Radiative Effect on MHD Fluid Flow in a Vertical Channel under Optically Thick Approximation

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Abstract

Many non linear problems that arise in real life situations defy analytical solution; hence numerical techniques are desirable to find the solution of such equations. In this study we use the Newton scheme method from Taylor series to solve fourth order non linear problem. The velocity profiles and temperature profile are studied for different physical parameters like, Magnetic parameter M, Radiation F and thermal Grashof number Ga. The results obtained after computation taking into cognizance of the parameters present shows that the Magnetic parameter M increases with increasing velocity, while the trend reverses with radiation and thermal Grashof number in a vertical channel under optically thick approximation. It also observed that temperature rise with increasing values of \( a_i \) and \( \tau_2 \) and trend of heat transfer coefficient due to thermal conduction \( h \) increases with the rate of change of \( \phi \).

Keywords: MHD, recurrence relation, vertical plate, radiation, flow rate


1. Introduction

The different models and approximations of the optical transfer operator are employed in multidimensional problems of radiative correction for remote sensing of ocean, cloudiness, in theories of light and image propagation in turbine media, and in theoretical and computational fundamentals of optical-electronic remote sensing systems. Boundary-value problem of radiation transfer in optically thick finite planar layers is considered which are represented, in particular, by: cloudiness, outbreaks of aerosol particles, dust plumes which are generated under influence of vast fires (forest, peat, steppe, anthropogenic), and the atmosphere with optically thick layers of trace gases.

Radiation and magnetic hydrodynamics (MHD) continues to attract the interest of engineering science and applied Mathematics researches owing to extensive applications of such flows in the context of aerodynamics. Radiation is the process by which energy can be transferred from one body to another through electromagnetic waves in the absence of an intervening medium. If an intervening medium is present. It must be at least partially transparent in order for the radiant energy transfer to take place. Energy is continuously radiated from all substances that are above absolute zero in temperature. If a blackbody is warmer than it radiates more energy than it receives and the environment receives more energy than it radiates. There is thus net transfer of energy from the warm blackbody to the cooler environment. Emission of electromagnetic energy results in a decrease in the energy level. However, there are some radiation phenomena that do not maintain equilibrium between the thermal motion of the molecules and the net gain or loss of radiant energy. Combustion products such as carbon dioxide, carbon monoxide and water absorb and emit radiation. Therefore, the heat transfer from the gas to the channel walls is by both convection and radiation, and a proper analysis of this heat transfer problem requires a simultaneous solution of convection and radiation problems. Heat transfer problems are classified in broad categories according to the variable that the temperature depends on. If the temperature is independent of time, the problem is called a steady or steady-state problem. If the temperature is a function of time, the problem is classified as unsteady or transient.

The study of flow of electrically conducting fluid, the so-called magneto-hydrodynamics (MHD) has a lot of attention due to its diverse applications. In astrophysics and geophysics, it is applied to the study of stellar and solar structures, interstellar matter, and radio propagation through the ionosphere. In engineering, it finds its application in MHD pumps, MHD bearings, nuclear reactors, geothermal energy extraction and in boundary layer control in the field of aerodynamics. A survey of MHD studies could be found in Crammer and Pai [4];

More recently, many researchers have focused attention on MHD applications where the operating temperatures are high. For example, at high temperatures attained in some engineering devices, gas can be ionized and so become electrically conducting. The ionized gas or plasma can be made to interact with the magnetic field and alter the heat and friction characteristics of the system. It is important to study the effect of the interaction of magnetic field on the temperature distribution and heat transfer when the fluid is not only electrically conducting but also when it is capable of emitting and absorbing thermal radiation. Heat transfer by thermal radiation is important when we are concerned with space technology applications and in power engineering. Thus, Grief et al. [5] obtained an exact solution for the problem of laminar convective flow in a vertical heated channel within the optically thin limit of Cogley et al. [3]. Makinde and Mhone [8] investigated the effect of thermal radiation on MHD oscillatory flow in a channel filled with saturated porous medium and non-uniform wall temperatures. Recently Taiwo and Ogunlaran [11] studied Numerical solution of fourth order linear ordinary differential equations by cubic spline collocation tau method.

This paper, is an extension of the work of Ibrahim [6] to include magnetic field. Hence it is proposed to study radiation effect on MHD fluid flow in a vertical channel under optically thick approximation.

2. Formulation of the Problem

The physical model above shows a stream of radiating flow velocity $u$ flowing over the plate. The radiating flow is at its temperature $T$. The temperature of the wall $T_w$. The gravity effect on the radiation and the laminar convection of the gas are also taken into account, the gravity effect is denoted by $g$.

Consider the fully developed laminar flow between parallel walls as shown in the above physical model i.e. the velocity will be a maximum at the center and zero at the walls, also the velocity distribution will be symmetric about the $y$ axis i.e. $y = 0$. With the width of the channel being equal to $2b$. Since in the assumption, the temperatures of the walls are the same and are maintained at a constant temperature gradient $\frac{T_b}{b}$ so that the wall temperature $T_w$ is given by $T_w = T_0 + \left(\frac{T_b}{b}\right)x$. Where $x$ is the coordinate in the vertical direction.

Here The problem of radiation effect on MHD fluid flow in a vertical channel under optically thick approximation, formulated, analysed and solved numerically. The $x$ - axis is taken along the plate in the vertically upward direction and also the $y$ -axis is taken normal to the plate. That the flow is fully developed, the velocity and temperature fields are symmetrical about the central line of the channel in a magnetic region. The temperature of the walls is the same and is maintained at a constant temperature. The viscosity, the thermal conductivity and specific heat are independent of temperature and the essential influence of the variation in density is included in the body force term. Steady flow equations are momentum and energy equation.

$$\begin{align*}
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{\sigma \beta^2 u^2}{\rho} \\
&+ g \beta (T_w - T) + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho_0 C_p} \frac{\partial q_r}{\partial y} 
\end{align*}$$

where $u$ and $v$ are the velocity of the fluid, $T$ is the fluid temperature, $\beta$ is the thermal expansion of the fluid $\rho_0$ is the fluid density, $C_p$ is the specific heat capacity, $v$ is the viscosity of the fluid and $k$ is the thermal conductivity.

In this research work the mathematical formulation has Magnetic field which is not included in the work of Ibrahim (1997) where $\alpha$ the thermal diffusivity and $q_r$ is the radiative, using the Rosseland differential approximation.

$$\frac{\partial q_r}{\partial y} = -\frac{16}{3\alpha} \sigma^2 T^3 \frac{\partial T}{\partial y}$$

The boundary conditions are;

$$\begin{align*}
T &= 0 \quad u = 0 \quad y = b \\
T &= 0 \quad u = 0 \quad y = -b \\
\text{as} \quad y &\rightarrow \infty
\end{align*}$$

On introducing the following non-dimensional quantities

$$\begin{align*}
y &= bY, \quad u = \frac{\alpha U}{b}, \quad T = -\phi \phi, \quad M = \frac{\sigma \beta^3 \alpha^2}{\rho v} \quad F = bL \phi \phi, \quad G = \frac{g \beta^3 \tau}{v \alpha}
\end{align*}$$

Substituting the non-dimensional quantities of equation (5) into (1) to (2), leads to
\[ \frac{d^2 U}{dY^2} - MU = G_u\phi - \gamma \]  
(6)

\[ U = F\phi^3 \frac{\partial \phi}{\partial Y} - \frac{d^2 \phi}{dY^2} \]  
(7)

Where \( \gamma = \frac{b^3}{\nu \alpha} \left( \frac{1}{\rho \chi} \right) \)

Equation (6) and (7) leads to

\[ \frac{\partial^2 \phi}{\partial Y^2} - F\phi^3 \frac{\partial \phi}{\partial Y} - \frac{d^2 \phi}{dY^2} - \left( 6F\phi^2 + M \right) \frac{d^2 \phi}{dY^2} = 0 \]  
(8)

Thus from (8) we have

\[ \frac{\partial G}{\partial \phi_n} = -3F\phi_n^2 \phi_n - 12F\phi_n^3 \phi_n - \left( 6F - 3MF^2 \phi_n^2 \right) \phi_n + G_u \]  
(10)

\[ \frac{\partial G}{\partial \phi_n} = -6F\phi_n^2 - M \]  
(11)

\[ \frac{\partial G}{\partial \phi_n} = -\frac{dF\phi_n}{dY} \]  
(12)

\[ \frac{\partial G}{\partial \phi_n} = -F\phi_n^3 \]  
(13)

\[ \frac{\partial G}{\partial \phi_n} = 1 \]  
(14)

Substituting equations (10) to (14) into equation (9) we obtain

\[ \Delta \phi_n \left( -3F\phi_n^2 \phi_n - 12F\phi_n^3 \phi_n - \left( 6F - 3MF^2 \phi_n^2 \right) \phi_n + G_u \right) \]

\[ + \Delta \phi_n \left( -6F\phi_n^2 - M \right) + \Delta \phi_n \left( -\frac{dF\phi_n}{dY} \right) + \Delta \phi_n \left( -F\phi_n^3 \right) + \Delta \phi_n (1) = 0 \]  
(15)

Expanding equations (15) where

\[ \Delta \phi_n = \phi_{n+1} - \phi_n \, , \, \Delta \phi_n = \phi_{n+1} - \phi_n \, , \, \Delta \phi_n = \phi_{n+1} - \phi_n \, , \, \Delta \phi_n = \phi_{n+1} - \phi_n \, , \, \Delta \phi_n = \phi_{n+1} - \phi_n \, , \, \Delta \phi_n = \phi_{n+1} - \phi_n \]

\[ \phi_{n+1} - \phi_n \left( -3F\phi_n^2 \phi_n - 12F\phi_n^3 \phi_n - \left( 6F - 3MF^2 \phi_n^2 \right) \phi_n + G_u \right) \]

\[ + (\phi_{n+1} - \phi_n) \left( -6F\phi_n^2 - M \right) + (\phi_{n+1} - \phi_n) \left( -\frac{dF\phi_n}{dY} \right) + (\phi_{n+1} - \phi_n) \left( -F\phi_n^3 \right) + (\phi_{n+1} - \phi_n) (1) = 0 \]  
(16)

Collecting the terms involving \( \phi_{n+1}, \phi_{n+1}, \phi_{n+1}, \phi_{n+1} \) and \( \phi_{n+1} \)

On the L.H.S and the terms involving \( \phi_n, \phi_n, \phi_n, \phi_n \) neglecting \( \phi_{n+1} \)

On the R.H.S gives

\[ \phi_{n+1} \left( -6F\phi_n^3 \phi_{n+1} - (6F - 3MF^2 \phi_n^2) \phi_{n+1} \right) \]

\[ - (6F\phi_n^2 + MF^2 \phi_n^2) \phi_{n+1} \]

\[ - (3F\phi_n^2 \phi_n^3 + 12F\phi_n^3 \phi_n^3 + \left( 6F - 3MF^2 \phi_n^2 \right) \phi_{n+1} + G_u \phi_{n+1} \]

\[ = -3F\phi_n^2 \phi_n^3 - 12F\phi_n^3 \phi_n^3 - \left( 6F - 3MF^2 \phi_n^2 \right) \phi_{n+1} \]

Where \( \phi_n \) is our initial solution and \( \phi_{n+1} \) is the assumed solution.

Let

\[ \phi_{N, n+1}(y) = \sum_{i=0}^{6} a_i y^i \]

Considering \( N=6 \)

\[ \phi_{n+1}(y) = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5 + a_6 y^6 \]

\[ \phi_{n+1}(y) = a_1 + 2a_2 y + 3a_3 y^2 + 4a_4 y^3 + 5a_5 y^4 + 6a_6 y^5 \]

\[ \phi_{n+1}(y) = a_2 + 6a_3 y + 12a_4 y^2 + 20a_5 y^3 + 30a_6 y^4 \]

(18)

\[ \phi_{n+1}(y) = 6a_3 + 24a_4 y + 60a_5 y^2 + 120a_6 y^3 \]

\[ \phi_{n+1}(y) = 24a_4 + 120a_5 y + 360a_6 y^2 \]

Putting (18) into (17)

\[ 24a_4 + 120a_5 y + 360a_6 y^2 \]

\[ - (6F\phi_n^3) \left( 6a_3 + 24a_4 y + 60a_5 y^2 + 120a_6 y^3 \right) \]

\[ - (6F\phi_n^2 - M) \left( 2a_2 + 6a_3 y \right) \]

\[ - (6F\phi_n + MF\phi_n^3) \left( 3a_3 y^2 + 4a_4 y^3 + 5a_5 y^4 + 6a_6 y^5 \right) \]

\[ - (6F\phi_n + MF\phi_n^3) \left( a_0 + a_1 y + a_2 y^2 \right) \]

\[ + (6F - 3MF^2 \phi_n^2) \left( a_0 + a_1 y + a_2 y^2 \right) \]

\[ + (a_0 + a_1 y + a_2 y^2) \left( a_0 + a_1 y + a_2 y^2 \right) \]

(19)

Simplifying by collecting terms in \( a_0, a_1, a_2, a_3, a_4, a_5, a_6 \), and \( a_0 \) we obtain.
The velocity distribution of the flow can be obtained from equation (6)

\[
\frac{d^2 U}{dY^2} - MU = G_0 \phi - \gamma
\]
We have our assumed solution to be

\[ \phi(y) = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5 + a_6 y^6 + \tau_2 e^y \]

\[ \phi'(y) = a_1 + 2a_2 y + 3a_3 y^2 + 4a_4 y^3 + 5a_5 y^4 + 6a_6 y^5 + \tau_2 e^y \]

\[ \phi''(y) = 2a_2 + 6a_3 y + 12a_4 y^2 + 20a_5 y^3 + 30a_6 y^4 + \tau_2 e^y \]

Substituting the expression \( \phi(y), \phi'(y) \) and \( \phi''(y) \) into (6) gives

\[ U = F \left( \frac{a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5 + a_6 y^6 + \tau_2 e^y}{M} \right)^3 \]

\[ \left( a_1 + 2a_2 y + 3a_3 y^2 + 4a_4 y^3 + 5a_5 y^4 + 6a_6 y^5 + \tau_2 e^y \right) \]

\[ \left( 2a_2 + 6a_3 y + 12a_4 y^2 + 20a_5 y^3 + 30a_6 y^4 + \tau_2 e^y \right) \]

Evaluating equation (6) in the interval \(-1 \leq y \leq 1\) will enable us to determine the effect of different radiation parameter.

### 3.2. Determination of the Non-dimensional Flow Rate

The non-dimensional flow rate through the channel per unit width is given by

\[ U = \left( \frac{(G_a \phi + \gamma) e^{-\sqrt{M}}}{M(e^{2\sqrt{M}} + e^{-2\sqrt{M}})} \right) e^{\sqrt{M}} \]

\[ + \left( \frac{(G_a \phi + \gamma) e^{-\sqrt{M}}}{M(e^{2\sqrt{M}} + e^{-2\sqrt{M}})} \right) e^{-\sqrt{M}} - \frac{1}{M}(G_a \phi + \gamma) \]

We have our assumed solution to be

\[ \phi(y) = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5 + a_6 y^6 + \tau_2 e^y \]

\[ \phi'(y) = a_1 + 2a_2 y + 3a_3 y^2 + 4a_4 y^3 + 5a_5 y^4 + 6a_6 y^5 + \tau_2 e^y \]

\[ \phi''(y) = 2a_2 + 6a_3 y + 12a_4 y^2 + 20a_5 y^3 + 30a_6 y^4 + \tau_2 e^y \]

Substituting the expression \( \phi(y), \phi'(y) \) and \( \phi''(y) \) into (6) gives

\[ U = F \left( a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5 + a_6 y^6 + \tau_2 e^y \right)^3 \]

\[ \left( a_1 + 2a_2 y + 3a_3 y^2 + 4a_4 y^3 + 5a_5 y^4 + 6a_6 y^5 + \tau_2 e^y \right) \]

\[ \left( 2a_2 + 6a_3 y + 12a_4 y^2 + 20a_5 y^3 + 30a_6 y^4 + \tau_2 e^y \right) \]

Evaluating equation (6) in the interval \(-1 \leq y \leq 1\) will enable us to determine the effect of different radiation parameter.

### 3.3. Determination of Heat Transfer Coefficient

Thus heat transfer coefficient due to thermal conduction is given by

\[ h = -\frac{d\phi}{dy} \]

\[ h = -a_1 - 2a_2 y - 3a_3 y^2 - 4a_4 y^3 - 5a_5 y^4 - 6a_6 y^5 - \tau_2 e^y \]

\[ = -7.7597 + 2(3.3273) y + 3(3.8442) y^2 + 4(0.3152) y^3 \]

\[ -5(1.3929) y^4 - 6(0.2906) y^5 - (0.4982) y^6 \]

### 4. Results and Discussion

The problem of radiation effect on MHD fluid flow in a vertical channel under optically thick approximation, formulated, analysed and solved numerically. In order to point out the effects of physical parameters namely; thermal Grashof number \( G_a \), radiation parameter \( F \), Magnetic parameter \( M \), on the flow patterns, the computation of the flow fields are carried out. The values of velocity and temperature are obtained for the physical parameters as mention.

The velocity profiles has been studied and presented in Figure 1 to 3. The effect of velocity for different values of Magnetic parameter \( M = 5, 10, 15, 20 \) is presented in Figure 1. The trend shows that the velocity increases with increasing Magnetic parameter. The effect of velocity for different values of radiation \( F = 2, 10, 15, 20 \) is also presented in Figure 2. It is then observed that the velocity decreases with increasing values of radiation. The effect of velocity for different values of thermal Grashof number \( (G_a = 0.5, 1, 2) \) is also presented in figure 3. It is then observed that the velocity decreases with increasing values of thermal Grashof number.
In Figure 4 It is observed that temperature rise with increasing values of $\alpha_i$ and $\tau_2$ while Figure 5 shows that trend of heat transfer coefficient due to thermal conduction $h$ increases with the rate of change of $\phi$.

5. Summary and Conclusion

Radiation effect on MHD fluid flow in a vertical channel under optically thick approximation has been studied. In order to point out the effects of physical parameters namely; thermal Grashof number $G_a$, radiation parameter $F$, Magnetic parameter $M$, are presented graphically. It is observed that the velocity increases with increasing Magnetic parameter, while radiation and thermal Grashof number decreases with increasing values of radiation and thermal Grashof number respectively.

It also observed that temperature rise with increasing values of $\alpha_i$ and $\tau_2$ and trend of heat transfer coefficient due to thermal conduction $h$ increases with the rate of change of $\phi$.

References

