Modelling of an Explosion of an Improvised Explosive Device a Vehicle

Turza Jozef*, Eliáš Jozef, Kopiláková Beáta, Rakúsová Danka

Faculty of Special Technology, Trenčín University of A. Dubček in Trenčín

*Corresponding author: jozef.turza@tnuni.sk

Received September 05, 2014; Revised September 15, 2014; Accepted October 09, 2014

Abstract An aim of the paper is to define behaviour of a combat vehicle based on an analysis of its movement from a point of a maximum vertical movement, acceleration and forces through development of a mathematical model of a vehicle during an operation of the vehicle in critical situations of a modern battlefield.

Keywords: combat vehicle, explosives, an explosion, a system, mathematical model, a torque bar, a rocker, a damper, spring and rubber buffer, a vehicle hull


1. Introduction

In deployment of a mechanized infantry an explosion of a charge under a vehicle may appear. The paper is aiming to make an analysis based on a developed mathematical model, what influence will it have on a vehicle’s dynamics. In the paper there is a simplified mathematical model of behaviour of the vehicle. The model can determine what acceleration (hence power) influence a vehicle and what large vertical movements may occur. An issue of a vertical movement of a vehicle after an explosion of a charge [1,7] and in running over a bump is rather complicated; therefore some simplifications have been applied for the first analysis.

2. Description of Dynamics of Vehicle Movement in Explosion under a Vehicle

The BMP-2 combat vehicle has been taken for description of dynamics of a vertical movement of the vehicle, depicted with main parts needed for description of its dynamic features in the Figure 1. The traverse wheels with rockers and torque bars are important for a description, where a torque bar is a spring element for a vertical movement [1,2,3,6].

An explosion of a charge under a vehicle is supposed in a lower part of a hull places under the gravity center of the vehicle.

As an issue of a movement of the vehicle is rather difficult, the following assumptions have been assumed for modelling [5,7]:
- an explosion will occur in the middle of the vehicle under its gravity center,
- the gravity center of the vehicle is situated symmetrically in a longitudinal as well as in a transverse direction,
- we consider no longitudinal, nor transverse vibration of the vehicle, we take only vertical movement of the vehicle into consideration,
- pressure during explosion is regularly distributed on a whole lower surface of the vehicle,
- distribution of forces is regular for all loaded wheels,
- a factor of dumpers absorption is considered as different for an engaging and disengaging of a dumper,
- dumpers are constantly connected with contact points,
- terrain under vehicle is considered flat, clearance of a vehicle is the same,
- vertical movement of the vehicle downwards is limited proportionally for four rockers through lamellar and for four rockers through rubber buffers.

2.1. The Vehicle as a Whole

Mathematical description of particular important parts of the vehicle is defined in a subsequent text. Particular important elements and groups needed for description of behaviour of the vehicle are described separately with certain simplification. There are independent units, as a hull of the vehicle, rockers with torque bars, dampers, and spring and rubber buffers of the vehicle. A separate chapter is a description of a force action from an explosion of an improvised charge on a vehicle bottom.
2.2. Vehicle Hull

With respect to the assumptions we are taking into consideration the forces are acting symmetrically only in a cross and longitudinal sections. The action of forces acting on the vehicle hull in a cross and longitudinal sections are illustrated in the Figure 2. There are forces from placing a torque bar $F_{e}$, from the weight of the vehicle $G_v$, a force from a vehicle acceleration $F_{mv}$, forces from dampers $F_{d}$, forces from a spring buffer $F_{pr}$, in the first and the sixth pair of wheels, forces from a spring buffer $F_{pr1}$, in the second and the fifth pair of wheels, forces from a spring buffer $F_{pr2}$, in the second and the fifth pair of wheels having the number of rockers with wheels $n_k=12$.

\[
G_v + F_{mv} - F_v - F_{d} = \frac{3}{n_k} - F_A \cdot n_k \\
- F_{pr1} \cdot \frac{3}{n_k} - F_{pr2} \cdot \frac{3}{n_k} = 0.
\]

Gravitational force of the vehicle having an $m_v$ weight is defined as
\[
G_v = m_v \cdot g
\]
and an accelerating force
\[
F_{mv} = m_v \cdot \dot{x}_v
\]

The other forces and relations will be defined in the following text.

2.3. A Rocker with a Traverse Wheel and a Torque Bar

We suppose that a road has a flat surface. Kinematics of a movement and main parameters of a traverse wheel rocker with a torque bar is illustrated in Figure 3. We can simplify it for purposes of a dynamic analysis with a simplified way.

Figure 3. Kinematics of a movement of a rocker, a damper, a rubber and a spring buffer

The other forces and relations will be defined in the following text.

In Figure 5 there are illustrated all stresses applied to the rocker in a vertical plane. The principle of a movement of a rocker, damper, a rubber and a spring buffer is delineated there. The movement will be explored only on the only one rocker. The number of rockers is $n=12$, number of spring buffers is four and number of rubber buffers is four. The weight of a vehicle per a rocker is reduced in $1/n_k$ ratio, therefore the number of buffers is also considered in $1:4$ ratio. An uplift of a rocker (vehicle) will be considered approximately
\[
x_v = a_1 \cdot \sin \phi
\]

The $M_f$ torque of a torque bar, having an $l_f$ length of a torque bar, the $G_t$ modulus of elasticity in shear and a $J_f$ polar square moment of a torque bar cross-section having a $d_f$ diameter causes twisting of a rocker by an angle
\[
\phi = \frac{M_1 \cdot l_i}{G_1 \cdot J_i} \Rightarrow M_1 = \frac{G_1 \cdot J_i}{l_i} \cdot \phi = k_1 \cdot \phi \tag{5}
\]

where a polar square moment of a torque bar cross-section is defined from a relation

\[
J_i = \frac{\pi \cdot d_i^4}{32} \tag{6}
\]

Stiffness of a rocker torque bar is defined using an equation (5)

\[
k_i = \frac{G_1 \cdot J_i}{l_i} = \frac{G_1 \cdot \pi \cdot d_i^4}{32 \cdot l_i} \tag{7}
\]

There are applied similar relations between a twisting \( \phi \) and a shift of a vehicle body in large and small deformations

\[
\begin{align*}
x_v &= a_1 \cdot \sin \phi \approx a_1 \cdot \phi \\
x_v &= a_1 \cdot \cos \phi \approx a_1 \cdot \phi \\
x_v &= a_1 \cdot (-\sin \phi \cdot \phi) \approx a_1 \cdot \phi
\end{align*} \tag{8}
\]

We suppose balanced loading of all rockers of traverse wheels with torque bars. A reduced \( I_r \) moment of inertia for a rotational movement of a rocker with a Wheel is calculated based on a relation

\[
I_r = I_a \cdot \phi \tag{9}
\]

where \( I_a \) is a mass moment of inertia of a rocker arm of a traverse wheel and a torque bar. For a vertical movement of an A point (Figure 5) the equation of a balance in a vertical direction is applied

\[
F_A + F_c - \frac{3}{n_k} \cdot F_d = \frac{3}{n_k} \cdot F_{pr1} - \frac{3}{n_k} \cdot F_{pr2} = 0 \tag{10}
\]

a torque equation to A point

\[
F_c \cdot a_1 + M_s + M_f - \frac{3}{n_k} \cdot F_d \cdot a_3 = 0 \tag{11}
\]

respectively to B point

\[
F_A \cdot a_1 - \frac{G_1}{n_k} \cdot a_1 - M_s - M_f - \frac{3}{n_k} \cdot F_d \cdot (a_1 - a_3) = 0 \tag{12}
\]

The buffer forces \( F_{pr1} \) and \( F_{pr2} \) start acting only after having finished an \( x_{max} \) uplift (Figure 3). A mass moment of inertia for \( I_{vr} \) rocker is computed based on a relation

\[
I_{vr} = \frac{1}{32} \cdot \rho \cdot \pi \cdot d_{i1}^4 \cdot l_i + m_{vah} \cdot r_{vah}^2 \tag{13}
\]

After having substituted real values for simulation we receive

\[
\begin{align*}
I_{vr} &= 7850 \cdot \pi \cdot 0.038^4 \cdot 2 / 32 + 25 \cdot 0.15^2, \\
I_{vr} &= 0.5657 \text{ kg} \times \text{m}^2.
\end{align*} \tag{14}
\]

2.4. Damper

It is a double action hydraulic damper, the manufacturer states from testing with 40 double lifts per minute its mean power 3.25 kN in piston rod disengagement and 13 kN in piston rod engagement at \( h_t = 0.1 \text{ m} \) uplift. As a speed of a piston rod uplift in testing is the same in engagement and disengagement, time of uplift is

\[
t_u = 60 / (2 \cdot 40) = 0.75 \text{ s} \tag{15}
\]

and a speed of uplift is

\[
v_t = h_t / t_u = 0.1 / 0.75 = 0.13333 \text{ m/s.} \tag{16}
\]

a coefficient of a damping the piston rod down in disengaging is

\[
f_{d_{12}} = F_2 / h_t = 3250 / 0.1 = 32.5 \text{ kN/m;} \tag{17}
\]

and a coefficient of a damping the piston rod down in engaging is

\[
f_{d_{11}} = F_1 / h_t = 13000 / 0.1 = 130 \text{ kN/m.} \tag{18}
\]

2.5. Spring Buffer

A plate spring has been used in form of an Archimedean spiral in conformity with Figure 7. Basic dimensions for computation were: number of convolutions \( n_l = 7 \), final angle of a spiral \( \phi_{p2} = 0.1 \text{ rad,} \phi_{p1} = 0.1833 \text{ round,} \)

\[
\begin{align*}
&h_{l2} = 92 \text{ mm, plate thickness} b_{l2} = 4 \text{ mm, maximum compression} x_{dm} = 86.6 \text{ mm, radius of a spiral origin} r_{s1} = 12 \text{ mm, radius of a spiral end} r_{s2} = 44 \text{ mm, modulus of elasticity of a plate material} E_{ps} = 210 \text{ GPa.}
\end{align*}
\]

2.6. Construction of a plate buffer in a shape of an Archimedean spiral

A force of a damper can be expressed by an equation

\[
F_d = f_{d1} \cdot \dot{x}_{d1} \cdot \left( \dot{x}_{d1} \geq 0 \right) + f_{d2} \cdot \dot{x}_{d2} \cdot \left( \dot{x}_{d2} < 0 \right) \tag{19}
\]

An uplift of a damper on a rocker will be approximately

\[
x_{d1} = a_3 \cdot \sin \alpha = a_3 \cdot x_v / a_1 \tag{20}
\]
\[ s = r_2 \left[ 1 + \pi \cdot n - \phi_{p2} / 2 \right] \] 
\[ s = 44 \left[ 1 + \pi \cdot 7 - 1.83333 / 2 \right] - 12 \left[ 1 + \pi \cdot 7 - 0.1 / 2 \right] \] 
\[ = 0.7347 \text{ m}. \]

Length of a plate of an Archimedean spiral
\[ L = \sqrt{s^2 + x_{dm}^2} = \sqrt{0.7347^2 + 0.0866^2} = 0.74 \text{ m}. \] 

Stiffness of a plate spring
\[ k_{pr1} = 3 \cdot E \cdot I_1 / L^3 \]
\[ = 3 \cdot 2.1 \cdot 10^{11} \cdot 25.9563 \cdot 10^{-8} / 0.7398^3 = 4.08 \times 10^5 \text{ N/m}. \]

After having finished an uplift of a spring, stiffness of a buffer will be
\[ k_{dor} = E_1 \cdot b \cdot L / h = 2.1 \cdot 10^{11} \cdot 0.004 \cdot 0.7398 / 0.923 \]
\[ = 8.4 \times 10^8 \text{ N/m}. \]

Course of a spring force will be
\[ F_{pr1} = k_{pr1} \cdot x_{pr1} \left( (x_v - x_{max}) \geq 0 \right) \]
\[ + k_{dor} \cdot (x_v - x_{max}) \left( (x_v - x_{max} - x_{dm}) \geq 0 \right) \]
in compression of a plate spring
\[ x_{pr1} = x_v - x_{max}. \]

Course of forces of a plate spiral spring and a fixed buffer is illustrated in the Figure 8.

![Figure 8](image)

**Figure 8.** Course of forces of a plate spring buffer and a fixed buffer

### 2.6. Rubber Buffer

Construction of a rubber buffer is illustrated in the Figure 9. A rubber is vulcanically connected on a segment. A segment is firmly attached to a vehicle hull with a bolted connection. The dimensions and data on a rubber in accordance with the manufacturer’s data are \( a_{g1} = 60 \text{ mm}, \) \( b_{g1} = 70 \text{ mm}, \) \( a_{g2} = 50 \text{ mm}, \) \( b_{g2} = 60 \text{ mm}, \) \( h_g = 53 \text{ mm}. \) A dynamic modulus of elasticity in pressure for a used rubber is considered to be \( E_g = 10 \text{ MPa}. \)

Compression of a rubber buffer is computed through a relation
\[ x_{pr2} = \int_{0}^{k_g} \frac{F_{pr2}}{E_g} \left[ \left( a_{g1} - a_{g2} - a_{g2} \cdot x \right) h_g \right] \left( b_{g1} + b_{g2} - b_{g2} \cdot x \right) dx \]
\[ = \int_{0}^{h_g} \frac{F_{pr2} \cdot k_g^{2}}{E_g \left( a_{g1} - a_{g2} \right) \left( b_{g2} - b_{g1} \right)} \left( k_g + x \right) \left( k_g + x \right) dx \]

Where \( k_1, k_2 \) are auxiliary constants
\[ k_1 = \frac{a_{g1} \cdot h_g}{a_{g2} - a_{g1}}, \]
\[ k_2 = \frac{b_{g1} \cdot h_g}{b_{g2} - b_{g1}}. \]

Through an integration of an equation 28 and using constants from an equation 29 we receive
\[ x_{pr2} = \frac{F_{pr2} \cdot k_g^{2}}{E_g \left( a_{g1} \cdot h_g / a_{g2} - a_{g1} \cdot b_{g2} / b_{g1} \right)} \left( a_{g1} \cdot b_{g2} / a_{g2} \cdot b_{g1} \right) \]
\[ \ln \left( a_{g1} \cdot b_{g2} / a_{g2} \cdot b_{g1} \right) \]

From equation 30 after an adjustment a stiffness of a rubber buffer can be obtained in a following form
\[ k_{pr2} = \frac{F_{pr2}}{x_{pr2}} = E_g \left( a_{g1} \cdot h_g / a_{g2} \cdot b_{g1} \right) \]
\[ h_g \cdot \ln \left( a_{g1} \cdot b_{g2} / a_{g2} \cdot b_{g1} \right) \]

After having substituted the dimensions and parameters of a rubber buffer into an equation 31 we receive an intensity of stiffness
\[ k_{pr2} = \frac{1 \cdot 10 \cdot \left( 0.06 - 0.06 \right) - 0.07 - 0.05}{0.053 \cdot \ln \left( 0.06 - 0.06 \right)} \approx 700 \text{ kN/m}. \]

Relation for computation of a force of a rubber buffer will be
\[ F_{pr2} = k_{pr2} \cdot x_{pr2}. \]

Course of a force of a rubber buffer is drawn by an equation 33 in the Figure 10.

![Figure 9](image)

**Figure 9.** Construction of a rubber buffer

![Figure 10](image)

**Figure 10.** Course of a force of a rubber buffer
2.7. Forces from an Explosion on a Vehicle

An overpressure spreads on a front of a blast wave in an explosion of a charge in spherical surfaces. The time of a rise and a discharge of a blast wave are very short, in order up to 20 ms. Makovička was dealing with modelling of a course of a blast wave in more details [7] and he defined the most probable course. His model has been used in the next text for our case. A value of an overpressure on a front of a blast wave will be calculated through following relations. A reduced spacing distance from an explosion epicenter will be calculated through a distance from an epicenter of an charge explosion in m, a total equivalent mass of a charge \( C_w \) in m/kg\(^{1/3} \) from a relation:

\[
(34)
\]

There is a different value of an overpressure (Figure 11) acting a each point P on a bottom of a vehicle having dimensions \( a \), \( b \) on an element of a surface \( dA = dx \cdot dy \) at \( R \) distance. This value will be calculated from a relation:

\[
(35)
\]

where \( R_0 \) is a vertical distance of a lower surface of the vehicle from a road, \( x \), \( y \) are coordinates of the P point in a coordinate system \( x \), \( y \).

Figure 11. Exerting pressure explosion charges under the vehicle

A total equivalent mass of a charge in kg TNT (trinitrotoluene) is defined from a relation:

\[
(36)
\]

Where \( C_N \) is a weight of a used charge of an explosive in kg, \( k_{NTp} \) is a pressure TNT equivalent, \( k_E \) coefficient of a charge sealing, \( k_G \) coefficient of a geometry of a blast wave spreading in a space (for a detonation in a free space \( k_G = 1 \), on a terrain surface \( k_G = 2 \)), \( k_B \) is a ballistic ratio (ratio of a case weight in kg/ to weight of explosives in kg). A sealing coefficient is defined based on relation:

\[
k_E = 0.2 + 0.8 \cdot (1 + k_B). \tag{37}
\]

A size of an overpressure in MPa on a front of a blast wave for a ground explosion in MPa at a ground explosion in external environment for \( R_p \leq 1 \) (in our case the term is applied)

\[
p_p = \frac{1.07}{R_p^3} - 0.1 = \frac{1.07 \cdot C_w}{R^3} = \frac{1.07 \cdot C_w (x^2 + y^2 + R_0^2)^{3/2}}{R^3} - 0.1. \tag{38}
\]

An overpressure in an overpressure phase of a blast wave in Mpa

\[
p_{pod} = \frac{0.035}{R_p^3} = \frac{0.035 \cdot C_w^{1/3}}{(x^2 + y^2 + R_0^2)^{1/2}}. \tag{39}
\]

Duration of an overpressure wave in s

\[
t_p = 1.6 \cdot 10^{-3} \cdot C_w^{1/6} \cdot R^{1/2}. \tag{40}
\]

Duration of an under pressure wave in s

\[
t_{pod} = 1.6 \cdot 10^{-3} \cdot C_w^{1/3}. \tag{41}
\]

Course of an overpressure and an under pressure wave in time by previous relations is illustrated in figure, where \( p_0 \) is a pressure of an environment. For different 4 values of a charge weight in kg of TNT a Figure 13 illustrates a maximum value of an overpressure for a vertical distance from a bottom surface of vehicle from a road \( R_0 = 0.5 \) m.

Figure 12. Values of a course of a maximum overpressure

Figure 13. Course of a maximum overpressure on a blast wave head under a vehicle

Figure 14. Course of maximum stresses in quadrant I for different sections in a cross direction \( x=(1÷10) \)
In a selected I quadrant a bottom of a vehicle is divided through a grid into ten sections in a cross direction of a vehicle \((x=1\to10)\) and also into ten sections in a longitudinal direction \((y=1\to10)\).

Figure 14 illustrates a course of maximum overpressures in a cross-section in \(t_p\) time and the Figure 15 in a longitudinal direction.

![Figure 15](image)

**Figure 15.** Course of maximum overpressures in the I quadrant for different sections in a longitudinal section \(y=(1\to10)\)

Based on courses we can note, that if a blast wave leaves in a longitudinal direction, the value of a maximum pressure is lower than if it leaves under a vehicle in a cross direction. A size of maximum acting force acting on a bottom part of the vehicle in a cross-section in \(t_p\) time can be defined based on a relation

\[
F_y = 4\cdot10^6 \int_0^{y/2} \int_0^{x/2} \frac{1.07\cdot C_w}{\left(x^2 + y^2 + R_0^2\right)^{3/2}} - 0.1\, dx \cdot dy. \tag{42}
\]

After integration and adaptations from the equation 42 we receive

\[
F_y = 4\cdot10^6 \cdot \frac{1.07\cdot C_w}{R_0} \cdot \frac{1 - \arccos\left(\frac{a_x - b_y}{\sqrt{a_x^2 + b_y^2 + 4R_0^2}}\right)}{4 - 0.1\cdot \frac{a_x - b_y}{4}}. \tag{43}
\]

In case, we would considered, that under a whole surface of a vehicle there is acting a maximum overpressure, a value of a maximum force in \(t_p\) time would be

\[
F_{y}^* = p_d \cdot a_x \cdot b_y. \tag{44}
\]

A course of a force is acting on a bottom of a vehicle in \(t_p\) time only with a maximum value of \(F_y\) in time \(t_p/2\), it means in a form of an isosceles pulse (Figure 12, respectively for particular values by a Figure 13). With respect to small dimensions of a vehicle we do not consider any regressive blast wave.

For combat vehicles we need to consider mostly the mass of a used charge up to \(C_{v}=5\) kg TNT placed just about a terrain surface, where \(k_G=2\).

### 3. Simulation Model

A simulation model has been made up in a computation model DYNAST based on above mentioned equations for a particular vehicle through impedance networks. A block simulation model is shown in the Figure 17. Stiffness of a torque bar with a rocker using equations 5 and 7 with a damper and buffers into consideration as follows

\[
k_i = g_i \cdot \pi \cdot d_i^4
\]

\[
= 78.5 \cdot 10^9 \cdot 0.038^4
\]

\[
= 78.828 \text{ kN/m.}
\]

In balanced stable conditions a \(\phi_{st}\) deformation of rockers is defined using equations 8 and 11, equations 11 and 12 and further adjustment without taking forces of a damper and buffers into consideration as follows

\[
\phi_{st} = \frac{32\cdot m_c \cdot g \cdot I_i \cdot a_1}{n_k \cdot G_t \cdot \pi \cdot d_i^4}
\]

\[
= 32-12.8 \cdot 10^{-3} \cdot 9.81\cdot 1.93 \cdot 0.3
\]

\[
= 0.377 \text{ rad.}
\]

with a corresponding angle \(\phi_{st}=2.202^\circ\), respectively a static compression in accordance with equation 8

\[
x_{ext} = a_1 \cdot \phi_{st} = 300 \cdot 0.377 = 113.1 \text{ mm.}
\]

This value will be an initial condition for a further solution in a simulation. The previous values were calculated for a length of a torque bar \(l_t=1.93\) m, diameter of a torque bar \(d_i=38\) mm, weight of a vehicle \(m_c=12800\) kg, gravity acceleration \(g=9.81\) m/s².

![Figure 17](image)

**Figure 17.** A block simulation model for an explosion of a charge under a vehicle
feature of a damper, an Lt acceleration resistance describing a mass moment of inertia of a rocket and attached masses, Ct capability of a torque bar observing stiffness of a torque bar, a Epr source respecting a feature of a spring buffer, a Epr2 observes a feature of a rubber buffer, Lv an acceleration resistance observing a mass of a vehicle, the Epm source observes a mass of a vehicle, Ev source of force from explosion, J1 and E1 represent a transformer of a vehicle movement and a rocker movement, Blv and BIt are integrating blocks to define uplifts of a vehicle xv and uplift at the end of a rocker xv. Theory in more details is mentioned in [8].

4. Results of a Simulation

Some results of a simulation of the explosion under a vehicle with a charge of size \( C_N = 5 \) kg TNT in accordance with data from a previous text are drawn in the Figure 18.

![Figure 18](image.png)

**Figure 18.** Results of simulation of a charge explosion under a vehicle

A course \( x_v \) of a vehicle uplift is drawn in the Figure 18, having a \( v_v \) speed of the uplift and an \( a_v \) acceleration in a vertical direction during an explosion of a charge under a centre of a vehicle. From courses it can be seen that a maximum uplift of a vehicle is 388 mm at 10.8 ms, a minimum uplift -115 mm, becoming stabilized after 1.5 s. Speed of uplift is maximum at 5.873 m/s in time 16.5 ms and minimum -1.419 m in time 1.5 s. A maximum value of acceleration is 6542 m/s² in time of 1 ms and minimum -46.92 m/s² in time of 56.5 ms. A value of a maximum acceleration for a charge being considered is significant, mainly a value of an acceleration of a vertical movement of a vehicle.

Based on verification by a created program we are presenting its improvements also for an explosion of a charge aside from a vehicle centre.

5. Conclusions

A generated simplified mathematical model and results of a simulation have shown that a construction of a chassis is very well designed.

It has been proved that action by force from an explosion of a charge of a size of \( C_N = 5 \) kg TNT is significant, mainly a value of an acceleration of a vertical movement of a vehicle.

Acknowledgement

This publication was created in the frame of the project "Alexander Dubček University of Trenčín wants to offer high-quality and modern education", ITMS code 26110230099, based on the Operational Programme Education and funded from the European Social Fund.

References