Numerical Modelling of RC Columns Subjected to Biaxial Horizontal Loading and Variable Axial Load

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Abstract A numerical study of RC columns subjected to variable axial load in conjunction with biaxial cyclic bending are presented in this article. The numerical models were built to simulate the response of all specimens using the computer program SeismoStruct and adopting three different modelling strategies, such as lumped plasticity and distributed plasticity (based in displacements and forces formulations) that were adopted in order to obtain the real behaviour of the RC columns. The efficiency of each modelling strategy is evaluated through the comparison between the experimental global hysteretic behaviour, stiffness degradation, columns capacity and energy dissipation and the numerical results.

Keywords: numerical modelling, rc columns, biaxial behaviour, variable axial load

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1. Introduction

The damage reports caused by last earthquakes have showed the need for further investigation in order to improve the seismic safety of existing constructions (Bertero, 1986, Pribadi *et al.*, 2008, Vicente *et al.*, 2012, Romão *et al.*, 2013), and beyond that provides an important source of information to upgrade new design methodologies in the evolution of existent codes and guidelines.

The majority of column failures observed after being submitted to seismic actions are caused by inadequate concrete confinement, high shear stresses, lack of concrete and also bidirectional load effects (Saatcioglu *et al.*, 1989). It has been also reported that the damages of reinforced concrete (RC) elements caused by earthquakes increased when submitted to multiaxial excitation (Takizawa *et al.*, 1976, Lejano, 2007, Rodrigues *et al.*, 2012, 2013). From the axial load-bending interaction diagrams, the yielding and ultimate moment's increases with the level of axial load until the balance point is achieved. The variation of the axial load during an seismic action can change all the hysteretic properties and inelastic response of the RC columns, in particular the strength, stiffness, and ultimate displacement capacity (Sfakianakis *et al.*, 1991, CEB, 1996, Bonet *et al.*, 2006). In many cases, columns under biaxial loads also result in the torsional response of the structures, namely as a result of in-plane structural irregularities.

Adequate non-linear models should reproduce the influence of the structural elements geometry and the behavior when submitted to cyclic loading in terms of stiffness, strength and energy dissipation. A significant number of models have been developed and tested to represent the materials non-linear behavior and they can be divided according to different approaches (Fardis, 1991, CEB, 1996): hysteresis modelling (Casciati, 1989, Kunnath *et al.*, 1990, Wang *et al.*, 2000, Romão *et al.*, 2004), Bouc-Wen (Wen, 1976), fibre models (Taucer *et al.*, 1991, Spacone *et al.*, 1992, Petrangeli *et al.*, 1999), bounding surface plasticity (Sfakianakis *et al.*, 1991, Sfakianakis *et al.*, 1991, Bousias *et al.*, 2002) and lumped damage models (Marante *et al.*, 2002, Marante *et al.*, 2003, Mazza *et al.*, 2008). Some of the examples among others (Melo *et al.*, 2011) (Scott *et al.*, 2008).

The main purpose of the present study is to compare three different numerical strategies to represent the behaviour of RC columns under horizontal biaxial cyclic load and variable axial load that were tested. The experimental campaign is briefly presented and the evaluation of the different numerical strategies efficiency is performed in terms of shear-drift response, shear-drift envelopes and stiffness degradation and energy dissipation.

2. Experimental Campaign

2.1. Specimen Description and Experimental Setup

The experimental campaign is composed by six rectangular RC columns were constructed with the same geometric characteristics, reinforcement detailing that were cyclically tested for different loading histories. The column specimens dimensions are: 1.70m high, with the cross section of $0.30 \times 0.50 \text{ m}^2$ and are cast in strong square concrete foundation blocks with dimensions $1.30 \times 1.30 \text{ m}^2$ in plan and 0.50m high, as can be observed in Figure 1a and 1b. The foundation block has four holes that are drilled to fix the specimen to the laboratory strong floor.

The setup is composed by two horizontal actuators that work independently in order to apply the lateral loads on the column specimen and one vertical actuator with the

capacity of 700kN is used to apply the axial load. The reaction system for the three actuators is composed by one concrete reaction wall and two steel reaction frames (illustrated in Figure 1c). The column specimens and the reaction frames were fixed to the strong floor of the laboratory with prestressed steel bars to avoid sliding or overturning of the specimen during testing, or sliding of the reaction frame. Since the axial load actuator remains in the same position during the test while the column specimen laterally deflects, a sliding device is used (placed between the top-column and the actuator), which was built to minimise spurious friction effects.

Figure 1. Testing setup: General view b) Specimen dimensions and cross section layout c) setup schematic layout (plan view)

2.2. Axial Load and Displacement Path

The main objective of this experimental campaign was the characterization of RC columns response under variable axial load. For this, the first two columns were only submitted to uniaxial load pattern (strong and weak direction) and the four others columns were submitted to different biaxial loading pattern, that are indicated in the Table 1 and illustrated in Figure 3. To characterize the column specimen's responses, cyclic lateral displacements

were imposed at the top of the column with progressively increasing demand levels. To understanding the column's behaviour three cycles were repeated for each lateral deformation demand level, and the following nominal peak displacement levels (in mm) were considered: 3, 5, 10, 4, 12, 15, 7, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80. A more detailed description of the specimens' materials properties and horizontal displacement path type are presented in Table 1 and Figure 2.

Figure 2. Horizontal displacement path type

Table 1. RC columns specimens description and testing conditions.

Specimen	Geometry [cmxcm]	f_{cm} [MPa]	f_{ym} [MPa]	Initial N [kN]	Horizontal Displacement path type
PC01-N19	30x50	27.92	575.6	300 (± 150 kN)	Uniaxial – Strong
PC02-N20					Uniaxial – Weak
PC12-N21					Rhombus
PC12-N22					45°
PC12-N23					Quadrangular
PC12-N24					Circular

Prior to the tests with varying axial loading, the peak capacities in terms of displacements and the strengths corresponding to the first yield was estimated. With this information the columns axial load were considered variable and proportional to the imposed lateral drift applied until the yielding drift. In the biaxial tests, the axial load variation is relative to the displacement observed in the strong direction. Beyond the yielding point the axial load was kept constant. The initial axial load was set on 300kN and variations of ± 150 kN were considered as can be observed in Figure 3.

Figure 3. Axial loading condition for tests under varying axial load

3. Numerical Modelling of RC Columns

In the present article, all the numerical analyses performed were achieved using the software Seismo Struct (SeismoSoft, 2004). To represent the RC columns

specimens, it was tested three different types of modelling strategies: elements with lumped-plasticity (with fixed length, so-called plastic-hinge) and elements with distributed inelasticity (force or displacement-based formulations) as illustrated in Figure 4.

Figure 4. RC columns modelling strategies with indication of the integration points: a) distributed inelasticity element with displacement-based formulation; b) distributed inelasticity element with force-based formulation; c) lumped-plasticity element **Figure 5.** a) Fiberbased modelling b) Section fiber discretization (Rodrigues, 2012)

In the present study it was also adopted fibre discretization with the main objective to represent the behaviour at section level (see Figure 5), where the section is divided in a defined number of fibers and each fiber is associated with the respective uniaxial stress-strain rule. With the integration of the non-linear uniaxial stress-strain response of each fiber the moment-curvature of the section is obtained.

3.1. Description of modelling strategies

The modelling strategies adopted in the present article were taken based on the results of previous parametric studies carried out by different authors (Taucer *et al.*, 1991, Calabrese, 2008, Calabrese *et al.*, 2010, Melo *et al.*, 2012, Rodrigues, 2012, Rodrigues *et al.*, 2012). The plastic hinge length (L_p) of the RC columns were considered half of the higher dimension of the cross-section. This decision was based on the reports provided by Priestley and Park (Priestley *et al.*, 1987) and Paulay and Priestley (Paulay *et al.*, 1992) and by other authors that have concluded based on experimental evidence that the plastic hinge length is not strongly affected by 2D loading (Tsuno *et al.*, 2004),

For the distributed inelasticity elements (force based formulation) it was considered seven integration points, based in previous results obtained by Calabrese *et al.* (Calabrese *et al.*, 2010). According to them least six integration sections are needed to obtain a completely stabilized prediction of the element local response. Relatively to the distributed inelasticity elements (displacement based formulation) a similar response can be achieved with a mesh discretization of at least four elements, with two Gauss-Legendre points per element, if

all elements have the same length. Considering this and taking into account the concentration of the non-linear response close to the fixed end of the column, plastic hinge length, a six elements discretization of the column with the length illustrated in Figure 4 was adopted.

3.2. Material Models Properties

In the present section are presented the concrete and reinforcing steel hysteretic models adopted and the values considered for each numerical model presented. In order to obtain accurate modelling of the uniaxial stress-strain hysteretic rules for the materials, required for the consideration of the material non-linearity, it is fundamental to find the best model that represents real behaviour of the elements.

3.2.1. Concrete Model

For the concrete model, it was adopted the Mander *et al.* (1988) model which is based on the Madas model (1992) uniaxial model. The confined and unconfined concrete follows the cyclic rules, included in this model, proposed by Martinez-Rueda and Elnashai (1997). The confinement effects provided by the transverse reinforcement were considered through the rules proposed by Mander *et al.* (1988), whereby constant confining pressure is assumed throughout the entire stress-strain range, indicated by the increase in the peak value of the compression strength and the stiffness of the unloading branch (SeismoSoft, 2004).. All the adopted values are in agreement with the properties obtained in the material tests and presented in Table 2.

Table 2. Concrete mechanical parameters for the numerical models

Specimens	Compressive strength f_c (MPa)	Tensile strength f_t (MPa)	Strain at peak strength ϵ_c (%)	Confinement factor*
PC01-N19-PC12-N24	27.92	3.94	0.36	1.11

* the confinement factor was obtained with the Mander *et al.* (Mander *et al.*, 1988) proposal.

3.2.2. Steel Model

For the steel reinforcement it was adopted the uniaxial model proposed by Menegotto and Pinto (1973), combined with the isotropic hardening rules proposed by Filippou *et al.* (1983). This model takes into account the

Bauschinger effect, which is relevant for the representation of the columns' stiffness degradation under cyclic loading. All the adopted values are in accordance with the properties obtained in the material tests and for the others parameters the default values summarized in Table 3.

Table 3. Steel mechanical parameters for the numerical models

Specimens	Elasticity modulus E_s (GPa)	Yield strength f_y (MPa)	Strain hardening parameter r (%)	Transition curve initial shape R_0	Transition curve shape a_1 a_2		Isotropic hardening a_3 a_4	
PC01_N19- PC12_N24	194.7	575.63	2.71	20	18.5	0.15	0.025	2

3.3. Modelling Strategies Results - Comparison

For each column analysed, the axial load and the corresponding horizontal displacement law were imposed in accordance with the experimental results. The numerical results for each modelling strategy were compared with the experimental results and between them. In this section is presented and discussed the shear-drift, shear-drift envelopes, maximum strength and initial stiffness ratio and accumulative energy dissipation results.

3.3.1. Shear-drift

The experimental shear-drift hysteretic behaviour of the columns PC01-N19NV, PC02-N20NV, PC12-N21NV and PC12-N23NV, presented in Figure 6, is compared with the numerical results. A good representation of the global columns' response is achieved, but for the largest drifts the numerical response may not capture well the strength degradation observed in the experimental test, which is normally associated to the buckling of the reinforcing steel bars. In particular it can be stated that the accuracy of the numerical models are better for the biaxial tests than the uniaxial tests.

It is also noted, in the majority of the cases that the reloading phase of the columns' cyclic response obtained

with the numerical models cannot capture the pinching effect observed experimentally.

Figure 6. Numerical shear-drift results: a) distributed inelasticity element with displacement-based formulation; b) lumped-plasticity element and c) distributed inelasticity element with force-based formulation

3.3.2. Shear-drift Envelopes

Based on the cyclic response the shear-drift envelopes obtained from the numerical analysis and the experimental tests, presented in Figure 7, a good agreement was found

between them. It is observed for small drift values that the numerical models can simulate the real behaviour of the RC columns, but otherwise for the largest drifts values, the numerical response may not capture well the strength degradation.

Figure 7. Shear-drift envelopes results: a) distributed inelasticity element with displacement-based formulation; b) distributed inelasticity element with force-based formulation; c) lumped-plasticity element

The force-based formulation is the only method that achieves a better approximation in terms of strength degradation, mainly for the biaxial tests.

3.3.2.1. Maximum Strength Ratio

In Figure 8 is illustrated the maximum strength ratio between the experimental and the numerical results. It is observed that the non-linear models have a good accuracy

Figure 8. Maximum strength ratio results: a) strong-direction b) weak direction

3.3.2.2. Initial Stiffness Ratio

The accuracy of the numerical models were evaluated in terms of initial using aratio between the experimental values and those obtained numerically as illustrated in Figure 9. From the results obtained, it can be stated as follows:

- For the uniaxial tests, the numerical models represents the initial stiffness well in the strong direction and with a variation of (5-15%) and in the weak direction is not so able to capture the value

observed in the experimental result, with a variation of 35-40%;

- In the uniaxial tests the force-based formulation gives better estimation of the initial stiffness;
- For the biaxial tests, a large variation in the initial stiffness of the columns is found, with over-evaluation in both directions of the columns (30-40% for the strong direction and 20-60% for the weak direction);
- In the biaxial tests, the plastic-hinge formulation was found as being the best approach to obtain the initial stiffness of the RC columns.

Figure 9. Initial stiffness ratio: a) strong-direction b) weak direction

3.3.3. Secant Stiffness

The evolution of the columns' secant stiffness with the drift demand was evaluated by comparing the peak-to-peak secant stiffness of the first cycle of each drift demand level, and is illustrated in Figure 10. The numerical models show less stiffness degradation in the uniaxial

in the strong direction of the RC columns, but not so well in the weak direction. A small difference is observed in the weak direction of the RC columns for the biaxial tests between the numerical and experimental results (5-15%).

The force-based formulation appears to be the method that is able to obtain the same maximum strength values than the experimental results.

tests than the experimental results. Furthermore, for the biaxial tests it is observed that the experimental result show less secant stiffness than the numerical models. It is also observed that all the three strategies obtained the same type of response in terms of stiffness degradation during the analyses.

Figure 10. Relative stiffness results: a) strong-direction b) weak direction

3.3.4. Cumulative Energy Dissipation

The evolution of the energy dissipated and the total energy dissipated for each column were determined and compared with the numerical results, as shown in Figure 11 and in Figure 12. From the obtained results, the following can be concluded:

- For the columns under uniaxial or biaxial loading, overestimation of the energy dissipation obtained from the numerical models is observed. This overestimation of the energy dissipated is justified by

the inadequacy of the numerical models in the strength degradation;

- Results for all the columns shows a good accuracy between the numerical models and the experimental results for small drift values (5-10%) and a large variation for larger values (25-50%), as consequence of the inadequacy of the numerical models in the strength degradation;
- For each studied column under biaxial conditions, similar values were found for all the three modelling strategies.

Figure 11. Evolution of the dissipated energy – Experimental and numerical results

4. Conclusions

The efficiency of three modelling strategies in the representation of the response of RC columns under uniaxial and biaxial horizontal loading and variable axial load was studied. The numerical results show similar levels of efficiency in the representation of the

experimental results using the distributed inelasticity (force and displacement formulations) and the lumped plasticity modelling strategies. Based on the results of each modelling strategy the following remarks can be drawn:

- The results obtained for the cyclic shear-drift response was satisfactory for the three modelling strategies, but difficulties were found in the representation of the strength degradation for the

higher drift demands. All the numerical models also show limitations in representing the pinching effect in the unloading-reloading stage. Nevertheless the

shear-drift envelopes show a good agreement in the capture of the numerical models for the experimental results.

Figure 12. Total dissipated energy – Experimental and numerical results

The distributed plasticity (force-based formulation) models show better results when compared with the rest of the formulations.

- The ratio between the maximum experimental and numerical strength was performed and the results shows a good accuracy of the numerical models to represent the observed behaviour of the RC columns. The same procedure was carried out for the initial stiffness and it was found that the biaxial tests present slight differences, when compared with the experimental values, in the range of 25% to 50%. It was observed that in the biaxial tests the numerical results are higher than the experimental results. The opposite was observed in the uniaxial tests where the results of numerical are lower.

All the numerical strategies show differences with the experimental secant stiffness, mainly for the higher drift demands.

- In terms of representation of the energy dissipation, it was observed differences between the numerical and

the experimental results between 25% and 50%. The main differences correspond to an underestimation of the dissipated energy for lower drift demands associated with a quasi-linear numerical response and to an overestimation for higher demands associated with the limitations of the numerical models in the representation of the strength degradation and pinching effect.

The modelling strategies analysed in this paper can accurately predict the cyclic response of the columns until the strength degradation begins. This aspect is not so relevant for uniaxial loading, since strength degradation starts for larger drift demands in comparison with biaxial loadings, mainly when submitted to variable axial load.

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