A Sample of Calculation of Anisotropy Loaded Beams

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Abstract In this paper, newly-derived equations, leaning on the canons of the classic durability of materials are used for calculation of leaf springs. Analytic relations between displacements, deformations, Young’s modulus and external loads are used. The correctness of the method was verified through theoretical and experimental research. Researches were done on composite leaf spring made from polymer and glass fibres. The main value of the new theory is its high similarity with the experimental data.

Keywords: strength, calculations, crooked beams


1. Introduction

In the paper [1,2,3,4] the construction of left and new dependencies for large deflections of curved beams was described. New equations are basis of analysis of perpendicular dislocations in the range of small linear diffractions or large non-linear diffractions of straight beams. To calculate beams about any curvature were used analytic connections between dislocation of axis of beam, deformation of beam ε, surface of transversal section A and module of Young and external loads.

The analytical dependences between load F and moment of bend M for burdened beam will be shown in [2,3]:

• deformations (geometrical relationship):

\[
\varepsilon = \frac{1}{1 - \chi} \left[ \frac{dV}{ds} - \chi U \right] - \frac{1}{\chi} \left[ \frac{d^2U}{ds^2} + \frac{d^2\chi}{ds^2} V + \frac{dV}{ds} \right]
\]  

(1.1)

• normal strength (physical relationship):

\[
F = E_A \left( \frac{dV}{ds} - \chi U \right) - E_J \left( \frac{d^2U}{ds^2} + \frac{d^2\chi}{ds^2} V + \frac{dV}{ds} \right)
\]  

(1.2)

• moment of bend (physical relationship):

\[
M = E_J \left[ \frac{d^2u}{ds^2} + \frac{d\chi}{ds} \bigg( \frac{dV}{ds} - \chi U \bigg) + \frac{d\chi}{ds} \bigg( \frac{d\chi}{ds} V + \frac{dV}{ds} \bigg) \right]
\]  

(1.3)

where: U- radial dislocation, V- circuit dislocation axis of beam, \( \chi \) - curvature of the beam, s - coordinate the length of the beam, F - normal force
Angle of turn (γ) section of beam, contain is between unit vectors (e1) contiguous to axis of spring Figure 1 and contiguous vector (e2) to deformed axis. We describe this by equation:

\[
\gamma = \left( \frac{dU}{ds} + \chi V \right) \left( 1 - \frac{1}{2} \left( \frac{dU}{ds} + \chi V \right)^2 \right). \tag{1.12}
\]

And increase of angle of turn of section of beam calculates by equation:

\[
\varepsilon = \left( \frac{dP}{ds} - \xi_2 \right) \left( 1 - \xi_2 \right) ds. \tag{1.13}
\]

Deformation ε we can to describe by equation:

\[
\varepsilon = \left( \frac{dP}{ds} - \xi_2 \right) \left( 1 - \xi_2 \right) ds. \tag{1.14}
\]

Geometrical relationship between deformation, dislocations and curvature as well as location of considered fiber describes equation:

\[
\varepsilon = \frac{\int \left( \frac{dV}{ds} \gamma U \right) ds}{1 - \xi_2}. \tag{1.15}
\]

Bending moment can be described by equation:

\[
\frac{F_0 + \chi \cdot M}{EA} = \frac{dV}{ds} - \chi U, \tag{1.16}
\]

\[
\frac{1}{EJ} \left[ \left( 1 + \frac{\chi^2}{J} \right) M + \frac{\chi^2}{J} F \right] = \frac{d}{ds} \left( \frac{dU}{ds} + \chi V \right) \left[ 1 + \left( \frac{dU}{ds} + \chi V \right)^2 \right]^{\frac{3}{2}}. \tag{1.17}
\]

This equations of binding large dislocation with dimensions of section of beam, curvature and physical properties.

2. Application of the Method

In the next part of this article the derived equations were applied to calculations of composite lift spring. Internal burdens of any element of spring we can count use well-known conditions. Sum of projections of strengths in studied section Figure 2:

\[
\sum F_y = 0, \quad T(\phi) \cdot \cos \phi + F_r(\phi) \cdot \sin \phi = \frac{P}{2}, \tag{2.1}
\]

\[
\sum F_x = 0, \quad F_r(\phi) \cdot \cos \phi - T(\phi) \cdot \sin \phi = 0, \tag{2.2}
\]

and from this :

\[
T(\phi) = \frac{P \cdot \cos \phi}{2}, \tag{2.3}
\]

\[
F_r(\phi) = \frac{P \cdot \sin \phi}{2}. \tag{2.4}
\]

Sum of moments:

\[
\sum M = 0, \quad M(\phi) = \frac{PR}{2} (\sin \alpha - \sin \phi). \tag{2.5}
\]

Equation 2.3, 2.4, 2.5 and 1.16, 1.17 give possibility to count radial dislocations (U) and circumferential (V) of spring axis in dependence from external loads Figure 2.
Relationships between radial dislocations \((U)\) and circumferential \((V)\) and vertical dislocations \((f)\) and horizontal \((f_p)\) are possible to get from equations:

Horizontal dislocations \(- f_p\)
\[
f_p = U \sin \phi + V \cos \phi.
\] (2.6)

Vertical dislocations \(- f\)
\[
f = U \cos \phi - V \sin \phi.
\] (2.7)

If we’ll put to equation of inertia moment of spring section (1.7):
\[
\xi_2 = -\frac{h(\phi)}{2},
\] (2.8)
and we’ll accept, that:
\[
f' = J = \frac{b(\phi) \cdot h^3(\phi)}{12},
\] (2.9)

We will get possibility of calculation of leaf spring at any shape.

For spring at solid widths \(- b\) and variable thickness \(- h(\phi)\) to have substituted (2.8) and (2.6) to the equation (1.2) we receive maximum squeezing tensions:
\[
\sigma_{max} = \frac{P \sin \alpha}{2 \cdot A(\phi)} + \frac{P R(\sin \alpha - \sin \phi)}{2 \cdot J(\phi)} + \frac{0.5h(\phi)}{1 + 0.5 \chi h(\phi)}.
\] (2.10)

3. Experimental Research

The equations presented above were used to calculate the composite curved leaf spring. In the further step the results of calculations were compared with the FEM calculations. The leaf spring has a regular width and a changing thickness. The scheme of leaf spring was shown on the Figure 3. It has the three-ply construction. The core was made from glass epoxy composite reinforced by textile. External layers have regular thickness, they are reinforced by continuous glass fibers. The width of layers is equal to the width of leaf spring.

Using the Algor system the calculations of composite leaf spring were conducted. The results of FEM were compared with the calculations made using the new author’s analytic method and with the results of laboratory research. Leaf springs were tested on the static load. The perpendicular and horizontal dislocation of center leaf was measured on various loading. The experimental data was compared with the results of calculations obtained using the theory of small deflections (TMU), the method FEM (MES) and the theory of large deflections (TDU). In Figure 4 the horizontal dislocation of the central part of one composite leaf spring was shown. Experimental results were marked by symbol “EXP”.

Similarly, the horizontal dislocation of central part of leaf spring was put together in Figure 5.

In case of the perpendicular dislocation, exactitudes of particular computational methods are similar. The dislocation calculated by the FEM method and by the method derived in the support of the theory of large dislocation (TDU) are approximate to measured. The horizontal displacements calculated by TDU better describes the actual situation. In Figure 4 and Figure 5 the line marked EXP measurement data indicated a line described TDU symbol calculation results obtained using the derived equations. It should be noted much less time-consuming the TDU method in comparison to the method of MES. The methodology described above, has been used to calculate stress in the selected cross-section spring. In Figure 6 the normal tension in external layers of spring was shown.
Methods give very similar sizes of tensions. In sections No. 1, 2 and 3 the differences between methods did not exceed eight percent. In faintly burdened sections (4 and 5) the differences are larger and achieve eighteen percent. Counted tensions value without help of a new method (TDU) are comparable to counted values of FEM method.

4. Conclusions

The equations 1.16 and 1.17 can be used to calculate large dislocations of beams with any curvature, composite beams as well.

The newly-introduced correlations have been based on the classical canons of strength, assuming the continuity of the structure of construction materials, in contrast to finite element methods. This tool enables verification of the correctness of the calculations obtained by commonly used MES methods. It can also be used as a standalone tool for strength calculations of anisotropic structures. The correctness of the method was verified by theoretical and experimental studies. Tests were carried out on leaf spring made from epoxy resin and glass fiber. Formed by the FEM methods the composite spring were analyzed using analytical methods based on the new author theory. A series of laboratory research have been conducted as experimental studies. Theoretical studies have shown the usefulness of the newly-developed theory as a tool for verifying the correctness of FEM calculations. Maximum differences in treatment outcomes between the FEM and the developed theory have not exceeded 20%. The error of theory relative to the experimental measurements have not exceeded 18%. The main value of the new theory is its high similarity with the experimental data. The theory by no means can replace the FEM methods. But, it can enable to monitor the correctness of the FEM calculations performed. The theory can also be used to create rods design. It should be noted that the analytical method is based on the new theory is much less time-consuming compared to the MES. The simplicity of the method is its big advantage.

References