Analysis of Least Square and Exponential Regression Techniques for Energy Demand Requirement (2013-2032)

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Abstract This paper considered a long term electric power load forecast for twenty years (20 years) projection, in Nigeria power system using least-square regression and exponential regression model. The model is implemented in Matlab platform with a plot in residential load demand, commercial load demand and industrial load demand in (MW). In the quest for analysis and predicting the energy (power) demand (MW) requirement for a projection period of (2013 - 2032), data are collected between (2000 - 2012), from the Central Bank of Nigeria (CBN), and National Bureau of statistics (NBS). The results obtained shows that energy generated from the respective generating station including Egbin thermal power station Lagos, Sapele thermal power station etc. are grossly inadequate. This mismatch is a major problem in power system planning and operation. The result also shows that there is deviation between predicted energy demand (MW) and available power (or capacity allocated). The predicted energy demand into the projected future of 20years is 45 5,870.2MW. The paper work also extended the prediction form into: least-square, exponential regression model. Evidently, the comparism plot for linear and exponential model which shows similar predicting pattern: particularly least-square exhibit linear behavior, while exponential shows non-linear behaviour, the linear model gives more accurate result as compared to the exponential.

Keywords: exponential regression, least square, energy demand, load, energy, demand, long term forecast


1. Introduction

The generating section are strategically located across the geopolitical-zone in Nigeria with different generating station capacity, people gradually drift from rural to urban cities which has led to an excessive demand of electricity due to the fast growing rate of industries, economic development and increasing population of the residence which has also led to epileptic power supply, power failure, fluctuation and total power outage. It has brought to a loss of energy utilization by the consumers and utility companies.

Peak load forecasting play major role in electrical power system operation, unit commitment and energy scheduling (Amin-Naseri and Soroush, 2008). Energy demand forecasting presents the firsts step in planning and developing for future generation, transmission and distribution facilities. One of the primary tasks of electric utility is to accurately predict energy demand requirements at all times, especially for a long term planning.

Based on the outcomes of such forecasts utilities coordinates their resources to meet the forecasted energy demand, thereby engaging a least-cost Energy management plan and follow-up which are subject to numerous uncertainty that is, in planning for future capacity resource needs an information and operation of the existing generation resources, in order to predicts future capacities and the power system serves one main function that is, to supply energy to the respective customers, which are residential, commercial and industrial consumers with electrical energy as economically and reliable condition. Another responsibility of power utilities is to recognize the needs of their customers Demand and the supply the necessary energies.

Evidently, limitations of energy resources in additions to environmental factors, requires electric energy to be used, more efficiently power plant and transmission lines to be constructed.

1.1. Aim of this Paper

The aim of this paper is to conduct the analysis of the load forecast and energy demand.

1.2. Objectives of this Paper

(i). The objective of this paper is to carry out using engineering techniques, to analyze the behaviour of the energy demand forecast with the aid of data collection.

(ii). To investigate Energy demand profile of the existing capacity and the energy consumption pattern
(iii). To analyse and forecast the energy demand response for a projection of 25 years ahead.

(iv). To recommends to the regulatory agencies for implementing mismatches between the load allocations and the required capacity.

The study shall consider the consumption pattern for residential, commercial and industrial Energy demand forecast, for a projection of twenty-five years ahead.

1.3. Significance of the Paper

The contents of this paper will be of great benefits particularly, to the electricity utilities, regulatory agencies and Nigeria at large, owing to the fact that development of electricity infrastructure is undoubtedly a capital intensive project that needs a serious attention. Hence, Energy demand forecast shall be taken as the first priority for future expansion planning program. Therefore to keep Nigeria abreast with other developing countries which have exhibited in every standard a substantial growth in economic development this means that, the existing gap between the electric power generation and energy demand requirement must be bridged.

2. Problem Statement

Sustainable supply of electric power is a prerequisite for energy generation, transmission and distribution to foster all forms of Economic development in the country. The Nigerian power system is not generating enough electric power, this inadequacy has led to:

(i) Extra ordinary line losses,
(ii) Load shedding,
(iii) Failure and collapse of power system,
(iv) Reduction in quality of electric power

Therefore there is need to overcome these challenges of poor supply of power to the end users at all times.

3. Material and Method

3.1. Least Square

The materials required for the analysis of electricity (demand) prediction in this paper is the load-capacity (allocation) and capacity-utilization data of previous years from Central Bank of Nigeria (CBN), National Bureau of Statistics (NBS) and Power Holding Company of Nigeria (PHCN).

The paper strongly need to investigate the deviation of the capacity allocated to that of the capacity utilization on the view to analyse the rate of load consumption pattern with respect to capacity allocation by the Central Controlling Body: National Control Centre of Nigeria.

The analysis of demand forecasts are required for expansion, controlling and scheduling of power systems. The forecast help in determining the optimal-mix of generating capacities and which power plant to operate in a given period, so as to minimize costs and secure demand. The study is essential to be able to predict/forecast the quantity of power needed by Nigeria owing to the declining nature of the Nigerian power system supply and plan for future network expansion, in order to reduce cost of energy generation, stop load shedding and reduce power outages to minimum. Energy consumption data are collected for resident, commercial and industrial:

<table>
<thead>
<tr>
<th>Year</th>
<th>Industrial</th>
<th>Commercial</th>
<th>Residential</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>1011.60</td>
<td>2346.00</td>
<td>4608.40</td>
<td>8688.90</td>
</tr>
<tr>
<td>2001</td>
<td>1987.20</td>
<td>2439.00</td>
<td>7714.80</td>
<td>9034.40</td>
</tr>
<tr>
<td>2002</td>
<td>1830.00</td>
<td>3297.60</td>
<td>7668.50</td>
<td>12842.40</td>
</tr>
<tr>
<td>2003</td>
<td>1659.80</td>
<td>3583.00</td>
<td>7668.50</td>
<td>12866.60</td>
</tr>
<tr>
<td>2004</td>
<td>1605.00</td>
<td>3830.30</td>
<td>7725.30</td>
<td>13160.60</td>
</tr>
<tr>
<td>2005</td>
<td>1615.50</td>
<td>3851.00</td>
<td>7760.00</td>
<td>13226.60</td>
</tr>
<tr>
<td>2006</td>
<td>1575.00</td>
<td>3900.80</td>
<td>7650.00</td>
<td>13125.80</td>
</tr>
<tr>
<td>2007</td>
<td>1530.50</td>
<td>3915.00</td>
<td>7860.30</td>
<td>13305.80</td>
</tr>
<tr>
<td>2008</td>
<td>1502.50</td>
<td>3852.00</td>
<td>7910.05</td>
<td>13264.55</td>
</tr>
<tr>
<td>2009</td>
<td>1585.00</td>
<td>3865.50</td>
<td>8075.00</td>
<td>13525.50</td>
</tr>
<tr>
<td>2010</td>
<td>1589.40</td>
<td>3925.80</td>
<td>8205.20</td>
<td>13720.40</td>
</tr>
<tr>
<td>2011</td>
<td>1615.50</td>
<td>4004.70</td>
<td>8285.60</td>
<td>13905.80</td>
</tr>
<tr>
<td>2012</td>
<td>1648.00</td>
<td>4025.40</td>
<td>8350.00</td>
<td>14023.40</td>
</tr>
</tbody>
</table>


Calculation and Analysis using Parabola (2nd degree polynomial)

This method employs the least-square technique used in developing a curve that describes the relationship between two or more variables. For example, capacity allocation (A), capacity utilization (U), and difference between the two capacities as errors (E) etc.

That is a given pair of polynomial data can be represented between two sets as:

\[ Y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots \ldots + a_n x^n \]  

(3.1)

This can be represented in other form as:

\[
\begin{align*}
Y_1 &= a_0 + a_1 x_1 + a_2 x_1^2 + a_3 x_1^3 + \ldots \ldots + a_n x_1^n \\
Y_2 &= a_0 + a_1 x_2 + a_2 x_2^2 + a_3 x_2^3 + \ldots \ldots + a_n x_2^n \\
Y_3 &= a_0 + a_1 x_3 + a_2 x_3^2 + a_3 x_3^3 + \ldots \ldots + a_n x_3^n \\
Y_n &= a_0 + a_1 x_n + a_2 x_n^2 + a_3 x_n^3 + \ldots \ldots a_n x_n^n
\end{align*}
\]  

(3.2)

On summing up the columns (equation. 3.2) we have:

\[ \sum_{i=1}^{n} y_i = na_0 + a_1 \sum_{i=1}^{n} x_i + a_2 \sum_{i=1}^{n} x_i^2 + a_3 \sum_{i=1}^{n} x_i^3 + \ldots \ldots + a_n \sum_{i=1}^{n} x_i^n \]  

(3.3)

The above equation from the basis for the least-square method of 2nd – degree polynomial curve fit. Considering (equation 3.3), their increasing order of sequences, it can be placed in matrix formation e.g. Hence multiplying equation (3.3) by x:
\[
\sum_{i=1}^{n} y_i x_i = a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3 + a_3 \sum x_i^4 + \ldots + a_n \sum x_i^{n+1} 
\]  
(3.4)

- Multiply (3.4) by x:
\[
\sum_{i=1}^{n} y_i x_i^2 = a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4 + a_3 \sum x_i^5 + \ldots + a_n \sum x_i^{n+2} 
\]  
(3.5)

Hence, in matrix form:
\[
\begin{bmatrix}
\sum_{i=1}^{n} x_i \\
\sum_{i=1}^{n} x_i^2 \\
\vdots \\
\sum_{i=1}^{n} x_i^{n+1}
\end{bmatrix}
\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}
= 
\begin{bmatrix}
\sum_{i=1}^{n} y_i \\
\sum_{i=1}^{n} y_i x_i \\
\vdots \\
\sum_{i=1}^{n} y_i x_i^{n+1}
\end{bmatrix}
\]  
(3.6)

Since the equation of a straight line:
\[
y = a + bx 
\]  
(3.7)

This can be obtained using, determinate by matrix operation; using Cramer rule:
\[
\begin{bmatrix}
\sum_{i=1}^{n} x_i \\
\sum_{i=1}^{n} x_i^2 \\
\vdots \\
\sum_{i=1}^{n} x_i^{n+1}
\end{bmatrix}
\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}
= 
\begin{bmatrix}
\sum_{i=1}^{n} y_i \\
\sum_{i=1}^{n} y_i x_i \\
\vdots \\
\sum_{i=1}^{n} y_i x_i^{n+1}
\end{bmatrix}
\]  
(3.8)

Similarly,
\[
a = \frac{\sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i (y_i x_i)}{n \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} 
\]  
(3.9)

This means that,
\[
\begin{bmatrix}
\sum_{i=1}^{n} x_i \\
\sum_{i=1}^{n} x_i^2 \\
\vdots \\
\sum_{i=1}^{n} x_i^{n+1}
\end{bmatrix}
\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}
= 
\begin{bmatrix}
\sum_{i=1}^{n} y_i \\
\sum_{i=1}^{n} y_i x_i \\
\vdots \\
\sum_{i=1}^{n} y_i x_i^{n+1}
\end{bmatrix}
\]  
(3.10)

Several methods or techniques are usually used and applied in the analysis of load forecasting for energy demand. The research work rely on the least-square and exponential regression techniques this is because of its numerous advantages, measurements error problems, the issues of large variation of data collection, missing observation of data etc. Therefore this work will consider the comparison between the two techniques that is preferred in the analysis.

The least square method is one of the mathematical tools used in developing a curve that describe the relationship between two variables. A polynomial of any degree can be established using least square method including the straight line form. A given pair of data can be represented by a polynomial that can best fit the relationship between two set of values, as given in the Table 2.
equations in expression 3.18 to get the values of $a_0$ and $a_1$.

Applying cramer’s rule, this becomes:

$$
\begin{bmatrix}
    \sum x_i & \sum x_i^2 \\
    \sum x_i^2 & \sum x_i^3 \\
    \sum x_i^3 & \sum x_i^4
\end{bmatrix}
\begin{bmatrix}
    a_0 \\
    a_1
\end{bmatrix} =
\begin{bmatrix}
    \sum y_i \\
    \sum y_i x_i \\
    \sum y_i x_i^2
\end{bmatrix}
$$

(3.19)

$$
a_0 = \frac{\sum y_i \sum x_i^2 - \sum x_i (\sum y_i x_i)}{n \sum x_i^2 - (\sum x_i)^2}
$$

(3.20)

$$
a_1 = \frac{n \sum y_i x_i - \sum x_i (\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2}
$$

(3.21)

$$
a_i = \frac{n \sum y_i x_i^2 - \sum x_i^2 (\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2}
$$

(3.22)

The polynomial of the second degree (quadratic equation) can be determined in a similar manner. The matrix formation for such is given as:

$$
\begin{bmatrix}
    n & \sum x_i & \sum x_i^2 \\
    \sum x_i & \sum x_i^2 & \sum x_i^3 \\
    \sum x_i^2 & \sum x_i^3 & \sum x_i^4
\end{bmatrix}
\begin{bmatrix}
    a_0 \\
    a_1 \\
    a_2
\end{bmatrix} =
\begin{bmatrix}
    \sum y_i \\
    \sum y_i x_i \\
    \sum y_i x_i^2
\end{bmatrix}
$$

(3.23)

The matrix is 3 x 3 since the polynomial of the second degree is of the form

$$
Y = a_0 + a_1 x + a_2 x^2
$$

(3.24)

**Commercial Demand**

This analysis and procedure is repeated in the same manner for the case of residential and industrial load demand forecast.

<table>
<thead>
<tr>
<th>Year</th>
<th>X</th>
<th>Commercial Demand (MW)</th>
<th>xy</th>
<th>x²</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>-6</td>
<td>2346.00</td>
<td>-14076.00</td>
<td>36</td>
</tr>
<tr>
<td>2001</td>
<td>-5</td>
<td>2439.00</td>
<td>-12195.00</td>
<td>25</td>
</tr>
<tr>
<td>2002</td>
<td>-4</td>
<td>3297.60</td>
<td>-13190.40</td>
<td>16</td>
</tr>
<tr>
<td>2003</td>
<td>-3</td>
<td>3583.00</td>
<td>-10749.00</td>
<td>9</td>
</tr>
<tr>
<td>2004</td>
<td>-2</td>
<td>3830.30</td>
<td>-7660.60</td>
<td>4</td>
</tr>
<tr>
<td>2005</td>
<td>-1</td>
<td>3851.00</td>
<td>-3851.00</td>
<td>1</td>
</tr>
<tr>
<td>2006</td>
<td>0</td>
<td>3900.80</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>2007</td>
<td>1</td>
<td>3915.00</td>
<td>3915.00</td>
<td>1</td>
</tr>
<tr>
<td>2008</td>
<td>2</td>
<td>3852.00</td>
<td>7704.00</td>
<td>4</td>
</tr>
<tr>
<td>2009</td>
<td>3</td>
<td>3865.50</td>
<td>11596.50</td>
<td>9</td>
</tr>
<tr>
<td>2010</td>
<td>4</td>
<td>3925.80</td>
<td>15703.20</td>
<td>16</td>
</tr>
<tr>
<td>2011</td>
<td>5</td>
<td>4004.70</td>
<td>20023.50</td>
<td>25</td>
</tr>
<tr>
<td>2012</td>
<td>6</td>
<td>4025.40</td>
<td>24152.40</td>
<td>36</td>
</tr>
<tr>
<td>Total</td>
<td>0</td>
<td>46836.10</td>
<td>21372.60</td>
<td>182</td>
</tr>
</tbody>
</table>

The gradient of the trend line

$$
b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = 117.43
$$

Trend line value when $x = 0$:

$$
a = \frac{\sum y}{n} - \frac{b \sum x}{n} = 3602.77
$$

**Table 4. Table of Values for Actual Commercial Demand**

<table>
<thead>
<tr>
<th>Year</th>
<th>Commercial Demand (MW)</th>
<th>Trend Value Y (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>2346.00</td>
<td>2898.19</td>
</tr>
<tr>
<td>2001</td>
<td>2439.00</td>
<td>3015.62</td>
</tr>
<tr>
<td>2002</td>
<td>3297.60</td>
<td>3135.00</td>
</tr>
<tr>
<td>2003</td>
<td>3583.00</td>
<td>3250.48</td>
</tr>
<tr>
<td>2004</td>
<td>3830.30</td>
<td>3367.91</td>
</tr>
<tr>
<td>2005</td>
<td>3851.00</td>
<td>3485.34</td>
</tr>
<tr>
<td>2006</td>
<td>3900.80</td>
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</tr>
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<td>2007</td>
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</tr>
<tr>
<td>2008</td>
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</tr>
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<td>2009</td>
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</tr>
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</tr>
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</tr>
<tr>
<td>Total</td>
<td>46836.10</td>
<td>46836.01</td>
</tr>
</tbody>
</table>

To calculate the accuracy of commercial forecast

The mean absolute deviation (MAD) =

$$
\frac{\sum Actual - Forecast}{N} = 0.00692 \text{ MW}
$$

**Table 5. Table of Values for Commercial Demand Forecasted**

<table>
<thead>
<tr>
<th>Year</th>
<th>x</th>
<th>Industrial Demand (MW)</th>
<th>xy</th>
<th>x²</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>-6</td>
<td>1011.60</td>
<td>-6069.00</td>
<td>36</td>
</tr>
<tr>
<td>2001</td>
<td>-5</td>
<td>1987.20</td>
<td>-9936.00</td>
<td>25</td>
</tr>
<tr>
<td>2002</td>
<td>-4</td>
<td>1830.00</td>
<td>-7320.00</td>
<td>16</td>
</tr>
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<td>2003</td>
<td>-3</td>
<td>1659.80</td>
<td>-4979.40</td>
<td>9</td>
</tr>
<tr>
<td>2004</td>
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<td>1605.00</td>
<td>-3210.00</td>
<td>4</td>
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<tr>
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<td>-1</td>
<td>1615.50</td>
<td>-1615.50</td>
<td>1</td>
</tr>
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<td>1615.50</td>
<td>8077.50</td>
<td>25</td>
</tr>
<tr>
<td>2012</td>
<td>6</td>
<td>1648.00</td>
<td>9888.00</td>
<td>36</td>
</tr>
<tr>
<td>Total</td>
<td>0</td>
<td>20755.00</td>
<td>483.10</td>
<td>182</td>
</tr>
</tbody>
</table>

**Table 6. Trend Values for Residential Load Demand**

<table>
<thead>
<tr>
<th>Year</th>
<th>Residential Demand (MW)</th>
<th>y</th>
<th>Trend Value Y (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>4608.40</td>
<td>y</td>
<td>6691.83</td>
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<td>2002</td>
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<td>y</td>
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<td>2003</td>
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<td>y</td>
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</tr>
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<td>2004</td>
<td>7725.30</td>
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<td>2005</td>
<td>7766.00</td>
<td>y</td>
<td>7492.33</td>
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<td>y</td>
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</tr>
<tr>
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<td>8075.00</td>
<td>y</td>
<td>8132.73</td>
</tr>
<tr>
<td>2010</td>
<td>8205.20</td>
<td>y</td>
<td>8292.83</td>
</tr>
<tr>
<td>2011</td>
<td>8285.60</td>
<td>y</td>
<td>8452.93</td>
</tr>
<tr>
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<td>8350.00</td>
<td>y</td>
<td>8613.03</td>
</tr>
<tr>
<td>Total</td>
<td>99481.68</td>
<td>y</td>
<td>99481.59</td>
</tr>
</tbody>
</table>
• To calculate the accuracy of residential forecast

The mean absolute deviation (MAD)

\[
\text{MAD} = \frac{\sum |\text{Actual} - \text{Forecast}|}{N}
\]

\[
= \frac{0.09}{13} = 0.00692
\]

• The Predicted Residential Demand

The forecast value can be determined by adding the trend line value (160.10MW) to the preceding load demand to get the current years forecast demand also by using the trend equation.

The gradient of the trend line

\[
b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = 2.65,
\]

\[
a = \frac{\sum y - b \sum x}{n} = 1596.53
\]

Trend equation

\[
Y = a + bx = 1596.53 + 2.65x
\]

The trend values and actual industrial demand are shown in Table 7 below;

<table>
<thead>
<tr>
<th>Year</th>
<th>Industrial Demand y(MW)</th>
<th>Trend value Y(MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>1011.60</td>
<td>1580.63</td>
</tr>
<tr>
<td>2001</td>
<td>1987.20</td>
<td>1583.28</td>
</tr>
<tr>
<td>2002</td>
<td>1830.00</td>
<td>1591.23</td>
</tr>
<tr>
<td>2003</td>
<td>1590.00</td>
<td>1593.88</td>
</tr>
<tr>
<td>2004</td>
<td>1575.00</td>
<td>1596.53</td>
</tr>
<tr>
<td>2005</td>
<td>1530.50</td>
<td>1599.18</td>
</tr>
<tr>
<td>2006</td>
<td>1502.50</td>
<td>1601.83</td>
</tr>
<tr>
<td>2007</td>
<td>1585.00</td>
<td>1604.48</td>
</tr>
<tr>
<td>2008</td>
<td>1589.40</td>
<td>1607.13</td>
</tr>
<tr>
<td>2009</td>
<td>1615.50</td>
<td>1609.78</td>
</tr>
<tr>
<td>2010</td>
<td>1648.00</td>
<td>1612.43</td>
</tr>
<tr>
<td>Total</td>
<td>20755.00</td>
<td>20754.89</td>
</tr>
</tbody>
</table>

• To calculating the accuracy of industrial forecast

The mean absolute deviation (MAD)

\[
= \frac{\sum |\text{Actual} - \text{Forecast}|}{N} = 0.00846 \text{ MW}
\]

Year 2000: \( \frac{4608.40}{99481.68} \times 100\% = 4.63\% \)

Year 2001: \( \frac{7714.80}{99481.68} \times 100\% = 7.75\% \)

Year 2002: \( \frac{7668.50}{99481.68} \times 100\% = 7.71\% \)

Year 2003: \( \frac{7668.50}{99481.68} \times 100\% = 7.71\% \)

Year 2004: \( \frac{7781.30}{99481} \times 100\% = 7.77\% \)

Year 2005: \( \frac{7760.00}{99481} \times 100\% = 7.80\% \)

Year 2006: \( \frac{7650.00}{99481.68} \times 100\% = 7.69\% \)

Table 7. Table of Values for Actual Industrial Demand

<table>
<thead>
<tr>
<th>Year</th>
<th>Residential Energy Demand (MW)</th>
<th>Percentage of Residential Demand Contribution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>4608.40</td>
<td>4.63</td>
</tr>
<tr>
<td>2001</td>
<td>7714.80</td>
<td>7.75</td>
</tr>
<tr>
<td>2002</td>
<td>7668.50</td>
<td>7.71</td>
</tr>
<tr>
<td>2003</td>
<td>7668.50</td>
<td>7.71</td>
</tr>
<tr>
<td>2004</td>
<td>7725.30</td>
<td>7.77</td>
</tr>
<tr>
<td>2005</td>
<td>7650.00</td>
<td>7.90</td>
</tr>
<tr>
<td>2006</td>
<td>7650.00</td>
<td>7.95</td>
</tr>
<tr>
<td>2007</td>
<td>7860.30</td>
<td>8.12</td>
</tr>
<tr>
<td>2008</td>
<td>7910.08</td>
<td>8.25</td>
</tr>
<tr>
<td>2009</td>
<td>8075.00</td>
<td>8.33</td>
</tr>
<tr>
<td>2010</td>
<td>8205.00</td>
<td>8.39</td>
</tr>
<tr>
<td>Total</td>
<td>99481.68</td>
<td>100</td>
</tr>
</tbody>
</table>

Regression Exponential Analysis Method

In the case of least-square method, approximation are conducted between two variables, using polynomial of any degree (linear, quadratic, cubic etc). The two variables are expected to fit the function to the set of data, when the prediction equation is given as:

\[
y_t = a_0 + a_1 x (3.7)
\]

Where:

\[a_0\text{ } = \text{specify intercepts}
\]

\[a_1\text{ } = \text{specific slope}
\]

\[y_t\text{ } = \text{specify dependent variable}
\]

\[x\text{ } = \text{independent variable}
\]

\[y_t : \text{The estimated trend value for a given period t,}
\]

By the application of least square regression through Crammers rule, the constants, \(a\) and \(b\) can be determined from equation (3.25)

\[
\left( \frac{\sum x_i}{\sum x_i^2} \right) \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum y_ix_i \end{bmatrix} \]

(3.25)

Which are obtained from the matrix formation as:

\[
\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{\sum y_i}{\sum x_i^2} \begin{bmatrix} \sum x_i \\ \sum x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{\sum y_i}{\sum x_i^2} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} (3.26)
\]

or
the least-square method is used in fitting trend line to a time series. The technique is one of the basic tools used in developing a curve that describes the relationship between variables. This technique can be applied to any $n$-degree polynomials.

- Exponential Regression Analysis

Considering the curve behaviour, while plotting capacity allocation, capacity utilization and error, the curve of the data trend is non-linear in nature. Therefore there is need to apply the non-linear model: Exponential regression model which can give a good result between the exponential relationship of the variables considered in the study.

This is therefore achieved by estimating the base-load and the annual growth rate, given as:

\[ Y_i = A e^{b x} \]  \hspace{1cm} (3.32)

Converting equation (3.32) to a straight line on (linear), we take a rational logarithm of both sides of equation (3.32).

That is,

\[ \ln Y = \ln a + b x \]  \hspace{1cm} (3.33)

To solve ‘a’ and ‘b’, make a summation to both sides,

\[ \sum \ln Y = \sum \ln a + b \sum x \]  \hspace{1cm} (3.34)

Multiply, equation (3.34) by variable “x”

Hence

\[ \sum x \ln Y = n \ln a \sum x + b \sum x^2 \]  \hspace{1cm} (3.35)

From equation (3.34) and (3.35) form your matrix-function through cramer’s rule, we have:

\[ \left( \begin{array}{c} n \\ \sum x \end{array} \right) \begin{bmatrix} \sum \ln y \\ \sum \ln y \\ \sum \ln y \end{bmatrix} = \left( \begin{array}{c} \sum \ln y \\ \sum \ln y \end{array} \right) \]  \hspace{1cm} (3.36)

3.2. Using Exponential Regression Analysis Method

- To determine the values of ‘a’ and “b” respectively, according to the given equations.

<table>
<thead>
<tr>
<th>YEAR (N)</th>
<th>Year Index (X)</th>
<th>(X²)</th>
<th>Residential (Y) capacity allocation (MW)</th>
<th>In Y</th>
<th>X In Y</th>
<th>XY</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>−6</td>
<td>4608.40</td>
<td>8.4356</td>
<td>0</td>
<td>27670.70</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>−5</td>
<td>7714.80</td>
<td>8.9508</td>
<td>0</td>
<td>44201.25</td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>−4</td>
<td>7668.50</td>
<td>8.9449</td>
<td>0</td>
<td>38667.60</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>−3</td>
<td>7668.50</td>
<td>8.9449</td>
<td>0</td>
<td>32300.50</td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td>−2</td>
<td>7720.30</td>
<td>8.95160</td>
<td>0</td>
<td>15450.60</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>−1</td>
<td>7760.00</td>
<td>8.9567</td>
<td>0</td>
<td>7760.00</td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>0</td>
<td>7650.00</td>
<td>8.9425</td>
<td>0</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>1</td>
<td>7860.30</td>
<td>8.9696</td>
<td>0</td>
<td>7860.30</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>2</td>
<td>7910.08</td>
<td>8.9759</td>
<td>0</td>
<td>15820.16</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>3</td>
<td>8075.00</td>
<td>8.9965</td>
<td>0</td>
<td>32826.80</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>4</td>
<td>8205.20</td>
<td>9.02277</td>
<td>0</td>
<td>41428.00</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>5</td>
<td>8285.60</td>
<td>9.04227</td>
<td>0</td>
<td>50100.00</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>6</td>
<td>8350.00</td>
<td>9.0300</td>
<td>0</td>
<td>50100.00</td>
<td></td>
</tr>
</tbody>
</table>

Table 9. Parameter determination using Analysis of Exponential Regression
Substituting, the values obtained in the Table 9 into equation (3.36)

\[
\left( \sum \frac{x_i}{x_i} \right) \ln a = \left( \sum \frac{x_i}{x_i} \right) \ln y
\]

Where:

\[
\begin{align*}
(\sum x_i) &= 0, \\
(\sum x_i^2) &= 182, \\
(\sum y_i) &= 99481.68, \\
(\sum x_i y_i) &= 29139.76,
\end{align*}
\]

\[
\sum \ln y = 116.13379 \\
\sum x \ln y = 4.410458 \\
n = 13
\]

The system equation becomes:

\[
13 \ln a + (0 \times b) = 116.13379 \\
0 \times \ln a + 182 \times b = 4.410458
\]

or

\[
13 \ln a = 116.13379 \\
182b = 4.410458
\]

or

\[
b = \frac{4.410458}{182}
\]

\[
b = 0.02423
\]

\[
\frac{13 \ln a}{13} = 116.13379
\]

\[
\ln a = 8.9333
\]

Therefore,

\[
Y = Ae^{Bx} \\
\ln Y = \ln A + Bx
\]

or

\[
\ln Y = 8.9333 + 0.02423X
\]

\[
Y = \ell \times p (8.9333 + 0.02423X)
\]

But,

\[
\ell \times p (8.9333) = 7570.95
\]

Therefore,

\[
a = \frac{7570.95}{MW}
\]

Hence, substituting the values of “a” and “b” into the equation as:

\[
Y = Ae^{Bx} = 7570.95e^{0.02423x}
\]

Where, “b” is the growth rate is obtained from equation (3.52) as:

\[
\ln y = \ln a + bx
\]

But, slope is the antilog of the value (0.02423) = 1.02452

Hence, \(b = (1.02452-1.0) = 0.02452 \times 100\%

Therefore, \(b = \text{growth rate} = 2.5\% \)

This means substituting values of “x” into the relation \(y = a + bx\) to obtain other value of y.

The predicted load “Y”, using exponential regression function \(Y = Ae^{BX}\) are presented as:

\[
Y = \text{predicted value will be: } Y = a + bx
\]

When growth rate \(b = \frac{2.5}{100} \times 7570.95 = 189.2738\, MW\)

For 2013 - New predicted value (MW) = 7570.95 + 189.2738 = 7769.22MW

For 2014 - New predicted value (MW) = 7760.22 + 189.2738 = 7949.497MW

For 2015 - New predicted value (MW) = 7949.498 + 189.2738 = 8138.7718MW

For 2016 - New predicted value (MW) = 8138.7718 + 189.2738 = 8328.0456MW

For 2017 - New predicted value (MW) = 8328.0456 + 189.2738 = 8517.319MW

For 2018 - New predicted value (MW) = 8706.5932MW

For 2019 - New predicted value (MW) = 8706.5932 + 189.2738 = 8895.867MW

For 2020 - New predicted value (MW) = 8895.867 + 189.2738 = 9085.1408MW

For 2021 - New predicted value (MW) = 9085.1408 + 189.2738 = 9274.4146MW

For 2022 - New predicted value (MW) = 9274.4146 + 189.2738 = 9463.6884MW

For 2023 - New predicted value (MW) = 9463.6884 + 189.2738 = 9652.9622MW

For 2024 - New predicted value (MW) = 9652.9622 + 189.2738 = 9842.236MW

For 2025 - New predicted value (MW) = 9842.236 + 189.2738 = 10031.5098MW

For 2026 - New predicted value (MW) = 10031.5098 + 189.2738 = 10220.7836MW

For 2027 - New predicted value (MW) = 10220.7836 + 189.2738 = 10410.0574MW

For 2028 - New predicted value (MW) = 10410.0574 + 189.2738 = 10599.3312MW

For 2029 - New predicted value (MW) = 10599.3312 + 189.2738 = 10788.605MW

For 2030 - New predicted value (MW) = 10788.605 + 189.2738 = 10977.8788MW

For 2031 - New predicted value (MW) = 10977.8788 + 189.2738 = 11167.1526MW

For 2032 - New predicted value (MW) = 11167.1526 + 189.2738 = 11356.4264MW
Predicted load ($Y$) in the case of using least-square method ($Y = a + bx$)

The gradient of the trend-line:

$$Y = a + bx$$

Where $a = 7652.43\text{MW}$

$b = 160.10$ (which is 2.092% of the growth rate)

That is,

$Y = 7652.43 + 160.10x$

$2.092 \times 7652.43\text{MW} = 160.10$ (capacity addition)

For 2013 - New predicted value: $= 7652.43 + 2.093\% \times 1$

For 2014 - New predicted value: $= 7652.43 + 2.093\% \times 2$

For 2015 - New predicted value: $= 7652.43 + 2.093\% \times 3$

For 2016 - New predicted value: $= 7652.43 + 2.093\% \times 4$

For 2017 - New predicted value: $= 7652.43 + 2.093\% \times 5$

For 2018 - New predicted value: $= 7652.43 + 2.093\% \times 6$

For 2019 - New predicted value: $= 7652.43 + 2.093\% \times 7$

For 2020 - New predicted value: $= 7652.43 + 2.093\% \times 8$

For 2021 - New predicted value: $= 7652.43 + 2.093\% \times 9$

For 2022 - New predicted value: $= 7652.43 + 2.093\% \times 10$

For 2023 - New predicted value: $= 7652.43 + 2.093\% \times 11$

For 2024 - New predicted value: $= 7652.43 + 2.093\% \times 12$

For 2025 - New predicted value: $= 7652.43 + 2.093\% \times 13$

For 2026 - New predicted value: $= 7652.43 + 2.093\% \times 14$

For 2027 - New predicted value: $= 7652.43 + 2.093\% \times 15$

For 2028 - New predicted value: $= 7652.43 + 2.093\% \times 16$

For 2029 - New predicted value: $= 7652.43 + 2.093\% \times 17$

For 2030 - New predicted value: $= 7652.43 + 2.093\% \times 18$

For 2031 - New predicted value: $= 7652.43 + 2.093\% \times 19$

For 2032 - New predicted value: $= 7652.43 + 2.093\% \times 20$

Modified form of Exponential Regression Analysis

$$Y = At^{Rx}$$

$$In Y = In A + In t^{Rx}$$

Therefore the modified form of exponential demand equation may be expressed as:

$$Y = P_{Di} = t^{a+b}(x_i - x_0)$$

Where:

$$X_i$$ becomes;

$$X_i = x_i - x_0$$

Now, taking natural log of equation (3.5):

We obtain as:

$$In P_{Di} = In \left(t^{a+bX_i}\right)$$

or

$$In P_{Di} = a + bX_i$$

or

$$Y_i = In P_{Di}$$

$$Y_i = a + bX_i$$

Where:

In order to predicts the demand correctly, the sum of square of error should be minimum:

$$Summation \ (S) = \sum_{i=1}^{n} [Y_i - (a + b \times i)]^2$$

For S to be minimum, the conditions are:

$$\frac{\partial S}{\partial a} = 0 \quad and \quad \frac{\partial S}{\partial b} = 0$$
Differentiating equation (3.14) with respect to 'a' we have:

\[ O = \frac{\partial \tilde{S}}{\partial a} = \sum_{i=1}^{n} 2 \left[ Y_i - (a + bX_i) \right] (-1) = 0 \]

or

\[ \sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} (a + bX_i) \]  \hspace{1cm} (3.71)

\[ \sum_{i=1}^{n} Y_i = na + b \sum_{i=1}^{n} X_i \]  \hspace{1cm} (3.72)

- Similarly, differentiating equation 3.14) with respect to 'b' we have:

\[ \frac{\partial \tilde{S}}{\partial b} = \sum_{i=1}^{n} 2 \left[ Y_i - (a + bX_i) \right] (-1)X_i = 0 \]  \hspace{1cm} (3.73)

or

\[ \sum_{i=1}^{n} Y_iX_i = \sum_{i=1}^{n} (a + bX_i)X_i \]  \hspace{1cm} (3.74)

\[ \sum_{i=1}^{n} Y_iX_i = a \sum_{i=1}^{n} X_i + b \sum_{i=1}^{n} X_i^2 \]  \hspace{1cm} (3.75)

- Hence, the conditions for the sum of least square of the error (deviation), to be minimum are given by the two equations:

Let us consider

\[ \sum_{i=1}^{n} X_i = 0, \]  \hspace{1cm} (3.76)

in order to determine 'a' and 'b'

- Using the equation (3.21) into equation (3.17) and (3.20):

We have:

\[ a_{\text{New}} = \frac{1}{n} \sum_{i=1}^{n} Y_i \]  \hspace{1cm} (3.77)

or

\[ \sum_{i=1}^{n} Y_i = na + b \sum_{i=1}^{n} X_i = 0 \]  \hspace{1cm} (3.77)

\[ a = \frac{1}{n} \sum_{i=1}^{n} Y_i = \frac{1}{13} \sum_{i=1}^{13} Y_i \]

\[ = \frac{1}{13} \times 26.333456 \]

\[ = 2.02565 \]

\[ a = 2.02565 \]
Modified Form of Exponential Regression Calculation and Analysis for load demand for the year 2013, when our base year is equal 2006

Case 1: Residential

\[ Y_i = PD_i = e^{a+b(X_i)} \]

\[ Y_i = PD_i = e^{a+b(x_i-x_0)} \]

Where \( X_i = (x_i-x_0) \)

But \( x_i = \) predicted energy demand = 2013.

\( x_0 = \) based - year = 2006

Note: \( \ell = 2.718 \)

We can find our peak load demand from

\[ PD_2013 = 1000 \]

But \( PD = 1000e^{a+b(x_i-x_0)} \)

That is when \( x_0 =2006 \) (datum case) or base year \( x_i=2013 \) (prediction case)

Hence;

\[ PD_{2013} = 1000 \ell^2.02565+0.0130249(2013–2006) \]

\[ PD_{2013} = 1000 \ell^2.02565+0.0130249(7) \]

\[ PD_{2013} = 1000 \ell^2.02565+0.0911743 \]

\[ PD_{2013} = 1000 \ell^2.1168243 \]

or

\[ PD_{2013} = 1000 \ell^2.718^{2.01168243} \]

\[ PD_{2013} = 1000 \ell^2.8302899728 \]

\[ PD_{2013} = 8302.899728 \]

\[ PD_{2013} = 8305MW \text{ capacity} \]

Modified Form of Exponential Regression Calculation and Analysis For load demand for the year 2014, when our base year is equal 2006

That is;

When \( x_0 =2006 \) (datum case) or base year \( x_i=2014 \) (prediction case)

Therefore;

\[ PD_{2014} = 1000 \ell^2.02565+0.0130249(2014–2006) \]

\[ PD_{2014} = 1000 \ell^2.02565+0.0130249(8) \]

\[ PD_{2014} = 1000 \ell^2.02565+0.0041992 \]

\[ PD_{2014} = 1000 \ell^2.718^{2.1298492} \]

But \( \ell = 2.718 \)

\[ PD_{2014} = 1000 \times 8411.74016 \]

\[ PD_{2014} = 8411.74016 \]

\[ PD_{2014} \approx 8415MW \text{ capacity} \]

This process is continued in the same similar manner

Total summation of predicted load in the case of residential load (2000 – 2012)

That is,

\[ \Sigma \text{ predicted residential energy demand} = (7009.723+7101.6150 + 7298.02 + 7384.571 + 7481.37 + 7579.44+7678.80 + 7779.46+7881.44 +7984.76+ 8089.43+ 8195.47) \]

\[ \sum_{\text{predicted}} = 98.649,807 \]

Similarly, the actual allocated Residential load between (2000 – 2012) becomes:

\[ \Sigma \text{Actual} = (4608.40+7714.80 + 7668.50 + 7668.50 + 7725.30 + 7760.00 + 7650.00 + 7860.30 + 7910.05 + 8075.00 + 8205.20 + 8285.60 + 8350.00) = \]

\[ \sum_{\text{Actual}} = 99.481.65 \]

\[ M_{AD} = \sum_{N} (\text{Actual} – \text{Forecast}) \]

\[ M_{AD} = \frac{99481.65 – 98649.807}{13} \]

\[ M_{AD} = 831.843 \]

\[ M_{AD} = 63.9879MW \]

Case 1: Validation: Residential load demand Capacity Allocation @ 2000 = 4608.40MW (or available)

Load forecast (prediction) @ 2000;

\[ Y_i,2000 = PD_{2000} = \ell^{a+b(x_i)} \]

\[ Y_i,2000 = PD_{2000} = 2.718^{2.02565(1.754837982)} \]

\[ Y_i,2000 = PD_{2000} = 2.718^{2.025187111+10.52902789} \]

Where \( X_i = x_i-x_0 \)

\( a = 2.025187111, b = -1.754837982 \)

\( x_0 = \) base year = 2006

\( x_i = \) predicted year = 2000

\[ Y_i,2000 = PD_{2000} = 283049.2437MW \]

@ 2001 (Prediction)

\[ PD_{2001} = 1000 \ell^2.025187111+10.52902789 \]

or

\[ PD_{2001} = 1000 \ell^2.025187111+8.77418991 \]

or

\[ PD_{2001} = 1000 \ell^2.70937702 \]
\[ P_{D2001} = 1000 \times 2.718^{0.79937702} \times 48935.44671 = 48935446.17 \text{ MW} \]

**@ 2002 (Prediction)**

- \[ P_{D2002} = 1000 \times e^{2.025187111 - 1.7548379821} \times (2002 - 2006) \]  
  or \[ P_{D2002} = 1000 \times e^{2.025187111 - 1.7548379821} \times (-4) \]  
  or \[ P_{D2002} = 1000 \times e^{2.025187111 + 7.019351928} \]  
  or \[ P_{D2002} = 1000 \times e^{2.025187111} \times 0.226653029 \]  
  or \[ P_{D2002} = 8464204.057 \text{ MW} \]

- \[ P_{D2002} = 1000 \times e^{2.025187111} \times 13750.46693 \text{ MW} \]

**@ 2006 (Prediction)**

- \[ P_{D2006} = 1000 \times e^{2.025187111 - 1.7548379821} \times (2006 - 2006) \]  
  or \[ P_{D2006} = 1000 \times e^{2.025187111 - 1.7548379821} \times (-0) \]  
  or \[ P_{D2006} = 1000 \times e^{2.025187111 + 0} \]  
  or \[ P_{D2006} = 1000 \times e^{2.025187111} \times 7.581120857 \]  
  or \[ P_{D2006} = 7581.120857 \text{ MW} \]

- \[ P_{D2007} = 1000 \times e^{2.025187111 - 1.7548379821} \times (2007 - 2006) \]  
  or \[ P_{D2007} = 1000 \times e^{2.025187111} \times 0.270349129 \]  
  or \[ P_{D2007} = 1000 \times e^{2.025187111} \times 1.310385145 \]  
  or \[ P_{D2007} = 1310.385145 \text{ MW} \]

**@ 2008 (Prediction)**

- \[ P_{D2008} = 1000 \times e^{2.025187111 - 1.7548379821} \times (2008 - 2006) \]  
  or \[ P_{D2008} = 1000 \times e^{2.025187111 - 1.7548379821} \times (2) \]  
  or \[ P_{D2008} = 1000 \times e^{2.025187111 + 3.509675964} \]  
  or \[ P_{D2008} = 1000 \times e^{2.025187111} \times -1.48488853 \]  
  or \[ P_{D2008} = 1000 \times e^{2.025187111} \times -1.48488853 \]  
  or \[ P_{D2008} = 1000 \times e^{2.025187111} \times 226.6530296 \]  
  or \[ P_{D2008} = 226.6530296 \text{ MW} \]

**@ 2009 (Prediction)**

- \[ P_{D2009} = 1000 \times e^{2.025187111 - 1.7548379821} \times (2009 - 2006) \]  
  or \[ P_{D2009} = 1000 \times e^{2.025187111} \times 1000.000000 \]  
  or \[ P_{D2009} = 1000 \times e^{2.025187111} \times 226.6530296 \]  
  or \[ P_{D2009} = 226.6530296 \text{ MW} \]
$P_{D2009} = 1000 \times e^{0.025187111 - 1.7548379821(3)}$

or

$P_{D2009} = 1000 \times e^{0.025187111 - 5.264513946}$

or

$P_{D2009} = 1000 \times 2.718^{-3.239326835}$

or

$P_{D2009} = 1000 \times 0.039203432$  
$P_{D2009} = 39.20343268MW$

@ 2010 (Prediction)

$P_{D2010} = 1000 \times e^{0.025187111 - 1.7548379821(2010-2006)}$

or

$P_{D2010} = 1000 \times e^{0.025187111 - 1.7548379821(4)}$

or

$P_{D2010} = 1000 \times e^{-4.994164817}$

or

$P_{D2010} = 1000 \times 2.718^{-4.994164817}$

or

$P_{D2010} = 1000 \times 6.780889434 \times 10^{-3}$

or

$P_{D2010} = 6.780889434MW$

@ 2011 (Prediction)

$P_{D2011} = 1000 \times e^{0.025187111 - 1.7548379821(2011-2006)}$

or

$P_{D2011} = 1000 \times e^{0.025187111 - 8.77418991}$

or

$P_{D2011} = 1000 \times e^{-6.749002799}$

or

$P_{D2011} = 1000 \times 2.718^{-6.749002799}$

or

$P_{D2011} = 1000 \times 1.17286825 \times 10^{-3}$

or

$P_{D2011} = 8089.43MW$

@ 2012 (Prediction)

$P_{D2012} = 1000 \times e^{0.025187111 - 1.7548379821(2012-2006)}$

or

$P_{D2012} = 1000 \times e^{0.025187111 - 10.52902789}$

or

$P_{D2012} = 1000 \times e^{-8.503840774}$

or

$P_{D2012} = 1000 \times 2.718^{-8.503840774}$

or

$P_{D2012} = 1000 \times 2.028671829 \times 10^{-4}$

or

$P_{D2012} = 0.202867182MW$

Total summation of predicted load in the case of Residential load (2000 – 2012)

That is,

$\sum_{predicted} \text{Predicted Residential energy demand} = (7009.723+7101.6150 + 7194.708 + 7289.02 + 7384.571 + 7481.37 + 7579.44 + 7678.80 + 7779.46 + 7881.44 + 7984.76 + 8089.43 + 8195.47)\approx 98.649,807$ predicted MW

Similarly, the actual allocated Residential load between (2000 – 2012) becomes:

$\sum_{Actual} \text{Actual} = (4608.40 + 7714.80 + 7668.50 + 7668.50 + 7725.30 + 7760.00 + 7650.00 + 7860.30 + 7910.05 + 8075.00 + 8205.20 + 8285.60 + 8350.00) = 99.481.65$ Actual MW

$MAD = \frac{\sum (Actual - Forecast)}{N} = \frac{99481.65 - 98649.807}{13} = \frac{831.843}{13} = 63.9879 MW$

Standard Error Prediction

Standard error of prediction

$\sum Y_x = \sqrt{\frac{\sum (Y-Y_r)^2}{n}}$

$\sum Y_x = \sqrt{\frac{\sum (56,907,966.86)}{13}} = \sqrt{4377535.992} = 2092.256MW$

Analysis of Allocation (Available demand) in MW wrt to energy demand prediction (forecast)

$Y_i = P_{Di} = e^{a+b(x_i)}$

$Y_i = P_{Di} = e^{a+b(x_i-x_0)}$

Where $x_i = (x_i-x_0)$

Where $x_i = $ Predicted Energy Demand

$x_0 = $ base year = 2006

$x_i = 2000$

$\ell = 2.718$

$\sum_{i=1}^n Y_i = 1 \sum_{i=1}^{13} Y_i = 1.267639604$

$a = 1.267639604$
\[
\sum_{i=1}^{n} Y_i X_i = \frac{13}{n} \sum_{i=1}^{n} Y_i X_i \\
= \frac{13}{12} \sum_{i=1}^{12} Y_i X_i \\
b = 9368497571 \times 0.051475261 \\
b = 0.051475261
\]

\[x_0 = \text{base year (Reference year)} \ 2006\]
\[B = \text{base} - \text{MW} = 1 \times 10^3 = 1000\text{MW}\]

**Commercial Demand**

Load forecast (prediction) @ 2000

\[Y_i,2000 = P_{D_i,2000} = e^{a+b(x_i)}\]

\[Y_i,2000 = P_{D_i,2000} = 2.718\]

Where \( X_i = x_i - x_0 \)

\[a = 1.267639604, b = 0.051475261\]

\[x_0 = \text{base year} = 2006\]

\[x_i = \text{Predicted year} = 2006\]

\[Y_i,2000 = 1000 \times P_{D_i,2000} = 2.718\]

1.267639604+0.051475261(2000–2006)

\[Y_i,2000 = 1000 \times P_{D_i,2000} = 2.718\]

1.267639604+0.051475261(2000–2006)

\[Y_i,2000 = P_{D_i} = 1000 \times 2.718\]

1.267639604+0.051475261(2000–2006)

2001 (Prediction)

\[P_{D_2001} = 1000 \times 1.267639604+0.051475261(2001–2006)\]

\[P_{D_2001} = 1000 \times 1.267639604+0.051475261(5)\]

\[P_{D_2001} = 1000 \times 1.267639604+0.051475261(2001–2006)\]

\[P_{D_2001} = 1000 \times 1.010263299\]

\[P_{D_2001} = 1000 \times 2.718\times 1.010263299\]

\[P_{D_2001} = 1000 \times 2.746036366 = 2746.036366\]

\[P_{D_2001} = 2746.036366\text{MW}\]

2002 (Prediction)

\[P_{D_2002} = 1000 \times 1.267639604+0.051475261(2002–2006)\]

\[P_{D_2002} = 1000 \times 1.267639604+0.051475261(4)\]

\[P_{D_2002} = 1000 \times 1.267639604+0.061901044\]

\[P_{D_2002} = 1000 \times 1.267639604\]

\[P_{D_2002} = 1000 \times 2.718\times 1.267639604\]

\[P_{D_2002} = 1000 \times 3.551990568 = 3551.990568\]

\[P_{D_2002} = 3551.990568\text{MW}\]

2003 (Prediction)

\[P_{D_2003} = 1000 \times 1.267639604+0.051475261(2003–2006)\]

\[P_{D_2003} = 1000 \times 1.267639604+0.051475261(3)\]

\[P_{D_2003} = 1000 \times 1.267639604+0.046425783\]

\[P_{D_2003} = 1000 \times 2.718\times 2.21213821\]

\[P_{D_2003} = 1000 \times 2.718\times 2.21213821\]

\[P_{D_2003} = 1000 \times 3.390872288\]

\[P_{D_2003} = 3390.872288\text{MW}\]

2004 (Prediction)

\[P_{D_2004} = 1000 \times 1.267639604+0.015475261(2004–2006)\]

\[P_{D_2004} = 1000 \times 2.718\times 1.267639604+0.051475261(2)\]

\[P_{D_2004} = 1000 \times 1.267639604+0.030950522\]

\[P_{D_2004} = 1000 \times 2.718\times 2.36689082\]

\[P_{D_2004} = 3443.749528\text{MW}\]

2005 (Prediction)

\[P_{D_2005} = 1000 \times 1.267639604+0.015475261(2005–2006)\]

\[P_{D_2005} = 1000 \times 2.718\times 1.267639604+0.051475261(1)\]

\[P_{D_2005} = 1000 \times 2.718\times 1.267639604+0.01347526\]

\[P_{D_2005} = 1000 \times 2.718\times 2.52164344\]

\[P_{D_2005} = 1000 \times 3.497451339\]

\[P_{D_2005} = 3497.451339\text{MW}\]

2006 (Prediction)

\[P_{D_2006} = 1000 \times 1.267639604+0.015475261(2006–2006)\]

\[P_{D_2006} = 1000 \times 2.718\times 1.267639604+0.051475261(0)\]

\[P_{D_2006} = 1000 \times 2.718\times 1.267639604\]

\[P_{D_2006} = 1000 \times 1.267639604\]

\[P_{D_2006} = 3551.990568\text{MW}\]

2007 (Prediction)

\[P_{D_2007} = 1000 \times 1.267639604+0.015475261(2007–2006)\]

\[P_{D_2007} = 1000 \times 2.718\times 1.267639604+0.05147526(1)\]

\[P_{D_2007} = 1000 \times 2.718\times 1.267639604+0.01347526\]

\[P_{D_2007} = 1000 \times 2.718\times 2.83114864\]

\[P_{D_2007} = 1000 \times 3.607380282 = 3607.380282\text{MW}\]

\[P_{D_2007} = 3607.380282\text{MW}\]

2008 (Prediction)

\[P_{D_2008} = 1000 \times 1.267639604+0.01547526(2008–2006)\]

\[P_{D_2008} = 1000 \times 2.718\times 1.267639604+0.05147526(2)\]

\[P_{D_2008} = 1000 \times 2.718\times 1.267639604+0.03095052\]

\[P_{D_2008} = 1000 \times 2.718\times 2.98590124\]

\[P_{D_2008} = 1000 \times 3.66363742\]

\[P_{D_2008} = 3663.63742\text{MW}\]

2009 (Prediction)

\[P_{D_2009} = 1000 \times 1.267639604+0.01547526(2009–2006)\]}
INDUSTRIAL DEMAND

Analysis of Allocation (Available demand) in MW wrt to energy demand prediction (forecast)

\[ Y_i = P_{Di} = e^{a+b(x_i)} \]

Where \( x_i = (x_i - x_0) \)

Where \( x_i = \text{Predicted Energy Demand} \)

\[ x_0 = \text{base year} = 2006 \]

\[ x_i = \text{Predicted year} = 2006 \]

\[ a = \frac{1}{n} \sum_{i=1}^{n} Y_i = \frac{1}{13} \sum_{i=1}^{13} Y_i = 0.457859173 \]

\[ b = \frac{1}{13} \sum_{i=1}^{13} Y_i X_i \]

\[ X_i = \text{predicted year} = 2006 \]

\[ X_0 = \text{base year (Reference year)} = 2006 \]

\[ B = \text{base } \text{MW} = 1 \times 10^3 = 1000 \text{MW} \]

Capacity allocation @ 2000 = 1011.60 (or available)

Load forecast (prediction) @ 2000

\[ Y_{i,2000} = P_{Di} = e^{a+b(x_i)} \]

\[ Y_{i,2000} = P_{D_{2000}} = 2.718 \]

Where \( X_i = x_i - x_0 \)

\[ a = 0.457859173, \quad b = 5.519517819 \times 10^{-3} \]

\[ x_0 = \text{base year} = 2006 \]

\[ x_i = \text{Predicted year} = 2006 \]

\[ Y_{i,2000} = 1000 \times P_{Di,2000} = 2.718^{0.457859173 + 5.519517819 \times 10^{-3}} \]

\[ Y_{i,2000} = P_{D_{2000}} = 2.718 \]

\[ Y_{i,2000} = 1000 \times P_{Di,2000} \]

\[ X_0 = \text{base year (Reference year)} = 2006 \]

\[ B = \text{base } \text{MW} = 1 \times 10^3 = 1000 \text{MW} \]

Load forecast (prediction) @ 2000

\[ Y_{i,2000} = P_{Di} = e^{a+b(x_i)} \]

\[ Y_{i,2000} = P_{D_{2000}} = 2.718 \]

Where \( X_i = x_i - x_0 \)

\[ a = 0.457859173, \quad b = 5.519517819 \times 10^{-3} \]

\[ x_0 = \text{base year} = 2006 \]

\[ x_i = \text{Predicted year} = 2006 \]

\[ Y_{i,2000} = 1000 \times P_{Di,2000} = 2.718^{0.457859173 + 5.519517819 \times 10^{-3}} \]

\[ Y_{i,2000} = P_{D_{2000}} = 2.718 \]

\[ Y_{i,2000} = 1000 \times P_{Di,2000} \]

\[ X_0 = \text{base year (Reference year)} = 2006 \]

\[ B = \text{base } \text{MW} = 1 \times 10^3 = 1000 \text{MW} \]

Capacity allocation @ 2000 = 1011.60 (or available)

Load forecast (prediction) @ 2000

\[ Y_{i,2000} = P_{Di} = e^{a+b(x_i)} \]

\[ Y_{i,2000} = P_{D_{2000}} = 2.718 \]

Where \( X_i = x_i - x_0 \)

\[ a = 0.457859173, \quad b = 5.519517819 \times 10^{-3} \]

\[ x_0 = \text{base year} = 2006 \]

\[ x_i = \text{Predicted year} = 2006 \]

\[ Y_{i,2000} = 1000 \times P_{Di,2000} = 2.718^{0.457859173 + 5.519517819 \times 10^{-3}} \]

\[ Y_{i,2000} = P_{D_{2000}} = 2.718 \]

\[ Y_{i,2000} = 1000 \times P_{Di,2000} \]

\[ X_0 = \text{base year (Reference year)} = 2006 \]

\[ B = \text{base } \text{MW} = 1 \times 10^3 = 1000 \text{MW} \]

Capacity allocation @ 2000 = 1011.60 (or available)

Load forecast (prediction) @ 2000

\[ Y_{i,2000} = P_{Di} = e^{a+b(x_i)} \]
$P_{D2001} = 1000 \times 0.457859173 + 0.027597589$

$P_{D2002} = 1000 \times 0.4030261584$

$= 1000 \times 2.718^{1.537591103}$

$P_{D2001} = 1537.591103 MW$

2002 (Prediction) Industrial Demand

$P_{D2002} = 1000 \times 0.457859173 + 5.519517819 \times 10^{-3}$

$P_{D2002} = 1000 \times 2.718^{0.457859173} + 0.022078071$

$P_{D2002} = 1000 \times 2.718^{0.435781102}$

$= 1000 \times 1.546100444$

$P_{D2002} = 1546.100444 MW$

2003 (Prediction) Industrial Demand

$P_{D2003} = 1000 \times 0.457859173 + 5.519517819 \times 10^{-3}$

$P_{D2003} = 1000 \times 2.718^{0.457859173} + 0.016558553$

$P_{D2003} = 1000 \times 2.718^{0.459227364}$

$= 1000 \times 1.58277518$

$P_{D2003} = 1582.77518 MW$

2004 (Prediction) Industrial Demand

$P_{D2004} = 1000 \times 0.457859173 + 5.519517819 \times 10^{-3}$

$P_{D2004} = 1000 \times 2.718^{0.457859173} + 0.011663415$

$P_{D2004} = 1000 \times 2.718^{0.463746882}$

$= 1000 \times 1.585994005$

$P_{D2004} = 1589.94405 MW$

2005 (Prediction) Industrial Demand

$P_{D2005} = 1000 \times 0.457859173 + 5.519517819 \times 10^{-3}$

$P_{D2005} = 1000 \times 2.718^{0.457859173} + 0.005383506$

$P_{D2005} = 1000 \times 2.718^{0.470266365}$

$= 1000 \times 1.600342399$

$P_{D2005} = 1600.342399 MW$

2006 (Prediction) Industrial Demand

$P_{D2006} = 1000 \times 0.457859173 + 5.519517819 \times 10^{-3}$

$P_{D2006} = 1000 \times 2.718^{0.457859173} + 0.003383506$

$P_{D2006} = 1000 \times 2.718^{0.457859173}$

$= 1000 \times 1.609199074$

$P_{D2006} = 1609.199074 MW$

2007 (Prediction) Industrial Demand

$P_{D2007} = 1000 \times 0.457859173 + 5.519517819 \times 10^{-3}$

$P_{D2007} = 1000 \times 2.718^{0.457859173} + 0.0027597589$

$P_{D2007} = 1000 \times 2.718^{0.457859173}$

$= 1000 \times 1.665337791$

$P_{D2007} = 1663.377641 MW$

Total summation of predicted load in the case of Industrial load (2000 – 2012)

That is,
\[ \sum \text{Predicted Commercial energy demand} = 1529.128596 + 1537.591103 + 1546.10044 + 1582.77518 + 1589.944005 + 1600.342399 + 1609.199074 + 1618.104703 + 1627.058416 + 1636.064105 + 1645.118416 + 1654.222836 + 1663.377641 = 20,839.02692 \text{MW} \]

Similarly, the actual allocated Commercial load between (2000 – 2012) becomes:

\[ \sum \text{Actual} = 1011.60 + 1987.20 + 1830.00 + 1659.80 + 1605.00 + 1615.50 + 1575.00 + 1502.50 + 1585.00 + 1589.40 + 1615.50 + 1648.00 = 17,756.70906 \text{MW} \]

\[ \text{MAD} = \frac{\sum (\text{Actual} - \text{Predicted})}{N} = \frac{17,756.70906 - 20,839.02692}{13} = -3082.31786 \approx -237.1013738 \text{MW} \]

**Flow Chart Showing the Activities of Least and Exponential Regression Analysis**

- **Start**
- Read in values of year, allocation utilization, and year of forecast
- \( y = \text{allocation assigned values for } x = 6 \text{ to } -6 \) \( x = 7 \) to number of years of forecast
- No (exponential regression)
  - \( y = \beta \gamma^x \) (Allocation) \( n = \text{Exponential forecast; } y \)
- Yes
  - \( y = \text{Allocation} \)
  - \( \beta = \text{number of data in } y \)
  - Compute: \( \alpha = \frac{\sum \gamma \sum y - \sum \gamma \sum y}{\sum \gamma^2 - (\sum \gamma)^2} \)
  - Compute, \( \alpha = e^{\beta \gamma} \)
  - Exponential trend: \( y_\gamma = \alpha e^{\beta \gamma} \)
  - Exponential forecast: \( y_\gamma = \alpha e^{\beta X} \)
- No (exponential regression)
- Yes
  - Compute, linear trend: \( y_1 = \alpha_0 + \alpha_1 x \)
  - Linear forecast: \( Y_1 = \alpha_0 + \alpha_1 X \)
- Display table of year, Allocation, Utilization; \( y_1 \) and \( y_2 \)
- Display table of year forecast; \( Y_1 \) and \( Y_2 \)
- Display graph of year vs Allocation, utilization, \( y_1 \) and \( y_2 \)
- Display graph of year of forecast vs \( Y_1 \) and \( Y_2 \)
- **Stop**

*Figure 2. Flow chart showing the activities of least and exponential regression analysis*
4. Results and Discussion

The maximum load calculated with exponential regression, \( Y = Ae^{BX} \) function is given as:
\[
( Y = 7570.95 e^{0.02423x} )
\]
with percent growth rate of 2.5%, while the maximum load calculated for least-square regression function is given as: \( Y = a + bx \) with growth rate of 2.093%. Therefore the maximum load: (7570.95MW) with 2.5% growth rate from exponential regression analysis method is more realistic and appropriate because of its ability to capture the energy requirement for consumers at the receiving end, on the view with steady energy growth-rate.

Load forecasting analysis is a major problem in power system planning and operations. This is because it provides the necessary information for: customer service and billing, electricity pricing and tariff planning etc. thereby giving an insight into future expansion planning.

The result obtained shows that the utilization capacity (MW) does not give positive reflection of the capacity allocations (MW) when conducted in Matlab/Java platform especially in the case of residential, commercial and industrial plot (MW).

This mean that there is a deviation between installed capacity and that of utilization capacity of the consumer at the receiving end which need to be match in order to avoid overload and system collapse.

The paper work also extended to the linear model (lgast-square regression) and exponential regression plot which exhibit and confirms similar relationship in residential, commercial and industrial plot.

Evidently, the comparism plot for linear model and exponential regression show different behaviour, the least square model display linear graph while the exponential graph exhibit non-linear behaviour which is recommend because of the nonlinear relations between the capacity allocation (MW) and that of capacity in utilization.

**EXponential Regression Analysis**

**Figure 3.** Chart of Residential energy consumption versus years

**Figure 4.** Chart of Commercial energy consumption versus years
Figure 5. Chart of Industrial energy consumption versus years

![Chart of Industrial energy consumption versus years]

Figure 6. Chart showing the comparison of Available power (MW) for different components

![Chart showing the comparison of Available power (MW) for different components]

Figure 7. Graph showing the comparison of Predicted energy demand for different locations

![Graph showing the comparison of Predicted energy demand for different locations]
Graph showing the comparison of Error for different components

Figure 8. Graph showing the comparison of Predicted energy demand for different locations

LEAST SQUARE REGRESSION ANALYSIS

Chart of Commercial energy demand versus trend value

Figure 9. Chart of Commercial energy demand versus trend value

Chart of Residential energy demand versus trend value

Figure 10. Chart of Residential energy demand versus trend value
Figure 11. Chart of Industrial energy demand versus trend value

Figure 12. Chart showing the comparison of Predicted energy demand for different components

Figure 13. Comparison graph of Forecasted Energy demand for different components
5. Conclusion

The electricity load forecast is a comprehensive survey of electrical demand and supply at the receiving end, in order to identify areas of inadequacies resulting to mismatches, which may negatively impact an overloads on transmission and distribution network. This work carried out load forecast using the analysis of least-square regression and exponential regression and validated in Matlab and Java programming platform.

Data are collected from (2000 - 2012) Central Bank of Nigeria (CBN) and National Bureau of statistics which serves as the input data into the least-square and exponential regression model for prediction into the projected future 2032. The result obtained shows that there is mismatch between installed capacity (MW) and utilization capacity (MW) in the case of residential, commercial and industrial load demand. Evidently, the result suggested that there is need to bridge the gap in order to enhance high reliability and efficiency in power system operation.

This work also identified the relationship between least-square model and exponential model plot on Matlab and Java program environment. Since the deviation between the installed and utilization capacity (MW) are non-linear, the exponential regression plot is therefore recommended, because of the non-linear behaviour of the capacity and load demand requirement.

Therefore, strategies for expansion programme is required for engaging distribution generator (DG) or decentralized generator via distribution outlet, on the view to solve overloads problems. This suggestion and recommendation if implemented will improve the electricity supply and demand in Nigeria.
6. Recommendation

Owing to the finding obtained in the course of this research the following recommendation are made:

(i) To meet the energy demand requirement, additional capacity generation will be required.

(ii) Additional injection station and substation must be build to cater and care for rapid increase in load demand.

(iii) Generation expansion program must be in place to accommodate annual growth of energy consumption.

References


