difficulty that it encountered and would not arrive an answer to make a decision."

Firstly, this response could be classified as a tendency to attribute to temporal dilation the need for autonomous robots on Mars. But the final part, which we shouted, suggests the cause of the "mismatch of time" is, for Osvaldo, the distance between the planets.

In this same line of reasoning, Thales began to work his explanation from the concept of temporal dilation, but then referred to the delay of sending the signal. For the student, robots need to be automatons because "Earth time is different from Mars time (...). Thus, the coordinates dictated by scientists on Earth would take a while to reach Mars (...)"

The other students who proposed an explanation mentioned the time dilation. In this group are, for example, the following answers:

"Because of the way in which time is perceived and lived in different environments, each will take as reference the time in its environment and is only perceived the difference when compared. So, it is necessary that when we control or analyze bodies that are in different states we make adjustments in time, because we will notice a difference." (Vívian)

"Time on Earth does not pass the same way as on Mars. Time is relative, we need to determine a benchmark to be able to analyze it. Speed is the one that influences in time causing variations of seconds, hours and even years from one place to another. So, robots must be autonomous because the real time here on Earth is not the same as there." (Mariana)

"Because time on Earth is different from a body's time at speeds close to the speed of light, this speed that robots have when they are in orbit" (Lara)

"Special Relativity predicts that a clock on board on Mars will not mark the same time interval as another clock resting on the Earth's surface à operating conditions and synchronized previously. This temporal dilation is not noticed by us or by clocks. In the theory of relativity each observer has his/her own measure of time." (Nayara)

The answers indicate that the students were able to use a time scheme that uses OI linked to temporal relativity and the relative time interval CIA was decisive in solving the situation. Because this CIA is not relevant to the situation, it led the students to present unsatisfactory answers to the problem. Therefore, the amplitude of action of each subject's scheme of time should also be tested by analyzing the other responses. This is what will be presented below.

The second question showed a totally idealized situation as it questioned whether a newborn child could arrive alive on a planet 10,000 light-years from Earth. This situation, to be answered correctly, required temporal dilation and a recognition of the relativity of movement. So, the student should realize, first, that the distance of 10,000 light-years is measured in relation to a reference system that is fixed on Earth and that, therefore, a journey to this planet would take a very long time when measured from Earth. But depending on the speed of a spaceship,

the proper time, measured by who is on the ship, could be equivalent to the lifetime of a human being.

The analysis that can be made from the students' answers is that, for most of them, the relative time is not yet consolidated because of an inadequate use of the concept of frame of reference. In other ways, the scheme of time works in solidarity with movement, whose organization of activities is oriented in the search for a privileged reference.

In this sense, Antonio says that

"Even with the speed of light, <<the new-born>> would spend a period of time that for humans it would be impossible to stay alive. And since there are no speeds greater than light, it is impossible."

The student makes no considerations about the reference used for the measurement of the mentioned distance and, therefore, attributes to this distance an absolute character. The time it takes for light to travel the distance between the two planets, for Antonio, is 10,000 years regardless of which reference is used in this measurement. This contradiction between this answer and that given to the first question reveals that the scheme of time was used together with the movement scheme and that it contains the frame of reference CIA as something absolute. Therefore, the student also shows the tendency to interpret the physical quantities time and space as absolute entities.

This also occurs with Osvaldo, who presents the idea that the situation addressed in this problem would not be possible "because assuming that it travelled at the speed of light, which is the maximum speed, it would still take 10 000 years to reach this planet, which would prevent it from arriving alive (...)"

This is also the explanation presented by Lara, who says, more forcefully than Osvaldo, that "distance is already measured considering that the trip is made at the speed of light". For her, the distance of 10 000 light-years has already been measured in relation to Earth, and therefore the minimum amount of time that would be spent on the journey is 10,000 years.

Two students presented similar answers regarding the consideration of two distinct references: Mariana and Thales.

Mariana, revealing that she considered the time on Earth, said that

"Because these 10,000 years are being considered as time on Earth. In space and with the speed of light, time passes differently, since it is relative. On Earth, the 10,000 years pass normally, but in space this time passes slowly, resulting in a smaller number of years."

For Thales,

"If such a trip were possible, the rocket would must to have a very high speed, so the temporal dilation would be very large too, and then ... he ... can reach the other planet still alive."

The difference that can be brought out between the two answers is that the first one does not yet reveal a total disconnect from the idea of absolute movement. This can be completed by two statements: "in space time passes differently" and "on Earth, the 10 000 years pass normally". The interpretation of the answer indicates that Mariana still uses a frame of reference CIA that identifies the Earth as a privileged system.

In Thales' answer, there is a gap in his understanding of the physical meaning of the 10,000 light-years, since the student does not address this theme in his response. But the idea that the new-born child could reach the other planet still alive is an indication that he recognizes the two different times (or the two different distances) as valid in their respective references.

6. Conclusion

The investigation of the ways in which an individual approaches and solves problem situations can reveal personal conceptions and how each person organizes himself/herself into action. In this process, the investigation using the notion of scheme is a theoretical framework that allows us to verify the organization of the action from teaching situations.

The episodes discussed in this work were a small part of the set of activities that allowed us to conclude an important point about the transition between Classical Mechanics and Special Relativity. Since the understanding of relative time is dependent on the concept of frame of reference, the same tendency to assign an absolute character to the reference will be verified in relation to time. Although the success in the resolution of situations involving the relativity of time is contingent on reciprocal assimilation between the schemes of movement and time, if the OI of the scheme of movement are too closely linked to the attribution of a pseudo-relativity, these characteristics are also present in the analysis of time intervals measured in different references. It seems to us, therefore, that the difficulty in deal with situations related to TSR is not an intrinsic characteristic of this conceptual field.

It is certain that there are common obstacles to the learning of CM and TSR that are linked to the relativity of movement. Some students who failed learning TSR already had difficulties in using the classic concepts. The diversity of situations can contribute to these students being able to expand their scheme of movement and use it in both conceptual fields. Therefore, teaching both conceptual fields together may improve learning by providing more contexts to the concepts studied.

These new contexts could contribute to the reciprocal assimilation between schemes of movement and time and, with this, build a more complex set of conceptual relationships for the phenomena involved in each conceptual field. The interpretation of our data indicates that this assimilation is possible.

In this sense, the possibility of learning the fundamentals of TSR is due to the understanding of a system of knowledge that cannot be verified by the most immediate experiences of ordinary life, as it is also true for much of what is taught in CM. Although learning the fundamentals of TSR requires a process of strong abstraction, the construction of a consistent theoretical model in this area, on the part of the learner, is made from logical considerations arising from the adoption of postulates.

Students in high school would be able to perform this task of abstraction and recognize, in the TSR models, a logic of their own relationship between the notions of time,

space and speed. This is what happened, for example, with Osvaldo and Thales throughout the research. These students were able to perform such logical operations and expand their explanatory models to relativistic phenomena.

However, not all students were able to complete such a construction. This fact leads us to the discussion of the second point—the conditions for learning. In this case, the learning of CM as a conceptual field does not seem to have been a determinant for students to appropriate the TSR concepts. Nor can it be said, from the data presented in this research, that CM is an obstacle to TSR.

What seems to us as essential is that the notion of relative time requires an understanding of the relativity of movement. The apparent tautology of this statement requires a broader discussion. The notion of time begins to become operational when coordinated with speed. In this process, the MC's own signification system admits, even in its underlying form, that time is conceived as an absolute magnitude related only to the duration of the analyzed events (displacements, for example) and independent of the reference system used.

Therefore, the schemes of movement and time are used to deal with situations linked to Galilean relativity, sharing the concept of speed but without the need for reciprocal assimilation between them. This is because there is an asymmetry between the OI linked to each of these schemes, i.e., the scheme of movement needs to deal with relativity while scheme of time does not.

The novelty presented by TSR that we investigated in this research is the expansion of this relative nature to time. Thus, in order to account for the situations related to the comparison between the durations of the events, the subject needs to proceed to reciprocal assimilation between the time and movement schemes.

Some of the students, such as Nayara, were unable to operate with TSR models, largely because they had not well developed the scheme of movement and treated all movements as absolutes. In this sense, Lara's case is interesting because she did not refuse to work with the notion of relative time, but her scheme of movement organizes the action from the establishment of a privileged reference to analyze displacements and velocities from it. Therefore, the non-totally relative characteristic of the movement, revealed by the student, can also be verified when she is treating time.

However, it is not enough to have the scheme of movement well developed so that learning TSR is possible. Cassiano, for example, revealed a good use of classic models. However, the student did not develop the relativistic models as well. From the moment the transition to TSR began to take place, Cassiano did not adhere to the model and continued to solve the problems with the same IO as before. In this case, it does not seem to have been destabilized by the problem presented by the teacher, nor did they recognize a model in TSR that could be used to solve the proposed situations.

The teaching approach for Modern Physics that is made from the exploration of tension points of Classical Physics has been reported in the literature and can be effective in showing the transitory character of scientific knowledge. This strategy proved to be good with some students, such as Thales, Osvaldo and Lara. They, at different levels,

were capable of understanding the crisis of classical models and how the new theory managed to overcome it.

This question may be discussed from the consideration of three different groups of students. It is only effective if students are already, in a way, familiar, at least with some basic foundations of CM, such as the relativity of movement and the indistinguishability of mechanical laws within inertial frames of reference. Students like Nayara, for example, were not destabilized because they did not perceive the points where conceptual tension occurred. Other students, such as Cassiano, also did not realize the need for a reformulation in explanatory models with respect to crediting classical models with an absolute truth.

The analysis of the data of the participations of these three groups of students allows us to infer, therefore, that the organization of the teaching of both CM and TSR together is advisable, showing that there are fields of validity so that each of the models can be applied. There are obstacles to learning that are common to the two conceptual fields, such as an egocentric analysis of movement and the attribution of an absolute reference to movement. These obstacles can be overcome when the two conceptual fields are put into perspective and more situations are dominated by students. In this sense, a greater variety of situations can contribute not only to an improvement in the use of the time scheme within the TSR, but also to an increase in the scope of the scheme of movement, covering more aspects related to relativity.

Therefore, from the data presented in this research, we defend the thesis that the parallel work of the two conceptual fields provides better elements for students to develop their own IO and know the validity domains of each theoretical formulation. This position is particularly important to deal with situations in which learners must use of the concept of frame of reference for the discussion of the relativity of movement and time, thus analyzing the limits of validity of each conceptual fields studied.

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