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Soil-structure Interaction Taken into Account in Characterizing the Behavior of Flexible Piles

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Abstract The aim of this study is to gain a better understanding of the behavior of flexible piles subjected to lateral loading and head moment in clay soil, taking into account soil-structure interaction. In order to establish the behavioral model of the assembly, soil-pile interaction models according to different authors were presented. According to several authors, the soil-pile interface is characterized by the soil reaction modulus E_s. This parameter, which characterizes the interaction between the soil and the structure, depends not only on the mechanical and geometric properties of the concrete and the foundation soil. In this context, the model of [1] was used to characterize E_s, which is a function of the rheological parameters of the soil (α and E_M) and the geometric characteristics of the pile. After establishing and solving the behavior model of the rigid soil-pile assembly, the Python programming tool was used to perform the parametric analysis. The results show the significant impact of the interaction model adopted. The analysis also shows that the soil pressure modulus, soil slenderness and pile head forces are more influential than the other model parameters.

Keywords: *Flexible pile, Characterization, Soil-structure interaction, Analytical calculation, Numerical modeling*

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1. Introduction

Traditionally, deep foundations are designed to withstand axial loads. However, these structures are sometimes subjected to lateral loads. These may be quasi-static (e.g., the impact of a ship docking, the sudden braking of a convoy on a bridge) or dynamic (e.g., swell, the effect of wind on structures, earthquakes). However, the calculation of flexible piles is often complex, since it involves soil-structure interaction. For this reason, a complete and rigorous characterization of the soil-pile interaction is essential to ensure proper control of the structure's behavior. This phenomenon of soil-pile interaction has seen the development of several research works [1,2,3,4,5,6,7,8]. It is in this context that this study focuses on the consideration of soil-structure interaction in the characterization of flexible piles under lateral loads. In the context of this work, the pile is considered as a beam and the soil is modeled as a set of horizontal springs with a reaction modulus E_s. The aim of this work is to characterize the soil-pile interaction, establish the behavioral model of the assembly and carry out a parametric study after resolution of the behavioral model.

2. Modeling the Behavior of Flexible Piles Under Lateral Loads and Characterizing Soil-Pile Interaction

Several studies have been carried out to characterize the behavior of piles under lateral loads. These studies have led to the development of several approaches that can be classified into four categories:

- Reaction modulus method [9],
- ***p-y*** curve method [10] & [11],
- Elastic continuum method [12],
- Finite element or finite difference numerical methods.

Although somewhat complex, finite element and finite difference methods are often used.

2.1. Behavior model

Winkler's analytical method is the oldest, and can be used to predict the lateral reaction of the soil. It involves modeling the interaction between the soil and the pile using a series of independent springs of varying stiffness.

The stiffness provides a direct link between the lateral reaction of the soil (***p***) and the lateral displacement of the pile (***y)*** under lateral loading (Figure 1). This method is the basis of the ***p-y*** curves, where the "springs" have a non-linear behavior.

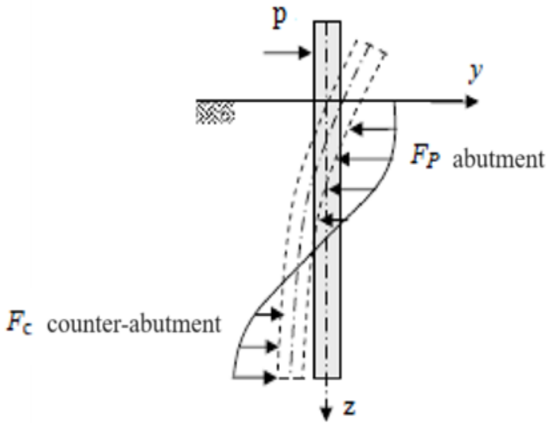

Figure 1. Pile mobilizing lateral soil reaction [9]

Winkler's model defines soil as a stack of independent slices. Each slice of soil is modelled by a lateral spring (Figure 2) on which the pile rests.

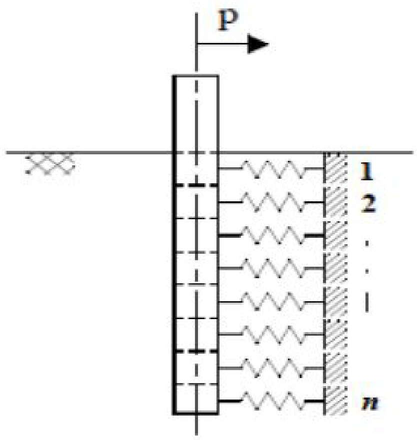

Figure 2. Representation of the Winkler Model [9]

The pressure ***p*** on a soil "slice" depends only on its lateral displacement and on a soil reaction coefficient, called $k_h\left(MN/m^3\right)$ in the case of lateral loading.

$$p = k_h\left(z\right).D.y \quad (1)$$

This equation is also expressed in the following form:

$$p = E_S.y \quad (2)$$

Where

P : Soil reaction per unit length of pile

E_S : Soil reaction modulus, ($E_S = k_h.D$)

D: Pile diameter or width

The pile is idealized as a laterally loaded elastic beam. The soil is modelled by horizontal springs, independent of each other, and of stiffness E_S. Thus, the pile's behavior is governed by the equation of a beam on elastic supports:

$$E_p I_p \frac{d^4 y}{dz^4} = -p.D \quad (3)$$

This leads to the following equation, which governs the behavior of the pile.

$$E_p I_p \frac{d^4 y}{dz^4} + k_h\left(z\right).D.y = 0 \quad (4)$$

The solutions to this equation can be obtained either analytically or numerically. The main advantage of this method is that at any point along the pile, the soil-pile interaction can be defined. But this definition is restricted by the assumption that the pressure at a point is a linear function of the displacement at that point, and by its dependence on the soil-pile interaction model characterizing the entire structure.

2.2. Characterization of Soil-structure Interaction

Defining the reaction modulus profile is the main difficulty in studying pile behavior. It depends on numerous parameters such as pile stiffness, loading level, soil type, etc. Pressiometric test results are commonly used for foundation design. [13] lists most of the models used to predict soil-structure interaction. He concludes that the reaction modulus E_S can be determined either from Young's modulus (E) or from the pressiometric modulus (E_M). Several authors have worked on the characterization of soil-structure interaction, namely (Table 1 to 3):

Table 1. Reaction modulus according to various authors

Authors	Relationship
[1]	$\frac{E_s}{E_M} = \begin{cases} \frac{3}{\frac{2}{3}\left(\frac{D_0}{D}\right)\left(2{,}65.\frac{D}{D_0}\right)^{\alpha} + \frac{\alpha}{2}} & pour\ D > D_0 \\ \frac{18}{4.2{,}65^{\alpha} + 3\alpha} & pour\ D < D_0 \end{cases}$
[2]	$\frac{E_S}{E} = \frac{1}{1{,}35} = 0{,}74$ $E = A.\gamma.z$
[3]	$\frac{E_s}{E} = 0{,}82$
[4]	$E_s = K\left(0{,}308 + 1{,}584\frac{D}{L}\right)\frac{z}{r.L}$ $r = D + D.tan\beta$
[10] & [11]	$E_s = 1{,}3\sqrt[12]{\frac{E.D^4}{E_p I_p}}\frac{E}{(1-\nu^2)}$

Table 2. A coefficient values from [2]

Sand density	Let go	Medium	Dense
A value	100-300	300-1000	1000-2000

Table 3. K parameter values [4]

Soil type	K
Dense sand	200-400
Medium-density gravel	150-300
Medium-dense sand	100-250
Fine sand	80-200
Stiff clay	60-180
Saturated stiff clay	30-100
Plastic clay	10-80
Clay	2-30

The different parameters involved in the relationships of Table 1 are defined below:

γ : Soil density

A: Dimensionless coefficient as a function of sand density given in Table 2.

D_0 : Reference diameter equal to 0.6

α : Rheological coefficient depending on soil type

E_M : Pressuremeter modulus

E : Soil modulus of elasticity

D : Pile diameter

E_pI_p : Flexural rigidity of the pile

ν : Poisson's ratio

L : Pile length

z : Depth

D : Pile diameter

β: Angle dispersion between φ/4 and φ

φ : Angle of ground friction

L : Pile length

K : Soil parameter (Table 3)

One of the disadvantages of this Poulos method [3] is that it cannot be extended to a stratified soil medium, nor can the influence factors be calculated using the equation of Mindlin [16]. Indeed, Mindlin's equation is not applicable to a non-homogeneous stratified medium. Furthermore, the assumption that the pile is a rectangular strip embedded in the soil is only approximately valid if the pile has a square or *I-shaped* cross-section. In the case of piles with a circular cross-section, this idealization needs to be brought closer, but seems reasonable. This method has been used in practice by several engineers.

Another approach is the ***P-y method***, a generalization of the Winkler model. It is a semi-empirical method, because the prediction and construction of curves for the study of an isolated pile is based on laboratory or in situ tests. Each soil is represented by a series of ***P-y*** curves. In effect, the soil is assimilated to linear or non-linear elastic supports (*commonly referred to as springs*). This is translated into (*P*, *y*) diagrams (Figure 3), *i*.e. relationships between lateral reaction, *P*, and lateral displacement, *y*.

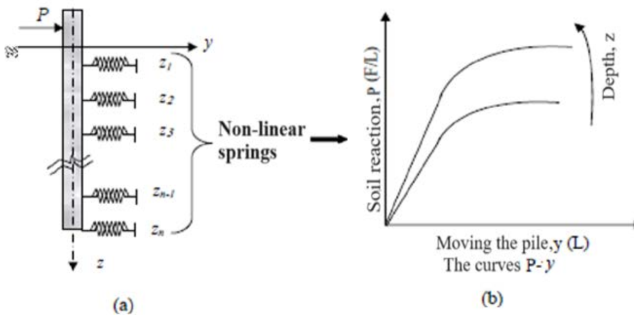

Figure 3. (a) Winkler model for lateral loading and (b) reaction curves (p-y) [9]

For a soil-pile system subjected to lateral loading, let's consider what happens at a section (or pile slice) located at depth z. At rest, after installation, the section is subjected to lateral earth pressure, the resultant of which is zero.

When the pile is subjected to lateral loading, the section under consideration is displaced laterally by y_i and the stress state is modified in such a way that the lateral resultant on the section under consideration has a direction opposite to the displacement y_i. Over the entire height of the pile, for a given depth, similar behavior with varying intensities can be observed. This makes it possible to study the entire pile for any loading and any soil type. Non-linear ***P-y*** curves that vary with depth and soil type are obtained along the entire length of the pile (Figure 4).

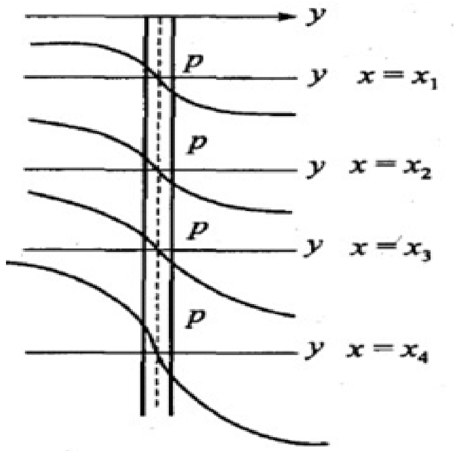

Figure 4. Series of p-y curves for a pile [14]

Since a *p-y* curve represents the behavior of a pile at a given cross-section, and therefore for different slabs for the whole pile, assuming that the cross-sections are independent, several researchers have proposed methods for determining them in order to dimension piles. For the shape of the pile cross-section, tests carried out by [15] show that the shape has very little influence on lateral pressure distribution and ultimate pile strength. The methods devised and developed by numerous researchers use a variety of approaches: in situ tests, laboratory tests, physical modelling or numerical modelling. The diversity of these approaches leads to as many *P-y* reaction curves. Among the various methods of calculating the reaction modulus E_s, two are applicable to clays, namely the formulas of [1] et [3]. We begin with a comparative analysis of these two models to determine which is closer to the exact results. Python was then used to plot curves representing displacements, bending moment and shear force as a function of all model parameters. To make the results more applicable and accessible, we made some of them dimensionless.

In this way, we have thus represented the displacements y(z), the bending moments M(z), and the shear forces V(z) as a function of the ratio between the depth and the length of the pile (z/l) for a value of the moment at the head M_0 = 3 MN.m and V_0 = 0.1 MN (Figure 5).

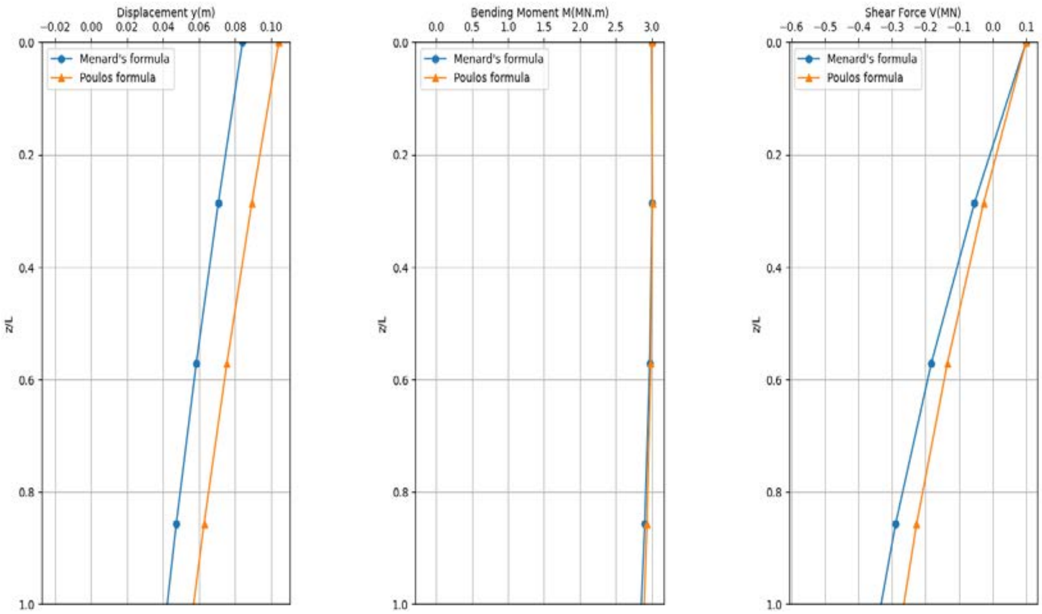

Figure 5. Comparative study of the Ménard and Poulos formulas

3. Resolution of the Behavioural Model of a Rigid Pile Subjected at the Head to a Moment M_0 and a Lateral Force V_0

The behavior of the foundation depends on both its own bending stiffness (E_p, I_p) and that of the soil E_s, i.e. the relative stiffness of the pile-soil. This is expressed hereafter as the transfer length l_0. Let's consider a flexible pile subjected at the head to a moment M_0 and a force V_0 in linearly elastic clay (Figure 6).

Considering a beam section loaded by a distributed load ***P*** and delimited by two infinitely adjacent cross-sections ***dz*** apart (Figure 7).

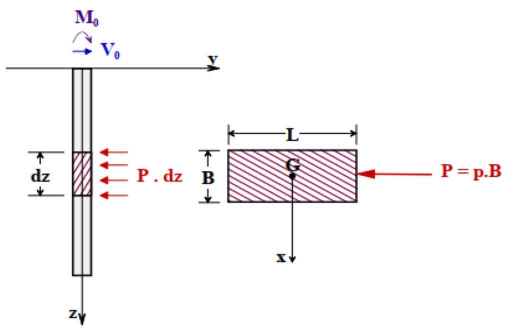

Figure 6. Diagram of a pile under M_0 and V_0

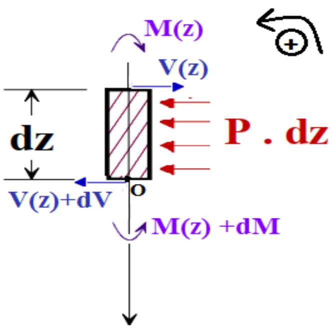

Figure 7. Diagram of an infinitesimal pile section under V and M

We obtain the following equilibrium equations:

$$\begin{cases} \Sigma F = 0 \\ \Sigma M_{/O} = 0 \end{cases} \Rightarrow$$

$$\begin{cases} -V(z) + V(z) + dV + P.dz = 0 \\ -M(z) - V(z).dz + P\dfrac{dz^2}{2} + M(z) + dM = 0 \end{cases} \quad (5)$$

By assuming $dz^2 = 0$

$$P = -\frac{dV}{dz} \quad (6)$$

$$V = \frac{dM}{dz} \quad (7)$$

We find the equations for straight beams subjected to a uniformly distributed force P (kN/m):

$$P = -\frac{dV}{dz} = -\frac{d^2M}{dz^2} = -E_P I_P \frac{d^4y}{dz^4} \quad (8)$$

$$V = \frac{dM}{dz} = E_P I_P \frac{d^3y}{dz^3} \quad (9)$$

$$M = E_P I_P \frac{d^2y}{dz^2} \quad (10)$$

E_pI_p: Flexural rigidity of the pile in relation to the main axis of inertia

$y^{(n)}$: Derivative $n^{ième}$ of displacement perpendicular to the mean fiber with respect to z

P(z) : Soil reaction distributed along the pile in kN/m (P = p × B)

B: Pile section width for a rectangular section (*B* = *D* for a circular section)

V(z) : Shear force; by convention, shear force is counted positively and the derivative of the moment with respect to z is equal to +V(z).

M(z) : Bending moment

In this work, the deferred Young's modulus ($E_p = E_{p,eff}$) was considered to take account of the effect of creep. If the soil reaction law can be considered as linear elastic : $P = E_s.y$. If the soil reaction law can be considered linear elastic, the fundamental relationship describing the behavior of the pile can be deduced, which is the 4th-order linear differential equation:

$$E_P I_P \frac{d^4 y}{dz^4} + E_s(z).y = 0 \quad (11)$$

The reaction modulus distribution is assumed to be of the form: $\boldsymbol{E_s(z) = a.z^n}$ (Gibson's Sol).

We are in the case of an over-consolidated homogeneous clay, so $n = 0$

The reaction modulus is therefore constant $E_s(z) = E_s$

Equation (11) becomes:

$$E_p I_p \frac{d^4 y}{dz^4} + E_s.y = 0 \quad (12)$$

$$\frac{E_p I_p}{E_s}.\frac{d^4 y}{dz^4} + y = 0 \quad (13)$$

Posing $l_0{}^4 = \frac{4E_p I_p}{E_s}$, we obtain the equation :

$$l_0{}^4 y^{(4)} + 4y(z) = 0 \quad (14)$$

$y^{(4)}$: 4^{th} derivative of displacement perpendicular to the mean fiber with respect to z.

3.1. Displacement and Load Calculations

The general solution to this equation [17] is given by:

$$y(z) = e^{-z/l_0}.\left(C_1 \cos\cos\frac{z}{l_0} + C_2 \sin\sin\frac{z}{l_0}\right) + e^{z/l_0}\left(C_3 \cos\cos\frac{z}{l_0} + C_4 \sin\sin\frac{z}{l_0}\right) \quad (15)$$

Where C_1, C_2, C_3, C_4 integration constants determined from the boundary conditions at the head and foot of the pile.

l_0: Transfer or elastic length. It can be defined as the minimum pile length for which lateral head loading exists. The remainder of the plug beyond about three times this length is mechanically inactive.

$$l_0 = \sqrt[4]{\left(\frac{4E_p I_p}{E_s}\right)} \quad (16)$$

This solution allows us to obtain the expressions for bending moment $\boldsymbol{M}$ and shear force $\boldsymbol{V}$ at any soil level, given respectively by the following expressions:

$$M(z) = \frac{2E_p I_p}{l_0^2}\left[e^{\frac{-z}{l_0}}\left(C_1 \sin\sin\frac{z}{l_0} - C_2 \cos\cos\frac{z}{l_0}\right) + e^{\frac{z}{l_0}}\left(-C_3 \sin\sin\frac{z}{l_0} + C_4 \cos\cos\frac{z}{l_0}\right)\right] \quad (17)$$

$$V(z) = \frac{2E_p I_p}{l_0^3}\left\{e^{\frac{-z}{l_0}}\left[C_1\left(\cos\cos\frac{z}{l_0} - \sin\sin\frac{z}{l_0}\right) + C_2\left(\cos\cos\frac{z}{l_0} + \sin\sin\frac{z}{l_0}\right)\right] + e^{\frac{z}{l_0}}\left[-C_3\left(\cos\cos\frac{z}{l_0} + \sin\sin\frac{z}{l_0}\right) + C_4\left(\cos\cos\frac{z}{l_0} - \sin\sin\frac{z}{l_0}\right)\right]\right\} \quad (18)$$

These results can be applied when the pile sheet $l > 3l_0$ for a flexible pile or $l < l_0$ for a rigid pile, they are intended to be used for simple cases where the soil is relatively homogeneous and for given head loads. When the flexible piles are loaded at the head, the conditions at the tip do not come into play, and the positive exponential terms are negligible. We are then reduced to a system of two equations with two unknowns, and the conditions at the head make it possible to determine the two remaining constants C_1 and C_2.

3.2. Calculating Integration Constants

A pile is said to be long when its length L is greater than or equal to three times the transfer length l_0. The general solution turns into:

$$y(z) = e^{-z/l_0}.\left(C_1 \cos\cos\frac{z}{l_0} + C_2 \sin\sin\frac{z}{l_0}\right) \quad (19)$$

For a free-heading pile subjected to a horizontal force V_0 and a moment noted M_0 at z = 0. Displacement and rotation are free at the head of the pile.

$$\frac{dy}{dz} = \frac{1}{l_0} e^{\frac{-z}{l_0}}\left((-C_2 - C_1)\sin\sin\frac{z}{l_0} + (C_2 - C_1)\cos\cos\frac{z}{l_0}\right) \quad (20)$$

$$\frac{d^2 y}{dz^2} = \frac{2}{l_0^2} e^{\frac{-z}{l_0}}\left(C_1 \sin\sin\frac{z}{l_0} - C_2 \cos\cos\frac{z}{l_0}\right) \quad (21)$$

$$\frac{d^2 y}{dz^2} = \frac{M(z)}{E_p I_p} = \frac{2}{l_0^2} e^{\frac{-z}{l_0}}\left(C_1 \sin\sin\frac{z}{l_0} - C_2 \cos\cos\frac{z}{l_0}\right) \quad (22)$$

$$\frac{d^3 y}{dz^3} = \frac{V(z)}{E_p I_p} = \frac{2}{l_0^3} e^{\frac{-z}{l_0}}\left((C_2 - C_1)\sin\sin\frac{z}{l_0} + (C_2 + C_1)\cos\cos\frac{z}{l_0}\right) \quad (23)$$

Boundary conditions at the top: at z = 0, $V(0) = V_0$ and $M(0) = M_0$. We obtain the expressions of the constants:

$$\begin{cases} C_1 = \dfrac{V_0 l_0^{\,3}}{2E_p I_p} + \dfrac{M_0 l_0^{\,2}}{2E_p I_p} \\ C_2 = -\dfrac{M_0 l_0^{\,2}}{2E_p I_p} \end{cases} \quad (24)$$

The solution to the differential equation is therefore:

$$y(z) = \frac{2}{l_0^{\,2} E_S} e^{\frac{-z}{l_0}} \left(\begin{matrix} V_0 l_0 \cos\cos\dfrac{z}{l_0} \\ +M_0 \left(\cos\cos\dfrac{z}{l_0} - \sin\sin\dfrac{z}{l_0} \right) \end{matrix} \right) \quad (25)$$

$$M(z) = e^{\frac{-z}{l_0}} \left(M_0 \left(\cos\cos\frac{z}{l_0} + \sin\sin\frac{z}{l_0} \right) + V_0 l_0 \sin\sin\frac{z}{l_0} \right) \quad (26)$$

$$V(z) = e^{\frac{-z}{l_0}} \left(V_0 \left(\cos\cos\frac{z}{l_0} - \sin\sin\frac{z}{l_0} \right) - 2\frac{M_0}{l_0} \sin\sin\frac{z}{l_0} \right) \quad (27)$$

4. Parametric Study of the Model

The behavior of the pile will be studied for different values for each soil-pile interaction parameter, while other data remains constant. Table 4 below shows the basic data used for the influence study.

Table 4. Data used for the influence study

D_0 (m)	L (m)	E_M (MPa)	α	f_{ck} (MPa)	V_0 (MN)	M_0 (MN.m)	E_p (MPa)
0.6	20	2.5	0.5	25	0.1	3	10492

f_{ck} represents here the characteristic compressive strength of the concrete.

Figure 8 shows that pile slenderness has a non-negligible influence on the stress state, especially over the upper 3/5 of the pile length. The results also show that variations in slenderness have a negligible influence on transverse pile displacements. These results highlight the significant role played by the soil mass in limiting the displacements generated by the lateral load in the clay.

Figure 9 and Figure 10 show that an increase in Young's modulus leads to a reduction in the displacements of the flexible pile. These observations highlight the significant impact of Young's modulus on the structural response of the flexible pile in clay. An increase in Young's modulus reduces deformations, while a decrease results in greater deformations for a flexible pile. It should be noted that the influence of Young's modulus is nevertheless quite weak (on a small scale), hence the zoom (figure 10) for a better appreciation.

The results in Figure 11 show that the lower the pressuremeter modulus, the greater the displacement. This is because the pressure modulus is a measure of soil stiffness and represents the relationship between the pressure applied to the soil and the resulting deformation. As a result, a higher pressuremeter modulus indicates a more rigid soil that is less likely to deform, resulting in reduced displacements. The influence of the pressiometric modulus is most noticeable in the upper 3/5ths of the pile length.

Figure 12 and Figure 13 show a significant influence of the lateral force applied at the head on the displacements of flexible piles, compared to the moment. The influence of pile head forces is most noticeable in the upper 2/5 of the pile. These forces play a decisive role in the pile's ability to deform and alter its interaction with the ground.

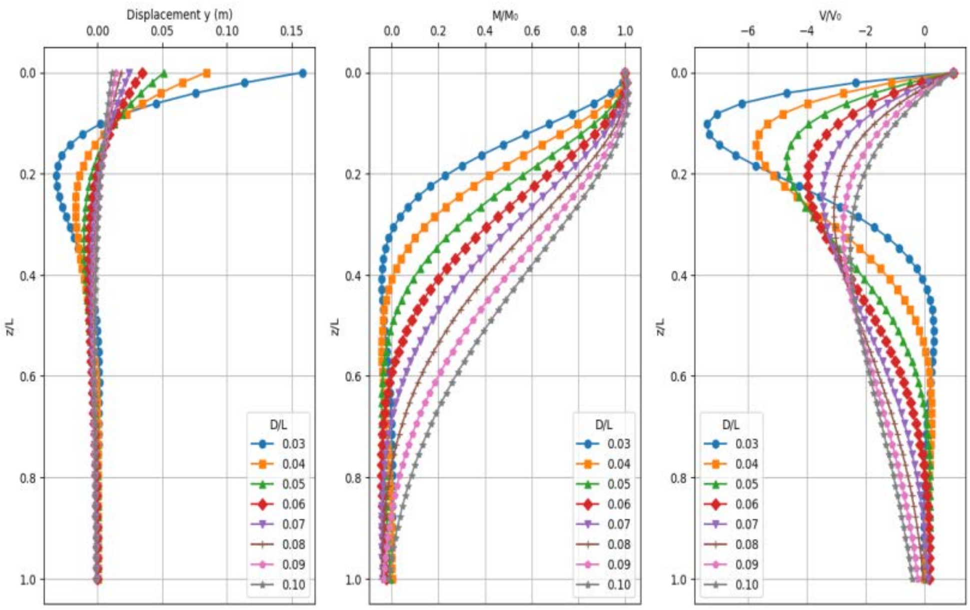

Figure 8. Influence of pile slenderness

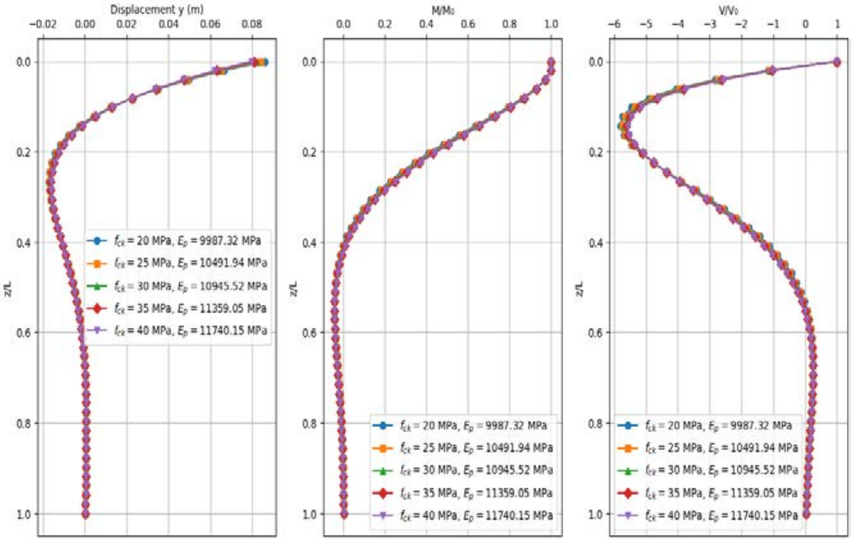

Figure 9. Influence of Young's modulus of pile

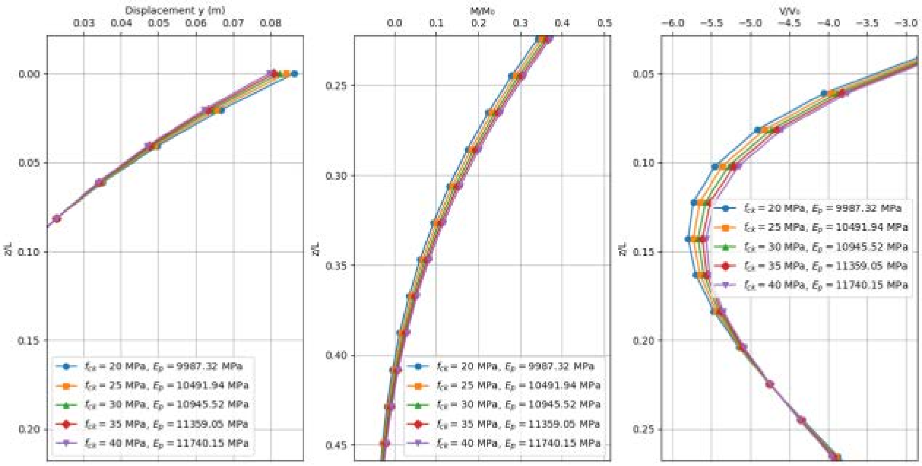

Figure 10. Zoom on the influence of Young's modulus of the pile

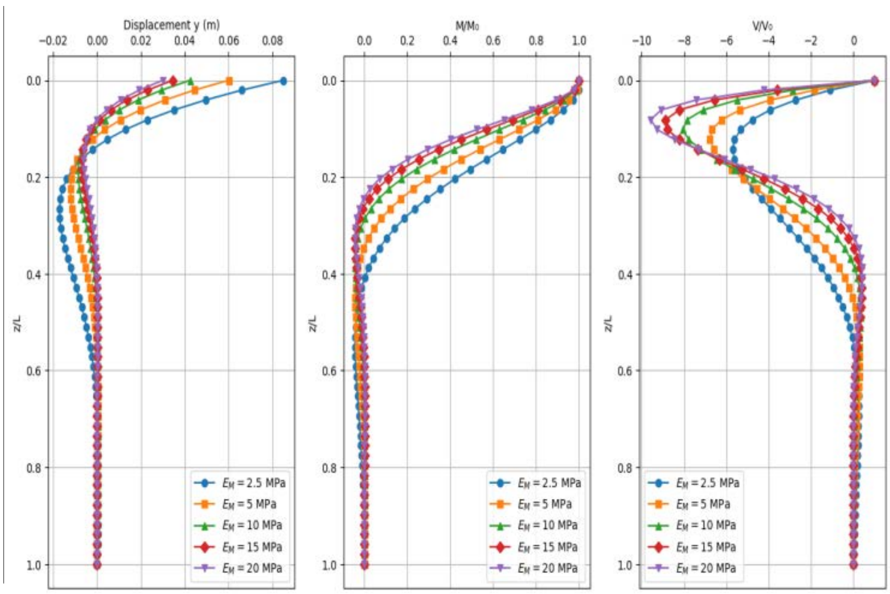

Figure 11. Influence of the Pressiometric Module

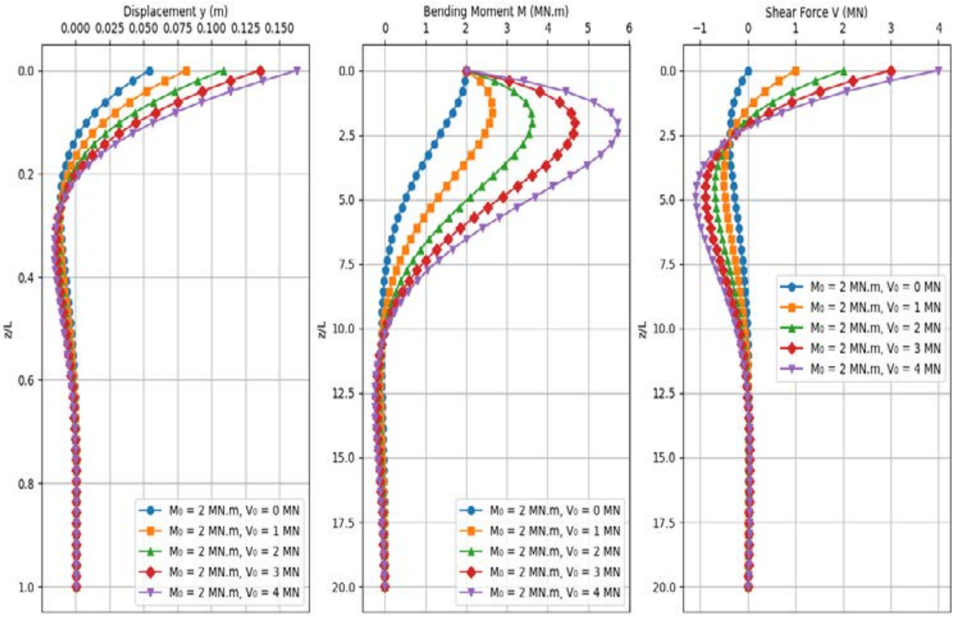

Figure 12. Influence of shear force at the head of the pile

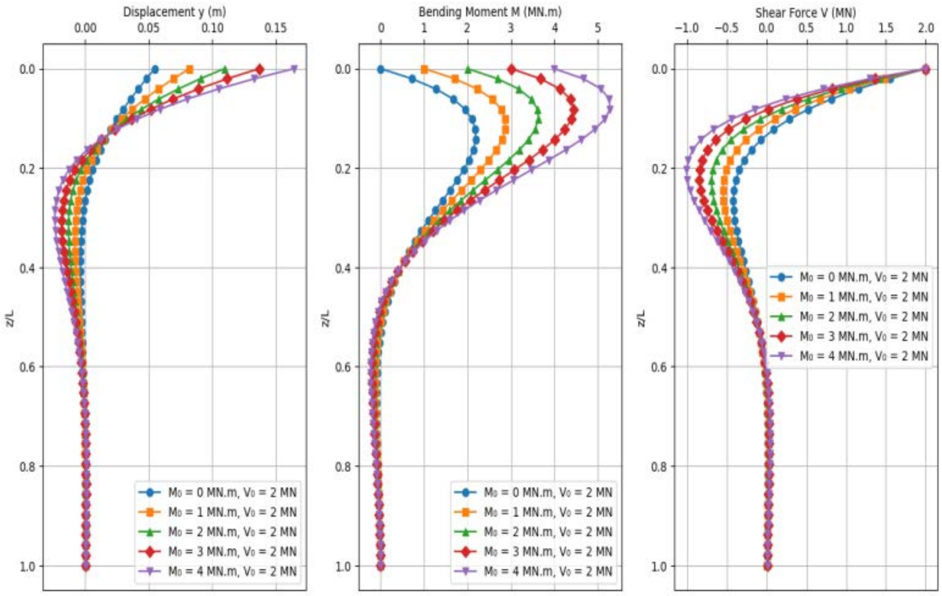

Figure 13. Influence of bending moment

5. Conclusion

In summary, the aim of this study is to gain a better understanding of the behavior of piles in a clay soil massif subjected to lateral loads applied at the head, taking into account soil-structure interaction, which is a complex phenomenon. After establishing the behavioral model, the influence of certain parameters, such as soil reaction modulus, pile diameter, Young's modulus and pressiometric modulus, on the behavior of rigid piles was studied. Analytical and numerical approaches were used to predict the response of flexible piles under lateral loads. The results showed the significant impact of the interaction model adopted. The analysis also showed that the soil pressure modulus, soil slenderness and pile head forces were more influential than the other model parameters.

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