

Challenging Photon Mass: from Scalar Quantum Electrodynamics to String Theory

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Abstract A massless photon, originated already through the Maxwell theory of electromagnetism, is one of the basic paradigms of modern physics, ideally supported throughout both the quantum electrodynamics and the Higgs mechanism of spontaneous symmetry breaking which lays the foundations of the Standard Model of elementary particles and fundamental interactions. Nevertheless, the physical interpretation of the optical experimental data, such like observations of total internal reflection of the beam shift in the Goos-Hänchen effect, concludes a photon mass. Is, therefore, light diversified onto two independent species - gauge photons and optical photons? Can such a state of affairs be consistently described through a unique theoretical model? In this paper, two models of a photon mass, arising from the scalar quantum electrodynamics with the Higgs potential, are discussed. The first scenario leads to a neutral scalar mass estimable throughout the experimental limits on a photon mass. In the modified mechanism, a neutral scalar mass is not affected throughout a photon mass and is determinable through the experimental data, while a massless dilaton is present and a non-kinetic massive vector field effectively results in the string theory of non-interacting invariant both a free photon and a neutral scalar, and the Aharonov-Bohm effect is considered. The Markov hypothesis on maximality of the Planck mass is applied.

Keywords: *scalar quantum electrodynamics, Higgs potential, scalar field, photon mass, dilaton, non-kinetic vector field, Aharonov-Bohm effect, Markov hypothesis, invariant particles, string theory*

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1. Introduction

The basic paradigm of physics is a massless photon, already established through the Maxwell theory of electromagnetism, Cf. the Refs. [1-15], solicited through quantum electrodynamics, Cf. the Refs. [16-26], the basis of quantum optics, Cf. the Refs. [27-38], and more general gauge field theories, Cf. the Refs. [39-49], which lay the foundations of the Standard Model of elementary particles and fundamental interactions, wherein a massless photon leads to the Higgs particle equipping the weak gauge fields, $W^\pm$ and Z^0 bosons, in a mass due to the spontaneous symmetry breaking mechanism [50,51,52]. Although a photon mass was early considered [53-62], this idea was explicitly implemented into Maxwell's electrodynamics through Alexandru Proca [63,64], whose academic mentor Louis

De Broglie made the grounds for this idea [65-78]. Furthermore, the physical interpretation of the optical experimental data, such like the Goos-Hänchen effect of the beam shift [79], through observations of total internal reflection [80], concludes a photon mass. This aspect was discussed in the context of quantized radiation and generalized to quantum theory of massive spin-1 photons [81], and then suggested to be untenable [82]. Furthermore,

many authors have considered the various aspects of a photon mass [83-185].

In modern physics, the Higgs potential, well-known in high energy physics, Cf. the Refs. [186-193], gives a particle mass. Scalar quantum electrodynamics, Cf. the Refs. [194,195,196,197,198], where a photon interacts with a charged scalar boson, exhibits this mechanism. It's both origin and the most remarkable physical application is the Ginzburg-Landau model [199], arising from L.D. Landau's model of the second order phase transitions [200] and formulating superconductivity near the critical temperature as a charged Bose-Einstein condensate, wherein for the 2 + 1-dimensional case, in the type-II superconductors, the magnetic flux is transported throughout the Abrikosov vortices carrying super current [201]. These vortices are point-like objects having a non-trivial topology of non-contractible circles created throughout the scalar fields [202].

Following the monograph [203], we consider two models based on the scalar quantum electrodynamics. The first one is based on the Higgs potential, whereas the alternative one deals with the modified Higgs potential, but both them involve existence of a neutral scalar boson and a dilaton, and differ from the Higgs mechanism through the resulting photon mass and a different scalar field mass. First, this mass is estimated throughout the present-day experimental limits on a photon mass, and

next remains a free parameter and has no relation to a photon mass. The Markov hypothesis [204,205], on the maximality of Planck's mass $M_p = \sqrt{\hbar c / G}$, is applied. In the second case, the Aharonov {Bohm effect [206] is included, and the effective string theory of invariant non-interacting both a free photon and a neutral scalar is obtained.

2. Higgs Potential

Scalar quantum electrodynamics is described through the Lagrangian

$$L = \frac{\hbar c}{2} (D_\mu \Phi)^\dagger D^\mu \Phi - V(|\Phi|) - \frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} - j^\mu A_\mu, \quad (1)$$

where $\hbar$ is the Planck constant and $c = 1/\sqrt{\varepsilon_0 \mu_0}$ is the speed of light in vacuum, with the magnetic permittivity of free space μ_0 and the electric permeability of free space ε_0 . $D_\mu \Phi = \partial_\mu \Phi - i \frac{e}{\hbar} A_\mu$, where e is the elementary charge and $\partial_\mu = [\partial_t / c, \nabla_i]$ is a covariant derivative of a charged (complex) scalar field $\Phi(x)$, $|\Phi|^2 = \Phi^\dagger \Phi$. $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the Faraday tensor of electromagnetic field, $F_{\mu\nu} = \eta_{\mu\alpha} \eta_{\nu\beta} F^{\alpha\beta}$, a photon is an abelian gauge field A_μ , $A^\mu = \eta^{\mu\nu} A_\nu$, $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is the Minkowski metric.

Moreover, Φ is expressed through neutral (real) scalar fields $\varphi_i(x)$

$$\Phi = \varphi_1(x) + i\varphi_2(x), \Phi^\dagger = \varphi_1(x) - i\varphi_2(x), \quad (2)$$

which have the vacuum expectation values

$$\langle 0 | \varphi_1(x) | 0 \rangle = \varphi_0, \langle 0 | \varphi_2(x) | 0 \rangle = 0, \quad (3)$$

where φ_0 is a real constant field and $|0\rangle$ the static Fock vacuum state. The Lagrangian (1) is invariant under the U(1) group transformations

$$\begin{aligned} \Phi' &= e^{-i\theta} \Phi, \Phi'^\dagger = e^{i\theta} \Phi^\dagger, \\ A'_\mu &= A_\mu + \frac{\hbar}{e} \partial_\mu \theta, A'^\mu = A^\mu - \frac{\hbar}{e} \partial^\mu \theta. \end{aligned} \quad (4)$$

where $\theta(x)$ is a local phase, and the conserved Noether current is

$$j^\mu = iec(\Phi^\dagger \partial^\mu \Phi - (\partial^\mu \Phi^\dagger) \Phi), \partial_\mu j^\mu = 0. \quad (5)$$

Let us consider the following effective O(2)-symmetric Higgs potential,

$$V(|\Phi|) = \frac{m^2 c^3}{2\hbar} |\Phi|^2 + \frac{\hbar c}{4} g |\Phi|^4, \quad (6)$$

where m is a mass parameter, g is a coupling constant, and the decomposition

$$\begin{aligned} \varphi_1(x) &= \varphi_0 + \chi(x), \langle 0 | \chi(x) | 0 \rangle = 0, \\ \varphi_2(x) &= \varphi(x), \langle 0 | \varphi(x) | 0 \rangle = 0, \end{aligned} \quad (7)$$

one receives the energy

$$\langle 0 | V(\Phi) | 0 \rangle = \frac{m^2 c^3}{2\hbar} \varphi_0^2 + \frac{\hbar c}{4} g \varphi_0^4 = \varepsilon(\varphi_0) \quad (8)$$

whose extremal values are established through the condition

$$\frac{d\varepsilon(\varphi_0)}{d\varphi_0} = \frac{m^2 c^3}{\hbar} \varphi_0 + \hbar c g \varphi_0^3 = 0, \quad (9)$$

which has two solutions, first $\varphi_0 = 0$ for which $\varepsilon(\varphi_0 = 0) = 0$, and

$$\varphi_0^2 = -\frac{1}{g} \frac{m^2 c^2}{\hbar^2}, \varepsilon(\varphi_0^2 = -\frac{1}{g} \frac{m^2 c^2}{\hbar^2}) = -\frac{1}{4g} \frac{m^4 c^5}{\hbar^3}. \quad (10)$$

The non-trivial solution is physical if and only if

$$g > 0, m^2 = -m_0^2, m_0^2 > 0. \quad (11)$$

In result, the Lagrangian (1) becomes

$$\begin{aligned} L &= \frac{\hbar c}{2} \left(\frac{m_0^2 c^2}{\hbar^2} - \frac{g}{2} \varphi_0^2 \right) \varphi_0^2 + \frac{\hbar c}{2} \theta_\mu \chi \theta^\mu \chi + \\ &\quad \hbar c \varphi_0 \left(\frac{m_0^2 c^2}{\hbar^2} - g \varphi_0^2 \right) \chi + \frac{\hbar c}{2} \left(\frac{m_0^2 c^2}{\hbar^2} - 3g \varphi_0^2 \right) \chi^2 \\ &\quad - \hbar c \varphi_0 g \chi^3 - \hbar c \frac{g}{4} \chi^4 - \hbar c \varphi_0 g \chi \varphi^2 - \hbar c \frac{g}{2} \chi^2 \varphi^2 \\ &\quad \frac{\hbar c}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{\hbar c}{2} \left(\frac{m_0^2 c^2}{\hbar^2} - g \varphi_0^2 \right) \varphi^2 - \hbar c \frac{g}{4} \varphi^4 \\ &\quad + ec \chi \partial^\mu \varphi A_\mu - ec \varphi \partial^\mu \chi A_\mu - \frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} \\ &\quad + \frac{e^2 c \varphi_0^2}{2\hbar} A_\mu A^\mu + \frac{e^2 c}{2\hbar} \varphi^2 A_\mu A^\mu + \frac{e^2 c \varphi_0}{\hbar} \chi A_\mu A^\mu, \end{aligned} \quad (12)$$

and its massive part written in the standard form

$$L_m = \frac{e^2 m_A^2 c^3}{2\hbar^3} A_\mu A^\mu - \frac{m_\chi^2 c^3}{2\hbar} \chi^2 - \frac{m_\varphi^2 c^3}{2\hbar} \varphi^2, \quad (13)$$

allows to establish the masses

$$m_A = \frac{\hbar}{c} \varphi_0, m_\varphi = \sqrt{g m_A^2 - m_0^2}, m_\chi = \sqrt{3g m_A^2 - m_0^2} \quad (14)$$

Application of the Markov hypothesis gives

$$\varphi_0 \leq \frac{1}{\sqrt{g}} \sqrt{\frac{c^3}{\hbar G}}, m_A \leq \frac{1}{\sqrt{g}} \sqrt{\frac{\hbar c}{G}}, g \geq 1. \quad (15)$$

The relations (14) lead to the coupling constant

$$g = \frac{m_\chi^2 - m_\varphi^2}{2m_A^2}. \quad (16)$$

Moreover, since a mass is physical when is a positive real number, one has

$$g m_A^2 - m_0^2 \geq 0, 3g m_A^2 - m_0^2 \geq 0 \quad (17)$$

and, therefore, one receives the lower bound for the coupling constant

$$g \geq \frac{m_0^2}{m_A^2}, \quad (18)$$

which applied to (16), leads to the bounds

$$m_0 \leq \sqrt{\frac{m_\chi^2 - m_\phi^2}{2}}, m \geq \sqrt{\frac{m_\phi^2 - m_\chi^2}{2}}, \quad (19)$$

and, through the Markov hypothesis, the squared-mass difference satisfies

$$\Delta m_{\chi\phi}^2 = m_\chi^2 - m_\phi^2 \leq \frac{2\hbar c}{G}. \quad (20)$$

Also, the field equations for the Lagrangian (12) are

$$\square A^\mu - \partial^\mu (\partial_\nu A^\nu) = -\frac{\mu_0 e^2 c}{\hbar} (\chi^2 + \phi^2 + \frac{2m_{AC}}{\hbar} \chi + \frac{m_A^2 c^2}{\hbar^2}) A^\mu + \mu_0 e c (\phi \partial^\mu \chi - \chi \partial^\mu \phi), \quad (21)$$

$$\square \phi + \frac{m_\phi^2 c^2}{\hbar^2} \phi + g \phi^3 = -2 \frac{g m_{AC}}{\hbar} \chi \phi - g \chi^2 \phi + \frac{e^2}{\hbar^2} \phi A^\mu A_\mu - 2 \frac{e}{\hbar} \partial^\mu \chi A_\mu - \frac{e}{\hbar} \partial^\mu \chi A_\mu \quad (22)$$

$$\square \chi + \frac{m_\chi^2 c^2}{\hbar^2} \chi + 3 \frac{g m_{AC}}{\hbar} \chi^2 + g \chi^3 = -\frac{m_\phi^2 c^2}{\hbar^2} \phi_0 - \frac{g m_{AC}}{\hbar} \phi^2 - g \chi \phi^2 + \frac{e^2}{\hbar^2} \chi A^\mu A_\mu + \frac{e^2 m_{AC}}{\hbar^3} A^\mu A_\mu + 2 \frac{e}{\hbar} \partial^\mu \chi A_\mu + \frac{e}{\hbar} \partial^\mu \chi A_\mu \quad (23)$$

Where $\square = \partial^\mu \partial_\mu = \frac{1}{c^2} \partial_t^2 - \nabla^2$ is the D'Alembert operator.

In the case of the non-trivial solution (10),

$$m_A = \frac{m_0}{\sqrt{g}}, m_\phi = 0, m_\chi = \sqrt{2} m_0, \quad (24)$$

are the ground state masses. In this case, the coupling constant is

$$g = \frac{m_0^2}{m_A^2}. \quad (25)$$

Similarly, for $\phi_0 = 0$, one obtains

$$m_A = 0, m_\phi = i m_0, m_\chi = i m_0 \quad (26)$$

that is a massless photon and two scalar tachyons, whereas g is undetermined throughout the masses. Interestingly, for

$$\phi_0 = \frac{1}{\sqrt{3g}} \frac{m_0 c}{\hbar}, \quad (27)$$

one gets the masses

$$m_A = \frac{m_0}{\sqrt{3g}}, m_\phi = i \sqrt{\frac{2}{3}} m_0, m_\chi = 0, \quad (28)$$

that is a massive photon, a scalar tachyon, and a massless scalar. Then, the coupling constant is

$$g = \frac{m_0^2}{2m_A^2}. \quad (29)$$

A constant term in a Lagrangian does not affect the resulting field equations.

Interestingly, when the constant term of (12) vanishes, then

$$\phi_0 = \sqrt{\frac{2}{g}} \frac{m_0 c}{\hbar} \quad (30)$$

In this case, the masses (14) become

$$m_A = \sqrt{\frac{2}{g}} m_0, m_\phi = m_0, m_\chi = \sqrt{5} m_0, \quad (31)$$

and, in the light of the Markov hypothesis, one has

$$m_0 \geq \sqrt{\frac{g}{2}} \sqrt{\frac{\hbar c}{G}}, m_0 \leq \frac{1}{\sqrt{5}} \sqrt{\frac{\hbar c}{G}}, g \leq \frac{2}{5}. \quad (32)$$

Also, for this case the coupling constant is

$$g = \frac{2m_0^2}{m_A^2}, \quad (33)$$

whereas the relation (20) gives

$$m_0 \leq \frac{1}{\sqrt{2}} \sqrt{\frac{\hbar c}{G}}, \quad (34)$$

and, therefore, without loss of generality, one can take ad hoc the value

$$m_0 = \frac{1}{\sqrt{2}} \sqrt{\frac{\hbar c}{G}}, \quad (35)$$

which implies the following values of the masses

$$m_A = \frac{1}{\sqrt{g}} \sqrt{\frac{\hbar c}{G}}, m_\phi = \frac{1}{\sqrt{2}} \sqrt{\frac{\hbar c}{G}}, m_\chi = \sqrt{\frac{5}{2}} \sqrt{\frac{\hbar c}{G}}. \quad (36)$$

The Markov hypothesis for m_χ is broken, that is χ is undetectable, while ϕ can be regarded as the Higgs particle. For m_A , one has $g \geq 1$, where

$$g = \frac{\hbar c}{G} \frac{1}{m_A^2}. \quad (37)$$

3. Modified Higgs Potential

Let us consider the $O(2)$ -invariant modified Higgs potential

$$V(|\Phi|) = \frac{m^2 c^3}{2\hbar} (|\Phi| - |\Phi_0|)^2 + \hbar c \frac{g}{4} (|\Phi| - |\Phi_0|)^4, \quad (38)$$

where $\Phi_0 = \langle 0|\Phi|0 \rangle$. Considering the decomposition

$$|\Phi(x)| = \chi_0 + \chi(x), \langle 0 | \chi(x) | 0 \rangle = 0, |\Phi_0| = \chi_0 \quad (39)$$

the potential (38) becomes

$$V(|\Phi|) = V(\chi) = \frac{m^2 c^3}{2\hbar} \chi^2 + \frac{\hbar c}{4} g \chi^4, \quad (40)$$

and has the extremal values at $\chi_0 = 0$ and

$$\chi_0^2 = -\frac{m^2 c^2}{\hbar^2 g}, \quad (41)$$

and, for consistency, either χ must be a tachyon, that is $m^2 = -m_0^2$ with $m_0^2 > 0$, or the coupling constant $g < 0$.

Let us present $\Phi(x)$ through the polar decomposition

$$\Phi(x) = |\Phi(x)| e^{i\theta(x)}, \quad (42)$$

$$\theta(x) = \arg \Phi(x) = \arctan \frac{\varphi_2(x)}{\varphi_1(x)} + 2\pi q, q \in \mathbb{Z}. \quad (43)$$

where $\theta(x)$ is a local phase, and we applied (2). In the most general situation

$$\theta(x) = \theta(x^\mu) = \int^{x^\mu} dx'^\mu k_\mu(x'), \quad (44)$$

where $x_\mu = [ct; x_i]$ is the position four-vector, $k_\mu = \left[\frac{\omega}{c}, k_i \right]$ is the wave four-vector, ω is an oscillation frequency, k_i is a wave vector.

Applying the formulas (39), (43) and (44), one obtains

$$\varphi_1(x) = \frac{\pm |\chi_0 + \chi(x)|}{\sqrt{1 + \tan\left(\int^{x^\mu} dx'^\mu k_\mu(x') - 2\pi q\right)}} \quad (45)$$

$$\varphi_2(x) = \pm |\chi_0 + \chi(x)| \frac{\tan\left(\int^{x^\mu} dx'^\mu k_\mu(x') - 2\pi q\right)}{\sqrt{1 + \tan\left(\int^{x^\mu} dx'^\mu k_\mu(x') - 2\pi q\right)}} \quad (46)$$

Considering the theory (1) according to the relations (39), (42) and (44)

$$\partial^\mu \Phi = \left(\partial^\mu \chi + i \chi k^\mu \right) e^{i\theta}, \theta_\mu \Phi^\dagger = \left(\partial_\mu \chi - i \chi k_\mu \right) e^{-i\theta} \quad (47)$$

one obtains

$$\begin{aligned} L = & \frac{\hbar c}{2} \theta_\mu \chi \theta^\mu \chi - \frac{m^2 c^3}{2\hbar} \chi^2 - g \frac{\hbar c}{4} \chi^4 - \frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} \\ & + \frac{e^2 c \chi_0^2}{2\hbar} A_\mu A^\mu + \frac{e^2 c \chi_0}{2\hbar} \chi A_\mu A^\mu + \frac{e^2 c}{2\hbar} \chi^2 A_\mu A^\mu \\ & + ec \chi_0^2 k^\mu A_\mu + 2ec \chi_0 \chi k^\mu A_\mu + ec \chi^2 k^\mu A_\mu \\ & + \chi_0^2 \frac{\hbar c}{2} k_\mu k^\mu + \chi_0 \hbar c \chi k_\mu k^\mu + \frac{\hbar c}{2} \chi^2 k_\mu k^\mu \end{aligned} \quad (48)$$

whereas the Noether current is

$$j^\mu = -ec(\chi_0 + \chi)^2 k^\mu, \quad (49)$$

and, moreover, the masses are

$$m_\chi = m, m_A = \frac{\hbar}{c} \chi_0, m_k = \sqrt{\frac{\hbar^3 G}{c^5}} \chi_0^2 = m_A^2 \sqrt{\frac{G}{\hbar c}}, \quad (50)$$

and, consequently

$$\chi_0 = \frac{m_{AC}}{\hbar} = \frac{c}{\hbar} \sqrt{m_k M_P}. \quad (51)$$

Applying the Markov hypothesis, one receives

$$\chi_0 \leq \sqrt{\frac{c^3}{\hbar G}}, m \leq \sqrt{\frac{\hbar c}{G}}. \quad (52)$$

Moreover, for the non-trivial solution (41) with $g > 0$, one has

$$g \leq 1, m_\chi = im_0, m_A = \frac{m_0}{\sqrt{g}}, m_k = \frac{m_0^2}{g} \sqrt{\frac{G}{\hbar c}}, \quad (53)$$

whereas for $g < 0$, one obtains

$$|g| \leq 1, m_\chi = m, m_A = \frac{m}{\sqrt{|g|}}, m_k = \frac{m^2}{|g|} \sqrt{\frac{G}{\hbar c}} \quad (54)$$

The Lagrangian (48) leads to the following field equations

$$0 = (\chi_0 + \chi)^2 \left(k_\mu + \frac{e}{\hbar} A_\mu \right), \quad (55)$$

$$\square \chi + g \chi^3 + \frac{m_\chi^2 c^2}{\hbar^2} \chi = \quad (56)$$

$$\left(k_\mu + \frac{e}{\hbar} A_\mu \right) \left(k^\mu + \frac{e}{\hbar} A^\mu \right) (\chi_0 + \chi),$$

$$\partial_\nu F^{\mu\nu} = \mu_0 ec \left(k^\mu + \frac{e}{\hbar} A^\mu \right) (\chi_0 + \chi)^2, \quad (57)$$

where $\partial_\nu F^{\mu\nu} = -\square A^\mu + \partial^\mu (\partial_\nu A^\nu)$ or, equivalently,

$$k_\mu = -\frac{e}{\hbar} A_\mu, \quad (58)$$

$$\square \chi + \frac{m_\chi^2 c^2}{\hbar^2} \chi + g \chi^3 = 0, \quad (59)$$

$$\partial_\nu F^{\mu\nu} = 0, \quad (60)$$

whereas the Noether current (49) has the form

$$j^\mu = \frac{e^2 c}{\hbar} (\chi_0 + \chi)^2 A^\mu. \quad (61)$$

Consequently, (48) describes free photon non-interacting with scalar field

$$L = \frac{\hbar c}{2} \theta_\mu \chi \theta^\mu \chi - \frac{m_\chi^2 c^3}{2\hbar} \chi^2 - g \frac{\hbar c}{4} \chi^4 - \frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} \quad (62)$$

whereas, although the photon mass term is cancelled through the current term, a photon mass has the value determined in (50). Also, then one receives

$$\varphi_1(x) = \frac{\pm |\chi_0 + \chi(x)|}{\sqrt{1 + \tan\left(\frac{e}{\hbar} \int^{x^\mu} dx'^\mu A_\mu(x') + 2\pi q\right)}} \quad (63)$$

$$\varphi_2(x) = \mp \left| \chi_0 + \chi(x) \right| \frac{\tan\left(\frac{e}{\hbar} \int^{x^\mu} dx'^\mu A_\mu(x') + 2\pi q\right)}{\sqrt{1 + \tan^2\left(\frac{e}{\hbar} \int^{x^\mu} dx'^\mu A_\mu(x') + 2\pi q\right)}} \quad (64)$$

and Φ becomes neutral, that is $\varphi_2 = 0$, if and only if the condition holds

$$\int^{x^\mu} dx'^\mu A_\mu(x') = (p - 2q) \frac{\pi \hbar}{e}, \quad p, q \in \mathbb{Z} \quad (65)$$

where $A_\mu = \left[\frac{\phi}{c}, A_i \right]$ is potential four-vector, $\phi(x)$ is the electric potential and $A_i(x)$ is the magnetic potential. Interestingly, the particular case

$$\int dx'^i A_i(x'^i, t'(x'^i)) = 2n \frac{\pi \hbar}{e}, \quad n \in \mathbb{Z}, \quad (66)$$

the magnetic flux quantization for the Aharonov {Bohm effect, gives

$$\int dt' \phi(x^i(t'), t') = (2n - 2q + p) \frac{\pi \hbar}{ec} \quad (67)$$

Interestingly, the formulas (43) and (44) allow to establish

$$k_\mu = \partial_\mu \theta = \frac{\varphi_1^2}{\varphi_1 \partial^\mu \varphi_2 - \varphi_2 \partial^\mu \varphi_1} \cos^2 \frac{\varphi_2}{\varphi_1}, \quad (68)$$

and throughout the gauge condition (58), one receives

$$A_\mu = \frac{\hbar}{e} \frac{\varphi_1^2}{\varphi_2 \partial^\mu \varphi_1 - \varphi_1 \partial^\mu \varphi_2} \cos^2 \frac{\varphi_2}{\varphi_1}, \quad (69)$$

The current conservation $\partial_\mu j^\mu = 0$, applied to (61), gives the solution

$$A^\mu = \frac{\hbar}{e^2 c} \frac{C^\mu}{(\chi_0 + \chi)^2}, \quad (70)$$

where C^μ is a constant current. Since a photon has spin 1, the Lorentz gauge $\partial_\mu A^\mu = 0$ holds for (70)

$$C^\mu \partial_\mu \chi = 0, \quad A^\mu \partial_\mu \chi = 0 \quad (71)$$

Joining (69) and (70), along with (39) and the Lorentz gauge, one receives

$$C^\mu \partial_\mu \varphi_1 = -\frac{\varphi_2}{\varphi_1} C^\mu \partial_\mu \varphi_2, \quad (72)$$

$$\varphi_1^2 + \varphi_2^2 = \frac{c}{e} \left(\frac{C^\mu \partial_\mu \varphi_1}{\varphi_1} - \frac{C^\mu \partial_\mu \varphi_2}{\varphi_2} \right) \frac{\varphi_2}{\varphi_1} \sec^2 \frac{\varphi_2}{\varphi_1}, \quad (73)$$

$$\left[\frac{1}{\frac{\varphi_2}{\varphi_1} - \frac{\partial_\mu \varphi_2}{\partial_\mu \varphi_1}} + \tan \frac{\varphi_2}{\varphi_1} - \frac{\frac{\varphi_2}{\varphi_1} \left(\frac{\partial_\mu \varphi_1}{\varphi_1} - \frac{\partial_\mu \varphi_2}{\varphi_2} \right)}{2 \left(\frac{\partial_\mu \varphi_1}{\varphi_1} - \frac{\partial_\mu \varphi_2}{\varphi_2} \right)^2} \right] \cos^2 \frac{\varphi_2}{\varphi_1} = 0 \quad (74)$$

Moreover, the solution (69) allows to establish the Faraday tensor

$$F^{\mu\nu} = \frac{2\hbar}{e^2 c} \frac{1}{(\chi_0 + \chi)^3} \left| C^\mu \partial^\nu \chi - C^\nu \partial^\mu \chi \right|. \quad (75)$$

Consequently, one can establish the electric field and the magnetic induction

$$E^i = c F^{i0} = \frac{2\hbar}{e^2 c} \frac{C^i \dot{\chi} - c C^0 \nabla^i \chi}{(\chi_0 + \chi)^3}, \quad (76)$$

$$B_i = -\frac{1}{2} \epsilon_{ijk} F^{jk} = -\frac{2\hbar}{e^2 c} \frac{\epsilon_{ijk} C^j \nabla^k \chi}{(\chi_0 + \chi)^3}, \quad (77)$$

and, moreover, the stress-energy tensor of the electromagnetic field

$$T^{\mu\nu} = \frac{1}{\mu_0} \left(\eta_{\alpha\beta} F^{\mu\alpha} F^{\nu\beta} - \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right) \\ = \frac{\hbar^2 \epsilon_0}{e^4} \frac{4C^2}{(\chi_0 + \chi)^6} \times \left[\left(\frac{C^\mu C^\nu}{C^2} - \frac{1}{2} \eta^{\mu\nu} \right) \partial_\alpha \chi \partial^\alpha \chi + \partial^\mu \chi \partial^\nu \chi \right], \quad (78)$$

where $C^2 = C_\mu C^\mu$. Therefore, one obtains the energy density

$$\epsilon = T_{00} = \frac{\hbar^2 \epsilon_0}{e^4} \frac{4C^2}{(\chi_0 + \chi)^6} \left[\left(\frac{C_0^2}{C^2} - \frac{1}{2} \right) \partial_\alpha \chi \partial^\alpha \chi + \frac{1}{c^2} \dot{\chi}^2 \right], \quad (79)$$

the Maxwell stress tensor

$$\sigma_{ij} = -T_{ij} = -\frac{\hbar^2 \epsilon_0}{e^4} \frac{4C^2}{(\chi_0 + \chi)^6} \left[\left(\frac{C^i C^j}{C^2} + \frac{1}{2} \delta^{ij} \right) \partial_\alpha \chi \partial^\alpha \chi + \nabla^i \chi \nabla^j \chi \right] \quad (80)$$

and the Poynting vector,

$$S_i = c T_{0i} = \frac{\hbar^2 \epsilon_0 c}{e^4} \frac{4C^2}{(\chi_0 + \chi)^6} \left(\frac{C^0 C^i}{C^2} \partial_\alpha \chi \partial^\alpha \chi + \frac{1}{c} \dot{\chi} \nabla^i \chi \right). \quad (81)$$

For the field (70), the first pair of the Maxwell equations (60), that is the Gauss and Ampere laws, give

$$C^\mu \left(\square \chi - \frac{3 \partial_\nu \chi \partial^\nu \chi}{\chi_0 + \chi} \right) = 0, \quad (82)$$

and applied in the equations (59), lead to the differential equation for χ

$$\partial_\nu \chi \partial^\nu \chi = -\frac{g}{3} \chi^4 - \frac{\chi_0 g}{3} \chi^3 - \frac{m_\chi^2 c^2}{3\hbar^2} \chi^2 - \frac{\chi_0 m_\chi^2 c^2}{3\hbar^2} \chi \quad (83)$$

The second pair of the Maxwell equations, that is the Faraday law and the Gauss law for magnetism, is given through the Bianchi identities

$$\partial_\mu G^{\mu\nu} = 0, \quad (84)$$

where

$$G^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\kappa\lambda} F_{\kappa\lambda} = \frac{2\hbar}{e^2 c} \frac{1}{(\chi_0 + \chi)^3} \varepsilon^{\mu\nu\kappa\lambda} C_\kappa \partial_\lambda \chi, \quad (85)$$

is the dual tensor of $F_{\mu\nu}$, and gives no more than (82)

$$\varepsilon^{\mu\kappa\lambda\nu} C_\kappa \left(\partial_\lambda \partial_\nu \chi - \frac{3\partial_\lambda \chi \partial_\nu \chi}{\chi_0 + \chi} \right) = 0 \quad (86)$$

For the case of the solution (70), the formulas (65), (66) and (67) become

$$\int^{\chi^\mu} \frac{dx'^\mu}{(\chi_0 + \chi(x'))^2} = (p - 2q) \pi e c \frac{C^\mu}{C^2}, \quad (87)$$

$$\int^x \frac{dx'^\mu}{(\chi_0 + \chi(x', t'))^2} = 2n\pi e c \frac{C^i}{C^2}, \quad (88)$$

$$\int \frac{dt'}{(\chi_0 + \chi(x', t'))^2} = (2n + 2q - p) \frac{\pi e}{C_0 c}, \quad (89)$$

Where $\bar{C}^2 = C_i C^i$. The equation (83) rewritten in the form

$$\sigma(x) - \sigma_0 = \pm \sqrt{3} \int^{\chi(x)} \frac{d\chi'}{\sqrt{g\chi'^4 + g\chi_0\chi'^3 + \frac{m_\chi^2 c^2}{\hbar^2} \chi'^2 + \frac{m_\chi^2 c^2}{\hbar^2} \chi_0 \chi'}}, \quad (90)$$

where

$$\sigma(x) = \int^x \sqrt{-dx'^\nu dx'_\nu} = \int^x \sqrt{-\eta_{\mu\nu} dx'^\mu dx'^\nu}, \quad (91)$$

is the proper distance and σ_0 is an integration constant, gives implicitly $\chi(\sigma)$, making both $\chi(\sigma)$ and $A_\mu(\sigma)$ the invariants. In the most general case

$$\sigma(x) - \sigma_0 = \frac{\mp 2\sqrt{3}}{\sqrt{\frac{m_\chi^2 c^2}{\hbar^2} - i\sqrt{g}\chi_0 \frac{m_\chi c}{\hbar}}} F \left[\sqrt{\frac{i\sqrt{g}\chi + \frac{m_\chi c}{\hbar}}{2i\sqrt{g}\chi}}; \frac{2i\sqrt{g}\chi_0}{i\sqrt{g}\chi_0 - \frac{m_\chi c}{\hbar}} \right], \quad (92)$$

where

$$F[\alpha; k] = \int_0^\alpha \frac{dt}{\sqrt{(1-t^2)(1-k^2 t^2)}}, \quad (93)$$

is the incomplete elliptic integral of the first kind, in the Jacobi form.

There are few special cases. For example, when $g < 0$ and $m_\chi^2 > 0$, then

$$\sigma(x) - \sigma_0 = \frac{\mp 2\sqrt{3}\lambda_\chi}{\sqrt{1 + \sqrt{|g|}\chi_0\lambda_\chi}} F \left[\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{|g|}\chi\lambda_\chi}}; \frac{2\sqrt{|g|}\chi_0\lambda_\chi}{1 + \sqrt{|g|}\chi_0\lambda_\chi} \right], \quad (94)$$

where $\lambda_\chi = \frac{\hbar}{m_\chi c}$. For $g > 0$ and $m_\chi^2 = -m_0^2$ with $m_0^2 > 0$, one obtains

$$\sigma(x) - \sigma_0 = \frac{\mp 2\sqrt{3}\lambda_\chi}{\sqrt{1 + \sqrt{g}\chi_0\lambda_\chi}} \quad (95)$$

$$F \left[\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{g}\chi\lambda_\chi}}; \frac{2\sqrt{g}\chi_0\lambda_\chi}{1 + \sqrt{g}\chi_0\lambda_\chi} \right],$$

where $\lambda_\chi = \frac{\hbar}{m_0 c}$. For the trivial extremum of the potential one has

$$\sigma(x) - \sigma_0 = \mp \sqrt{3}\lambda_\chi \operatorname{ar sinh} \left(\frac{1}{\sqrt{g}\lambda_\chi\chi} \right), \quad (96)$$

$$\chi(\sigma) = \mp \left[\sqrt{g}\lambda_\chi \sinh \left(\frac{\sigma - \sigma_0}{\sqrt{3}\lambda_\chi} \right) \right]^{-1}, \quad (97)$$

$$A^\mu(\sigma) = g\lambda_\chi^2 \frac{\hbar}{e^2 c} \sinh^2 \left(\frac{\sigma - \sigma_0}{\sqrt{3}\lambda_\chi} \right) C^\mu, \quad (98)$$

Similarly, for the non-trivial solution (41) with $g > 0$ one receives

$$\sigma(x) - \sigma_0 = \pm i\sqrt{6}\lambda_\chi F \left[\sqrt{\frac{\sqrt{g}\lambda_\chi\chi - 1}{2\sqrt{g}\lambda_\chi\chi}}; 1 \right], \quad (99)$$

where $\lambda_\chi = \frac{\hbar}{m_0 c}$, what for $-1 \leq \Re \sqrt{\frac{\sqrt{g}\lambda_\chi\chi - 1}{2\sqrt{g}\lambda_\chi\chi}} \leq 1$ becomes

$$\sigma(x) - \sigma_0 = \pm i\sqrt{6}\lambda_\chi \operatorname{ar tanh} \left[\sqrt{\frac{\sqrt{g}\lambda_\chi\chi - 1}{2\sqrt{g}\lambda_\chi\chi}} \right] \quad (100)$$

$$\chi(\sigma) = \frac{1}{\sqrt{g}\lambda_\chi} \left[1 + 2 \tanh^2 \left(\frac{\sigma(x) - \sigma_0}{\sqrt{6}\lambda_\chi} \right) \right]^{-1} \quad (101)$$

$$A^\mu(\sigma) = \frac{g\lambda_\chi^2}{4} \frac{\hbar}{e^2 c} \left[1 + \frac{\tanh^2 \left(\frac{\sigma(x) - \sigma_0}{\sqrt{6}\lambda_\chi} \right)}{1 + \tanh^2 \left(\frac{\sigma(x) - \sigma_0}{\sqrt{6}\lambda_\chi} \right)} \right]^2 C^\mu. \quad (102)$$

Let us consider two specific cases. First, for $g = 0$ one has

$$\sigma(x) - \sigma_0 = \pm \sqrt{3}\lambda_\chi \operatorname{ar sinh} \frac{\chi}{\chi_0}, \quad (103)$$

$$\chi(\sigma) = \pm \chi_0 \sinh \left(\frac{\sigma(x) - \sigma_0}{\sqrt{3}\lambda_\chi} \right), \quad (104)$$

$$A^\mu(\sigma) = \frac{\hbar}{e^2 c \chi_0^2} \left[1 \pm \sinh \left(\frac{\sigma(x) - \sigma_0}{\sqrt{3}\lambda_\chi} \right) \right]^{-2} C^\mu \quad (105)$$

Secondly, let us see what happens for a massless scalar, that is $m_\chi = 0$,

$$\sigma(x) - \sigma_0 = \pm \frac{2}{\chi_0} \sqrt{\frac{3}{g}} \sqrt{1 + \frac{\chi_0}{\chi}}, \quad (106)$$

$$\chi(\sigma) = \chi_0 \left[\frac{g\chi_0^2}{12} (\sigma(x) - \sigma_0)^2 - 1 \right]^{-1}, \quad (107)$$

$$A^\mu(\sigma) = \frac{\hbar}{e^2 c \chi_0^2} \left[1 - \frac{12}{g\chi_0^2 (\sigma(x) - \sigma_0)^2} \right]^2 C^\mu \quad (108)$$

Alternatively, the results can be presented in terms of the proper time

$$\tau(x) = \int^x \sqrt{dx'^\nu dx'_\nu} = \int^x \sqrt{\eta_{\mu\nu} dx'^\mu dx'^\nu}, \quad (109)$$

throughout a simple change of the parameters

$$\sigma(x) \rightarrow i\tau(x), \sigma_0 \rightarrow i\tau_0, g \rightarrow -g, m_\chi^2 \rightarrow -m_\chi^2 \quad (110)$$

Considering the effective theory (62) through the invariants, one obtains

$$\begin{aligned} \partial_\mu \chi \partial^\mu \chi &= \frac{1}{2} \left(\frac{\partial \chi}{\partial x_\mu} \frac{\partial \chi}{\partial x^\mu} + \frac{\partial \chi}{\partial x_\mu} \frac{\partial \chi}{\partial x^\mu} \right) \\ &= \frac{1}{2} \left(\frac{\partial \chi}{\partial \tau} \frac{\partial \chi}{\partial \tau} - \frac{\partial \chi}{\partial \sigma} \frac{\partial \chi}{\partial \sigma} \right), \end{aligned} \quad (111)$$

$$\begin{aligned} F_{\mu\nu} F^{\mu\nu} &= \frac{\partial A_\nu}{\partial x^\mu} \frac{\partial A^\nu}{\partial x_\mu} + \frac{\partial A_\mu}{\partial x^\nu} \frac{\partial A^\mu}{\partial x_\nu} - \frac{\partial A_\mu}{\partial x^\nu} \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A_\nu}{\partial x^\mu} \frac{\partial A^\mu}{\partial x_\nu} \\ &= \frac{\partial A_\nu}{\partial \tau} \frac{\partial A^\nu}{\partial \tau} - \frac{\partial A_\mu}{\partial \sigma} \frac{\partial A^\mu}{\partial \sigma} - 2 \left(\frac{\partial A^\mu}{\partial \tau} \frac{\partial x_\mu}{\partial \tau} \right). \end{aligned} \quad (112)$$

Since for the invariant fields the Lorentz gauge holds, one has

$$\partial_\mu A^\mu = \frac{\partial A^\mu}{\partial x^\mu} = \frac{\partial A^\mu}{\partial \tau} \frac{\partial x_\mu}{\partial \tau} = 0, \quad (113)$$

and, for this reason, the formula (112) takes the following form

$$\begin{aligned} F_{\mu\nu} F^{\mu\nu} &= \frac{\partial A_\mu}{\partial \tau} \frac{\partial A^\mu}{\partial \tau} - \frac{\partial A_\mu}{\partial \sigma} \frac{\partial A^\mu}{\partial \sigma} \\ &= 2 \frac{\partial A_\mu}{\partial \tau} \frac{\partial A^\mu}{\partial \tau} = -2 \frac{\partial A_\mu}{\partial \sigma} \frac{\partial A^\mu}{\partial \sigma}, \end{aligned} \quad (114)$$

whereas the effective theory (62) expressed becomes the string theory

$$\begin{aligned} L &= \frac{\hbar c}{4} \left[\left(\frac{\partial \chi}{\partial \tau} \right)^2 - \left(\frac{\partial \chi}{\partial \sigma} \right)^2 \right] - \frac{m_\chi^2 c^3}{2\hbar} \chi^2 - g \frac{\hbar c}{4} \chi^4 \\ &\quad - \frac{1}{4\mu_0} \left[\frac{\partial A_\nu}{\partial \tau} \frac{\partial A^\nu}{\partial \tau} - \frac{\partial A_\mu}{\partial \sigma} \frac{\partial A^\mu}{\partial \sigma} \right] \end{aligned} \quad (115)$$

$$L = \frac{\hbar c}{2} \left[\left(\frac{\partial \chi}{\partial \tau} \right)^2 - \frac{m_\chi^2 c^3}{2\hbar} \chi^2 - g \frac{\hbar c}{4} \chi^4 - \frac{1}{2\mu_0} \frac{\partial A_\nu}{\partial \tau} \frac{\partial A^\nu}{\partial \tau} \right], \quad (116)$$

$$L = \frac{\hbar c}{2} \left(\frac{\partial \chi}{\partial \sigma} \right)^2 - \frac{m_\chi^2 c^3}{2\hbar} \chi^2 - g \frac{\hbar c}{4} \chi^4 + \frac{1}{2\mu_0} \frac{\partial A_\mu}{\partial \sigma} \frac{\partial A^\mu}{\partial \sigma} \quad (117)$$

For the Lagrangian in the form (117), the field equations are

$$\frac{\partial^2 \chi}{\partial \sigma^2} + \frac{m_\chi^2 c^3}{2\hbar} \chi + g \chi^3 = 0, \quad (118)$$

$$\frac{\partial^2 A^\mu}{\partial \sigma^2} = 0, \quad (119)$$

and for the solution in the form (70), they reduce to

$$\left(\frac{\partial \chi}{\partial \sigma} \right)^2 = -\frac{g}{3} \chi^4 - \frac{\chi_0 g}{3} \chi^3 - \frac{m_\chi^2 c^2}{3\hbar^2} \chi^2 - \frac{\chi_0 m_\chi^2 c^2}{3\hbar^2} \chi, \quad (120)$$

what is the equation (83) written through the proper distance.

4. Summary

Scalar quantum electrodynamics, which throughout the Higgs mechanism of spontaneous symmetry breaking is the mental nucleus of the Standard Model of elementary particles and fundamental interactions, with help of the O(2)-symmetric Higgs potential, whose established physical significance is remarkable, produced two scenarios, similar to the Higgs mechanism through a neutral scalarfield identifiable with the Higgs boson, which include a dilaton field ' and, first of all, consistently elucidate a photon mass. In the first model, a neutral scalar field mass is estimable through the present-day experimental limits on a photon mass. In the modified model, a mass of non-kinetic vector field k_μ is given through a photon mass is present, dilaton is massless, while a neutral scalar field mass is a free parameter. Moreover, in this scenario the Aharonov {Bohm effect is possible, while the effective theory describes a free photon non-interacting with a neutral scalar, and a photon mass determines the modification to the Higgs potential. The second model opens the way for further research in the non-abelian Yang-Mills theories of the Standard Model, string theory and superconductivity physics.

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