Calculation of Radiant Section Temperatures in Fired Process Heaters

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Abstract Flame and effective gas temperatures are key variables that need to be accurately determined before analysis of heat transfer in the radiant section of fired heaters can be meaningfully undertaken. To facilitate the calculation of these temperatures, appropriate equations were derived using two Computer Matlab programmes specially written for the purpose. A third programme was also written for the solution of the derived equations using the Newton-Raphson method. The whole calculation procedure was illustrated by an example worked out for an actual process heater used in a crude oil topping unit.

Keywords: effective gas temperature, flame temperature, radiation section, fired heater, tubular heater


1. Introduction

Fired process heaters are furnaces in which a process fluid flowing inside tubes located in the furnace is heated by gases produced by the combustion of a liquid or gaseous fuel. Such heaters are widely used in oil refineries and petrochemical plants for heating purposes. They are of two basic types: vertical cylindrical or box-type heaters [1,2,3]. For the most part, tubular heaters consist of two main sections: a radiant section, variously called a combustion chamber or firebox, in which heat is transferred mainly by radiation, and a convection section followed by the stack. Roughly-speaking, about 45-55% of the total heat release in the furnace is transferred to the process fluid in the radiant section, leaving about 25-45% of the total heat release to be either transferred to the process fluid in the convection section or carried by the flue gases through the stack and is lost [4].

2. Heat Transfer Mechanisms in Fired Heaters

In the usual practice, the process fluid is first heated in the convection section preheat coil which is followed by further heating in the radiant section. In both sections heat is transferred by both mechanisms of heat transfer, viz. radiation and convection, where radiation is the dominant mode of heat transfer in the radiant section and convection predominates in the convection section as the average temperature in this section is much lower. The heat-absorbing surface in both sections is the outside wall of the tubes mounted inside the heater.

Total heat transfer to the process fluid can be estimated using the following equation:

\[ Q_{\text{total}} = U_C \times A \times \text{LMTD} \]  (1)

The radiant heat transfer follows the relationship:

\[ Q_r = \sigma \times \left( \alpha \times A_{cp} \right) \times F \times \left( T_g^4 - T_w^4 \right) \]  (2)

and convective heat transfer follows the relationship:

\[ Q_{\text{conv}} = h_{\text{conv}} \times A_t \times \left( T_g - T_w \right) \]  (3)

A number of key variables serve as a basis for the determination of heater performance. These include [5]:
1. Inlet and outlet process fluid temperatures.
2. Effective flue gas and flame temperatures.
3. Tube skin or tube wall temperature.
4. Heat transfer rates to the process fluid.
5. Flue gas composition.
7. Fuel flow rate.
9. Pressure profile in the heater and stack.

Of these variables, the effective gas and flame temperatures are most important for a comprehensive heat transfer analysis of the heater radiant section. Furthermore, efficiency calculations require prior calculation of these two temperatures [6].

3. Effective Gas Temperature (T_g)

The effective gas temperature is the temperature controlling radiant transfer in the heater radiant section. For a "well mixed" radiant section this temperature is
assumed to be equal to the bridgewall temperature, i.e. the exit temperature of the flue gases leaving the radiant section. For most applications, this is an acceptable assumption with the notable exception of high temperature heaters with tall narrow fireboxes and wall firing where the effective gas temperature may be 95 to 150°C higher than the bridgewall temperature. In this and other cases where the two temperatures differ widely and an adjustment may be necessary, the use of a more accurate gas temperature may have to be considered or the radiant section may have to be divided into zones for the energy balance calculations [7].

Furthermore, complete flue gas mixing in the firebox is normally assumed in most methods used for the estimation of the effective gas and other radiant section temperatures, including the widely-used Lobo-Evans method [8]. This ignores in effect the existence of longitudinal and transverse temperature gradients. Cross, on the other hand, evaluated the temperature gradients in fired heaters in order to predict the bridgewall temperature [9].

4. Derivation of Effective Gas Temperature Equation

There are two primary sources of heat input to the radiant section, the combustion heat of the fuel \( Q_{\text{rls}} \) and the sensible heat of the combustion air \( Q_{\text{air}} \), fuel \( Q_{\text{fuel}} \) and the fuel atomization fluid (for liquid fuel when applicable). Of this heat input some heat is absorbed in the radiant section by the radiant Q\( R \) and shield tubes Q\( shld \), while the gas temperature can then be calculated as follows [10]:

\[
Q_{\text{in}} = Q_{\text{rls}} + Q_{\text{air}} + Q_{\text{fuel}}
\]

\[
= Q_R + Q_{\text{shld}} + Q_{\text{losses}} + Q_{\text{flue gases}} = Q_{\text{out}}
\]

Where:

\[
Q_{\text{rls}} = m_{\text{fuel}} \times NCV
\]

\[
Q_{\text{air}} = m_{\text{air}} \times C_{\text{p,air}} \times (t_{\text{air}} - t_{\text{datum}})
\]

\[
Q_{\text{fuel}} = m_{\text{fuel}} \times C_{\text{p,fuel}} \times (t_{\text{fuel}} - t_{\text{datum}})
\]

\[
Q_R = Q_f + Q_{\text{conv}}
\]

\( Q_f \) is the radiant heat transfer

\[
Q_f = \sigma (\alpha \times A_{\text{cp}}) \times F \times (T_{g}^4 - T_{w}^4)
\]

and \( Q_{\text{conv}} \) is the convective heat transfer in the radiant section.

\[
Q_{\text{conv}} = h_{\text{conv}} \times A_f \times (T_g - T_w)
\]

\[
Q_{\text{shld}} = \sigma (\alpha \times A_{\text{cp}}) _{\text{shld}} \times F \times (T_g^4 - T_w^4)
\]

\[
Q_{\text{losses}} = (2 - 5\%) \times m_{\text{fuel}} \times NCV
\]

\[
Q_{\text{flue gases}} = m_{\text{flue gases}} \times C_{\text{p,flue gases}} \times (T_g - T_{\text{datum}})
\]

Substitution of the appropriate terms in Equation. 4:

\[
m_{\text{fuel}} \times NCV + m_{\text{air}} \times C_{\text{p,air}} \times (t_{\text{air}} - t_{\text{datum}}) + m_{\text{fuel}} \times C_{\text{p,fuel}} \times (t_{\text{fuel}} - t_{\text{datum}}) \]

\[
\times (T_g^4 - T_{\text{datum}}^4) + h_{\text{conv}} \times A_f \times (T_g - T_w)\]

\[
+ \sigma (\alpha \times A_{\text{cp}}) _{\text{shld}} \times F \times (T_g^4 - T_w^4) \]

\[
+ (2-3)\% \times m_{\text{fuel}} \times NCV
\]

\[
+ m_{\text{flue gases}} \times C_{\text{p,flue gases}} \times (T_g - T_{\text{datum}})
\]

The average tube wall temperature is given by [11]:

\[
T_W = 100 + 0.5 \frac{T_{\text{in}} + T_{\text{out}}}{2}
\]

The Newton-Raphson method [12] was used to solve the heat balance equation and determine the effective gas temperature, for which two Matlab programmes were written (Appendices 1 and 3).

5. Flame Temperature (\( t_f \))

Flame temperature is the temperature attained by the combustion of a fuel. This temperature depends essentially on the calorific value of the fuel. A theoretical or ideal flame temperature may be calculated assuming complete combustion of the fuel and perfect mixing. But even when complete combustion is assumed, the actual flame temperature would always be lower than the theoretical temperature. There are several reasons for this, chiefly dissociation of the combustion products at higher temperatures and heat loss. Up to a flame temperature of about 1370°C, the burned mixture generally includes such ordinary gases as \( \text{CO}_2, \text{N}_2, \text{SO}_2, \text{H}_2\text{O} \) and residual \( \text{O}_2 \) (from excess air). At higher temperatures, however, \( \text{CO}_2 \) appreciably dissociates to \( \text{CO} \) and \( \text{O}_2 \); \( \text{H}_2\text{O} \), to \( \text{O}_2 \) and \( \text{OH}^- \); \( \text{N}_2 \), to \( \text{N}^-3 \); and \( \text{NO} \), produced by \( \text{N}_2 \) and \( \text{O}_2 \), to \( \text{N}^3 \) and \( \text{O}^-2 \). These dissociation reactions absorb an enormous amount of energy, substantially lowering the flame temperature [13,14]. Further lowering of the flame temperature is also caused by radiation and conduction to the walls of the combustion chamber. Highly turbulent flames usually suffer an appreciable heat loss.

Some work has been done on the calculation of flame temperature, including work by Stehlik and others who studied furnace combustion and drew furnace temperature and enthalpy profiles [4]. Vancini wrote a programme in assembly language for the calculation of the average flame temperature, taking into account dissociation at higher temperatures [13].

6. Derivation of Flame Temperature Equation

In this paper, a simple heat balance serves as the basis for calculating the flame temperature. The increase in enthalpy between the unburned and burned mixtures is assumed to be equal to the heat produced by the
combustion. When the fuel is fired, the heat liberated raises the temperature of the combustion products from $t_1$ to $t_2$ so that the following relationship is satisfied:

$$Q_{\text{combustion}} = W_i \times \int_{t_1}^{t_2} C_{p_i} \, dt$$

(16)

Where:

- $Q_{\text{combustion}}$ = Heat of combustion of fuel.
- $W_i$ = Mass of a flue gas component.
- $C_{p_i}$ = Molar heat of a flue gas component.
- $t_1$ and $t_2$ = Initial and final temperatures.

The use of Equation (16) allows the calculation of the flame temperature $t_2$ by iteration using a programmable calculator. The variation of $C_{p_i}$ with temperature can be approximated by a polynomial, having the obvious advantage of being integrated easily. Using a third-degree polynomial, $C_{p_i}$ can be written as:

$$C_{p_i} = a_i + b_i \times t + c_i \times t^2 + d_i \times t^3$$

(17)

Where, $a_i$, $b_i$, $c_i$, and $d_i$ are constants dependent on the nature of the gas. Assuming $t_1$ to be negligible (= 0), Equation (16) thus becomes:

$$Q_{\text{combustion}} = W_i \times \int_0^{t_2} (a_i + b_i \times t + c_i \times t^2 + d_i \times t^3) \, dt$$

(18)

Integrating:

$$Q_{\text{combustion}} = W_i \times (a_i \times t + b_i \times \frac{t^2}{2} + c_i \times \frac{t^3}{3} + d_i \times \frac{t^4}{4})$$

(19)

It is customary to call the parenthetic term in Equation (19) the mean molar heat:

$$C_{p_{\text{m,i}}} = a_i \times t + b_i \times \frac{t^2}{2} + c_i \times \frac{t^3}{3} + d_i \times \frac{t^4}{4}$$

(20)

By taking mean molar heats instead of true molar heats, the integration of Equation (16) may be dispensed with. The molar heats at constant pressure for air and flue gases are given in Table 1.

### Table 1. Molar heats at constant pressure for air and flue gases [15]

<table>
<thead>
<tr>
<th>Gas</th>
<th>Molar heat (kJ / kmol .deg.)</th>
<th>Temp Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>33.915 +1.214 ×10^{-3} ×T</td>
<td>50-1500 K</td>
</tr>
<tr>
<td>CO₂</td>
<td>43.2936 +0.01147 ×T -818558.5/T²</td>
<td>273-1200 K</td>
</tr>
<tr>
<td>N₂</td>
<td>27.2155 +4.187 ×10^{-3} ×T</td>
<td>300-3000 K</td>
</tr>
<tr>
<td>O₂</td>
<td>34.63 +1.0802×10^{-3} ×T -785900/T²</td>
<td>300-5000 K</td>
</tr>
<tr>
<td>SO₂</td>
<td>32.24 +0.0222 ×T -3.475 ×10^{-6} ×T²</td>
<td>300-2500 K</td>
</tr>
<tr>
<td>*H₂O(g)</td>
<td>34.42 +6.281 ×10^{-4}×T +5.611×10^{-6}×T²</td>
<td>300-2500 K</td>
</tr>
</tbody>
</table>

*H₂O(g) is gas phase For heat transfer at constant pressure

Equation (21) allows the calculation of the theoretical flame temperature, $t_2$, by iteration using a programmable calculator. In order to compensate for the factors that tend to lower the theoretical flame temperature, the heat of combustion is usually multiplied by an empirical coefficient. The values normally used for this coefficient are only estimates; this is why the temperature calculated with any method can only approximate actual values. For an accurate calculation of the actual flame temperature, account must be taken of heat losses through the casing by setting up heat balance equation for fuel gas as follows:

$$Q_{\text{combustion}} - Q_{\text{losses}} = \sum_{i=1}^{5} W_i \times C_{p_{m,i}} \times (t_2 - t_1)$$

(21)

Where:

- $Q_{\text{combustion}} = M_{\text{fuel}} \times GCV$
- $Q_{\text{losses}} = 5\% \times Q_{\text{combustion}}$

The Newton-Raphson method [11] was used to solve the heat balance equation and determine the actual flame temperature, for which two Matlab programmes were written (Appendices 2 and 3).

To illustrate the use of the programme, an example is worked out for an actual crude oil heater used in an atmospheric topping unit at the Homs Oil Refinery (Cabin 43-5-16/21 N). In this example, fuel gas is fired with 25% excess air. Ambient temperature = 15°C, exit gas temperature = 400°C. Table 2 gives the furnace characteristics for the radiant section and fuel and combustion air.

The effective gas temperature equation, derived using programme (1), has the following form:

$$4F(T_g) = C T_g + D T_g - B$$

(25)

The first derivative of the effective gas temperature equation is:

$$\frac{dF(T_g)}{dT_g} = 4C T_g + D$$

(26)

where $B$, $C$, and $D$ are constants dependent on the type of fuel, percentage of excess air, operating conditions and geometrical characteristics of the fired heater. These constants can then be estimated using Programme 1 as follows (Table 3):

$$\frac{dF(T_g)}{dT_g} = 3.6299 \times 10^{-4} \times T_g^3 + 7.9153 \times 10^4$$

(27)

(28)
Table 2. Furnace characteristics

<table>
<thead>
<tr>
<th>Geometrical Characteristics of Radiant Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of passes</td>
</tr>
<tr>
<td>Number of tubes</td>
</tr>
<tr>
<td>Number of shield tubes</td>
</tr>
<tr>
<td>Overall tube length (m)</td>
</tr>
<tr>
<td>Effective tube length (m)</td>
</tr>
<tr>
<td>Tube spacing, centre-to-centre (mm)</td>
</tr>
<tr>
<td>centre-to-furnace wall (mm)</td>
</tr>
<tr>
<td>Outside diameter of tube (mm)</td>
</tr>
<tr>
<td>Wall thickness of tube (mm)</td>
</tr>
<tr>
<td>Tube materials</td>
</tr>
</tbody>
</table>

Fuel characteristics

<table>
<thead>
<tr>
<th></th>
<th>Natural gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow of fuel (kmol/h)</td>
<td>120</td>
</tr>
<tr>
<td>Molecular weight (kg/kmol)</td>
<td>19.99</td>
</tr>
<tr>
<td>Molar heat (kJ/kmol.K)</td>
<td>39.26</td>
</tr>
<tr>
<td>NCV (kJ/kmol)</td>
<td>927844.41</td>
</tr>
<tr>
<td>GCV (kJ/kmol)</td>
<td>976029.6</td>
</tr>
<tr>
<td>Film convective heat transfer coefficient (kJ/h.m$^2$.C)</td>
<td>30.7</td>
</tr>
<tr>
<td>Composition of fuel (% mol)</td>
<td>CH4 (80.43), C2H6 (9.02), C3H8 (4.54), iso-C4H10 (0.20), n-C4H10 (0.32), iso-C5H12 (0.04), n-C5H12 (0.02), CO2 (3.52), H2S (0.09), N2 (1.735).</td>
</tr>
</tbody>
</table>

Air characteristics

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Molar heat (kJ/kmol.K)</td>
<td>33.915+1.214×10$^{-3}$×T</td>
</tr>
<tr>
<td>Flow of air (kmol/h)</td>
<td>1589.014</td>
</tr>
<tr>
<td>Air temperature (°C)</td>
<td>25</td>
</tr>
<tr>
<td>Percentage of excess air</td>
<td>25%</td>
</tr>
</tbody>
</table>

Flue gas characteristics

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Molar heat (kJ/kmol.K)</td>
<td>29.98+3.157×10$^{-3}$×T</td>
</tr>
<tr>
<td>Specific heat (kJ/kg.K)</td>
<td>1.0775+1.1347×10$^{-4}$×T</td>
</tr>
<tr>
<td>Flow of flue gas (kmol/h)</td>
<td>1720.9</td>
</tr>
<tr>
<td>Molecular weight (kg/kmol)</td>
<td>27.8234</td>
</tr>
<tr>
<td>Composition (% mol)</td>
<td>CO2 (8.234), H2O (15.968), O2 (3.82), N2 (71.79), SO2 (0.188)</td>
</tr>
</tbody>
</table>

These equations were solved by the Newton-Raphson method in programme (3) to give the effective gas temperature in the fire box equal to 1278K.

The flame temperature equation, derived using programme (2), has the following form:

$$F(t_f) = a + b t_f + c t_f^2 + d t_f^3 + e$$

(29)

The first derivative of the flame temperature equation is:

$$\frac{dF(t_f)}{dt_f} = a + 2b t_f - c t_f^2 + 3d t_f^2$$

(30)

Where a, b, c, d and e are constants estimated dependent on the type of fuel and its gross calorific value, the percentage of excess air and the operating conditions of the fired heater. These constants can then be estimated using Programme 2 as follows (Table 4):

$$F(t_f) = 29.9825 + 0.0021 t_f^2 + 9.7421 \times 10^4 t_f^{-1} + 2.9648 \times 10^7 t_f^3 - 6.4657 \times 10^4$$

(31)

$$\frac{dF(t_f)}{dt_f} = 29.9825 + 0.0042 t_f - 9.7421 \times 10^4 t_f^2 + 8.8943 \times 10^7 t_f^3$$

(32)

These equations were solved by the Newton-Raphson method using programme (3) to give the actual flame temperature of 2128K.

Table 3. Data for determination of effective gas temperature equation

| Outlet temperature of process fluid (°C) | 355 |
| Stack temperature (°C) | 400 |
| Flow rate of combustion air (kmol/h) | 1589.014 |
| Flow rate of flue gases (kmol/h) | 1720.9 |
| Number of tubes in radiation section | 60 |
| Number of shield tubes | 8 |
| Effective tube length (m) | 20.024 |

Table 4. Data for determination of flame temperature equation

| Flow rate of fuel (kmol/h) | 120 |
| Flow rate of flue gases (kmol/h) | 1720.9 |
| Percentage of heat losses | 0.05 |
| Gross calorific value of fuel (kJ/kmol) | 976029.6 |
| Molar fraction of CO2 | 0.08234 |
| Molar fraction of H2O | 0.15968 |
| Molar fraction of N2 | 0.7179 |
| Molar fraction of O2 | 0.382 |
| Molar fraction of SO2 | 0.00188 |

7. Conclusion

Using Matlab programming and the Newton-Raphson method, it was possible to calculate simply and accurately both the effective gas temperature in a fired heater and the actual flame temperature. The calculation was based on
heat transfer analysis of the fired heater taking into account heat absorption and losses in the radiant section of the heater. This calculation can be an important tool, not only in the operation and daily running of fired heaters, but also and more importantly for their design.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Heat exchange surface area (m²).</td>
</tr>
<tr>
<td>( A_{qp} )</td>
<td>Cold plane area of tubes bank in radiation section (m²).</td>
</tr>
<tr>
<td>( A_{qp_shld} )</td>
<td>Cold plane area of shield tubes bank (m²).</td>
</tr>
<tr>
<td>( A_t )</td>
<td>Area of tubes bank in Radiation section (m²).</td>
</tr>
<tr>
<td>( C_{p_air} )</td>
<td>Molar heat of combustion air (kJ/kmol.K).</td>
</tr>
<tr>
<td>( C_{p_fuel} )</td>
<td>Specific heat of fuel (kJ/kg.deg).</td>
</tr>
<tr>
<td>( C_{p_flue_gas} )</td>
<td>Average specific heat of flue gases (kJ/kg.K).</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Molar heat of a flue gas component (kJ/kmol.K).</td>
</tr>
<tr>
<td>( D_i, D_o )</td>
<td>Inside and outside diameters of tube (mm).</td>
</tr>
<tr>
<td>e</td>
<td>Tube thickness (e = R_o - R_i) (mm).</td>
</tr>
<tr>
<td>F</td>
<td>Exchange factor = 0.97</td>
</tr>
<tr>
<td>( GCV )</td>
<td>Gross calorific value of fuel (kJ/h).</td>
</tr>
<tr>
<td>( h_{corr} )</td>
<td>Film convection heat transfer coefficient (kJ/m².K.h).</td>
</tr>
<tr>
<td>( L_{nube} )</td>
<td>Effective tube length (m).</td>
</tr>
<tr>
<td>( m_{air} )</td>
<td>Flow rate of combustion air (kg/h).</td>
</tr>
<tr>
<td>( m_{fuel} )</td>
<td>Flow rate of fuel (kg/h).</td>
</tr>
<tr>
<td>( m_{flue_gas} )</td>
<td>Flow rate of flue gas (kg/h).</td>
</tr>
<tr>
<td>( N_{CV} )</td>
<td>Nett calorific value of fuel (kJ/h).</td>
</tr>
<tr>
<td>( N_r )</td>
<td>Number of tubes in section.</td>
</tr>
<tr>
<td>( S_{tube_shld} )</td>
<td>Number of shield tubes.</td>
</tr>
<tr>
<td>( Q_{air} )</td>
<td>Sensible heat of combustion air (kJ/h).</td>
</tr>
<tr>
<td>( Q_C )</td>
<td>Total heat transfer (kJ/h).</td>
</tr>
<tr>
<td>( Q_{combustion} )</td>
<td>Combustion heat of fuel (kJ/h).</td>
</tr>
<tr>
<td>( Q_{conv.} )</td>
<td>Convective heat transfer in the radiant section, (kJ/h).</td>
</tr>
<tr>
<td>( Q_{fuel} )</td>
<td>Sensible heat of fuel (kJ/h).</td>
</tr>
<tr>
<td>( Q_{flue_gas} )</td>
<td>Sensible heat of gas leaving the radiant section (kJ/h).</td>
</tr>
<tr>
<td>( Q_{losses} )</td>
<td>Assumed radiation heat loss (kJ/h).</td>
</tr>
<tr>
<td>( Q_{nube} )</td>
<td>Total heat transferred to radiant tubes (heat absorbed by radiant tubes) (kJ/h).</td>
</tr>
<tr>
<td>( Q_{radiant} )</td>
<td>Radiant heat transfer (kJ/h).</td>
</tr>
<tr>
<td>( Q_{release} )</td>
<td>Heat release by burners (kJ/h).</td>
</tr>
<tr>
<td>( Q_{shld} )</td>
<td>Radiant heat to shield tubes (kW/h).</td>
</tr>
<tr>
<td>( R_i, R_o )</td>
<td>Inside and outside radii of tube (mm).</td>
</tr>
<tr>
<td>( S_i, S_o )</td>
<td>Inside and outside heat surface areas of tube (m²).</td>
</tr>
<tr>
<td>( S_{nube} )</td>
<td>Tube spacing (m).</td>
</tr>
<tr>
<td>( t_i )</td>
<td>Temperature of fuel and air (°C).</td>
</tr>
<tr>
<td>( t_f )</td>
<td>Flame temperature (°C).</td>
</tr>
<tr>
<td>( T_f )</td>
<td>Flame temperature (K).</td>
</tr>
<tr>
<td>( T_{g} )</td>
<td>Effective gas temperature in firebox (K).</td>
</tr>
<tr>
<td>( T_{in} )</td>
<td>Inlet process fluid temperature (K).</td>
</tr>
<tr>
<td>( T_{out} )</td>
<td>Outlet process fluid temperature (K).</td>
</tr>
<tr>
<td>( T_w )</td>
<td>Average tube-wall temperature (K).</td>
</tr>
<tr>
<td>( W_i )</td>
<td>Mass of flue gas component (kmol/h).</td>
</tr>
</tbody>
</table>

**Greek symbols**

- \( \alpha \) Relative effectiveness factor of the tubes bank.
- \( \sigma \) Stefan-Boltzman constant = \( 2.041 \times 10^{-7} \) kJ/h.m².K⁴.

**References**


**Appendix 1**

% Program for determination of effective gas temperature.

\[
\text{Input :}
\]
\[
t_i = \text{input}' \text{ Inlet temperature of process fluid (C) = } t_i + 273;
\]
\[
to = \text{input}' \text{ Outlet temperature of process fluid (C) = } to + 273;
\]
\[
\text{sigma} = 2.041 \times 10^{-7};
\]
\[
\text{Sigma} = \text{Stefan-Boltzman constant} = 2.041 \times 10^{-7} \text{ kJ/h.m².K}^4.
\]
\[ F = 0.97; \% \text{ Exchange factor} \]
\[ \alpha = 0.835; \% \text{ Relative effectiveness factor of the tubes bank} \]
\[ Q_{\text{rls}} = m_{\text{fuel}} \times NCV; \]
\[ Q = m_{\text{air}} \times C_{\text{pair}} \times (t_{\text{air}} - t_{\text{datum}}) \]
\[ t_{\text{air}} = 25; \]
\[ t_{\text{datum}} = 15; \]
\[ C_{\text{pair}} = 33.915 + 1.214 \times 10^{-3} \times (t_{\text{air}} + t_{\text{datum}})/2; \]
\[ Q_{\text{air}} = m_{\text{air}} \times C_{\text{pair}} \times (t_{\text{air}} - 15); \]
\[ t_{\text{fuel}} = 25; \]
\[ C_{\text{pfuel}} = 29.98 + 3.1157 \times 10^{-3} \times (T_{\text{g}} + T_{\text{datum}})/2; \]
\[ Q_{\text{fuel}} = m_{\text{fuel}} \times C_{\text{pfuel}} \times (t_{\text{fuel}} - t_{\text{datum}}); \]
\[ Q_{\text{in}} = Q_{\text{rls}} + Q_{\text{air}} + Q_{\text{fuel}}; \]
\[ Q_{\text{out}} = Q_{\text{R}} + Q_{\text{shld}} + Q_{\text{losses}} + Q_{\text{flue}}; \]
\[ Q_{\text{R}} = Q_{\text{r}} + Q_{\text{conv}}; \]
\[ Q_{\text{r}} = \sigma \times (\alpha \times A_{\text{cp}}) \times F \times (T_{\text{g}}^4 - T_{\text{w}}^4); \]
\[ T_{\text{w}} = \text{Average tube wall temperature in degrees Kelvin} \]
\[ T_{\text{g}} = \text{Effective gas temperature in degrees Kelvin} \]
\[ A_{\text{cp}} = N \times C \times L; \]
\[ Q_{\text{conv}} = h_{\text{conv}} \times A \times (T_{\text{g}} - T_{\text{w}}); \]
\[ h_{\text{conv}} = 30.66; \% \text{ Film convective heat transfer coefficient; (kJ/h.m2.c)} \]
\[ Q_{\text{shld}} = N_{\text{shld}} \times \sigma \times (\alpha \times A_{\text{cp}}) \times F \times (T_{\text{g}}^4 - T_{\text{w}}^4); \]
\[ \alpha = 1; \% \text{ for the shield tubes can be taken to be equal to one.} \]
\[ A_{\text{cpshld}} = N_{\text{shld}} \times C \times L; \]
\[ Q_{\text{loss}} = X \times Q; \]
\[ Qt = Q - Q_{\text{loss}}; \]
\[ C_{\text{pm}} = \int (X_{\text{CO2}} \times C_{\text{pCO2}} + X_{\text{H2O}} \times C_{\text{pH2O}} + X_{\text{N2}} \times C_{\text{pN2}} + X_{\text{O2}} \times C_{\text{pO2}} + X_{\text{SO2}} \times C_{\text{pSO2}}); \]
\[ H = C_{\text{pm}} - Qt/M_{\text{fluegas}}; \]
\[ M_{\text{fuel}} = \text{input}('\text{Flow rate of fuel (kmol/h)}=\prime); \]
\[ M_{\text{fluegas}} = \text{input}('\text{Flow rate of flue gases (kmol/h)}=\prime); \]
\[ X\% \text{ of heat losses}; \]
\[ GCV = \text{input}('\text{Gross calorific value of fuel (kJ/kmol)}=\prime); \]
\[ X_{\text{CO2}} = \text{input}('\text{Molar fraction of CO2}=\prime); \]
\[ X_{\text{H2O}} = \text{input}('\text{Molar fraction of H2O}=\prime); \]
\[ X_{\text{N2}} = \text{input}('\text{Molar fraction of N2}=\prime); \]
\[ X_{\text{O2}} = \text{input}('\text{Molar fraction of O2}=\prime); \]
\[ X_{\text{SO2}} = \text{input}('\text{Molar fraction of SO2}=\prime); \]
\[ t_{\text{d}} = 15; \% \text{ Datum temperature (C)} \]
\[ C_{\text{pCO2}} = 43.2936 + 0.01147 \times T - 818558.5 \times T^{-2}; \]
\[ C_{\text{pH2O}} = 34.42 + 6.281 \times 10^{-4} \times T + 5.611 \times 10^{-6} \times T^{2}; \]
\[ C_{\text{pN2}} = 27.2155 + 4.187 \times 10^{-3} \times T; \]
\[ C_{\text{pO2}} = 34.63 + 1.0802 \times 10^{-3} \times T^{2}; \]
\[ C_{\text{pSO2}} = 32.24 + 0.0222 \times T - 3.475 \times 10^{-6} \times T^{2}; \]
\[ t_{\text{d}} = 15; \% \text{ Datum temperature (C)} \]
\[ \% \text{ Molar heats at constant pressure for flue gases} \]
\[ \% C_{\text{pCO2}} = 43.2936 + 0.01147 \times T - 818558.5 \times T^{-2}; \]
\[ \% C_{\text{pH2O}} = 34.42 + 6.281 \times 10^{-4} \times T + 5.611 \times 10^{-6} \times T^{2}; \]
\[ \% C_{\text{pN2}} = 27.2155 + 4.187 \times 10^{-3} \times T; \]
\[ \% C_{\text{pO2}} = 34.63 + 1.0802 \times 10^{-3} \times T - 785900 \times T^{2}; \]
\[ \% C_{\text{pSO2}} = 32.24 + 0.0222 \times T - 3.475 \times 10^{-6} \times T^{2}; \]
\[ \% \text{ Heat evolved by fuel during combustion (kJ/h)} \]
\[ Q = GCV \times M_{\text{fuel}}; \]
\[ \% \text{ Heat losses (kJ/h)} \]
\[ Qt = Q - Qt/M_{\text{fluegas}}; \]
\[ Q_{\text{flue}} = m_{\text{flue}} \times C_{\text{pfuel}} \times (T_{\text{g}} - T_{\text{datum}}); \]
\[ C_{\text{t}} = \text{input}('\text{Molar fraction of CO2}=\prime); \]
\[ C_{\text{H}} = \text{input}('\text{Molar fraction of H2O}=\prime); \]
\[ C_{\text{N}} = \text{input}('\text{Molar fraction of N2}=\prime); \]
\[ C_{\text{O}} = \text{input}('\text{Molar fraction of O2}=\prime); \]
\[ C_{\text{S}} = \text{input}('\text{Molar fraction of SO2}=\prime); \]
\[ T_{\text{g}} = \text{input}('\text{Gross calorific value of fuel (kJ/kmol)}=\prime); \]
\[ t_{\text{d}} = 15; \% \text{ Datum temperature (C)} \]
\[ \% \text{ Molar heats at constant pressure for flue gases} \]
\[ % C_{\text{pCO2}} = 43.2936 + 0.01147 \times T - 818558.5 \times T^{-2}; \]
\[ % C_{\text{pH2O}} = 34.42 + 6.281 \times 10^{-4} \times T + 5.611 \times 10^{-6} \times T^{2}; \]
\[ % C_{\text{pN2}} = 27.2155 + 4.187 \times 10^{-3} \times T; \]
\[ % C_{\text{pO2}} = 34.63 + 1.0802 \times 10^{-3} \times T - 785900 \times T^{2}; \]
\[ % C_{\text{pSO2}} = 32.24 + 0.0222 \times T - 3.475 \times 10^{-6} \times T^{2}; \]
\[ % \text{ Integration of mean molar heats} \]
\[ C_{\text{pm}} = \text{input('X*C_{\text{pCO2}}+X_{\text{H}}*C_{\text{pH2O}}+X_{\text{N}}*C_{\text{pN2}}+X_{\text{O}}*C_{\text{pO2}}+X_{\text{S}}*C_{\text{pSO2}}');} \]
\[ H = C_{\text{pm}} - Qt/M_{\text{fluegas}}; \]
\[ \text{disp('Equation of actual flame temperature')} \]
\[ \text{func} = H; \%
\[ \% \text{ Finding of first derivative of flame temperature equation.} \]
\[ \text{disp('Finding of first derivative of flame temperature equation')} \]
\[ \text{dfun} = \text{diff(func)} \]

**Appendix 3**

\[ \% \text{ Solution of effective gas and flame temperature by Newton Raphson method} \]
\[ \text{function root=newtraph(func,dfun,es,maxit)} \]
\[ \text{newtraph(func,dfun,xguess,es,maxit)} \]
\[ \text{Uses Newton-Raphson method to find a function} \]
% input:
% func = name of function
% dfunc = name of derivative of function
% xguess = initial guess
% es = (optional) stopping maximum allowable iterations
% output:
% root = real root
% if necessary, assign default values
% if nargin<5, maxit=50; end % if maxit blank set to 50
% if nargin<4, es=0.001; end % if es blank set to 0.001
% Newton-Raphson
iter=0;
while (1)
    xrold=xr;
    xr=xr-func(xr)/dfunc(xr);
    iter=iter+1;
    if xr~0, ea=abs((xr-xrold)/xr)*100; end
    if ea<=es|iter>=maxit, break, end
end
root=xr;
if root>1300
    root=root+273;
    fprintf('The actual flame temperature is %8.0f K
',root)
else
    root=xr+273;
    fprintf('The Effective gas temperature is %8.0f K
',root)
end