A Decentralized Event-Based Model Predictive Controller Design Method for Large-Scale Systems

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Abstract This paper presents a new methodology to design decentralized event-based control strategy for large-scale systems under the general MPC framework. The method introduces an appealing perspective to effectively reduce the computing load and communication effort in computer-based networks by incorporating the MPC approach in an event-based design framework. The proposed methodology is shown to be capable of coping explicitly with multi-input, multi-output (MIMO) plants having constraints while preserving the control performance characteristics due to decentralized MPC method with less control computational effort. The proposed control architecture ensures the stability of the closed-loop system, optimal performance and significant reduction in computational load without sacrificing the performance. Performances of the proposed method are comparatively explored on a catalytic alkylation of benzene process plant as the benchmark case study. A diverse set of experiments has been conducted to clearly demonstrate superiority of the proposed methodology compared to the standard time-driven decentralized MPC scheme on the basis of mean-squared error and number of events or control actions measures.

Keywords: event-based control, model predictive control, large scale system, decentralized


1. Introduction

Event-based control strategy presents a new methodology in which the communications between different interconnected components of a control loop are invoked when something significant has occurred in the process. This is in contrast to the common periodic or time-driven control systems where control calculations are performed all the time at a fixed sampling rate. Practical experiences, however, indicate that it is not necessary to keep the control loop communication intact all the times especially when nothing significant has happened in the process. This is mainly due to the fact that it can lead to unnecessary waste of resources like processor usage and communication bus load. Moreover, strong and non-negotiable constraints are imposed to the control design requirements due to stringent real-time considerations.

Despite the significant improvements in communication network performance, limitation of available bandwidth is still the first difficulty for many applications. Event-based control policy has recently found considerable interest in research communities for efficient allocation of the limited bandwidth resources in networked control systems to substantially reduce the severe real-time constraints. The required event-based mechanism invokes transmission of the outputs when the difference between the current values of the outputs and their latest state values becomes large. In [1,2,3,4], it is proved that such an approach reduces the number of sampling instants while ensuring a desired control performance. Some recent papers have investigated event-based control for multivariable system [5,6,7]. This paper is aimed to address the important issue of utilizing model predictive control (MPC) in an event-based control configuration for multivariable systems.

MPC is usually studied from a centralized control point of view in which all the manipulated inputs of a control system are optimized with respect to an objective function in a single optimization problem [8,9,10,11]. However, when the number of the state variables and manipulated inputs of the process goes up, the computational load for the centralized optimization problem may significantly be increased, hindering the practicability of a centralized MPC. One possible substitute to overpower this problem is to use a decentralized MPC architecture in which the manipulated inputs are computed by more than one optimization problems in a coordinated mode [12,13,14,15]. In this work, we focus on decentralized MPC of linear large scale system in which several separate sets of manipulated inputs are used to regulate the process. For each set of manipulated inputs, a different MPC is used to compute the control actions. Generally, the computational load of these decentralized MPC methods is smaller compared to the one of the similar centralized MPC due to the formulation of optimization problems with a smaller number of decision variables. As
a result, configuring the event-based MPC in the centralized scheme can be undesirable especially for large scale systems and, therefore, a decentralized event-based MPC is proposed in this paper. The relevant event-based mechanism is designed to transmit the outputs in a subsystem whenever the difference between the current values of the outputs in the corresponding subsystem and their steady state values becomes large.

Computation load of the control law is always a critical characteristic especially for large-scale systems. A decentralized event-based framework is considered in this research work in order to diminish the number of times the control input should be calculated. Under this strategy, the MPC control law may not be updated at each sampling instant but rather, the already calculated control succession is performed to the plant until an event happens. It is proved that the proposed model architectures enforce practical stability in the closed-loop system and optimal performance. Performance evaluation of the proposed decentralized event-based control system is practically demonstrated through a catalytic alkylation of benzene process plant [16,17,18].

The paper is organized as follows. The proposed decentralized event-based MPC control structure is first developed in Section 2. A set of conducted test scenarios are presented in Section 3 to explore performances of the proposed control methodology in a simulated industrial process benchmark plant. Conclusions are finally summarized in Section 4.

2. Decentralized Event–Based MPC

Event-based control (EBC) architecture consists of two parts; the controller that computes the plant inputs and the event-based mechanism (EBM) that determines when and which outputs of the plants and the controller have to be transmitted, as shown in Figure 1. A typical event is generated whenever absolute value of the difference between the real vector \( v(t) = [y_j(t) \ u(t)]^T \) and the set point vector \( v_s(t) = [y_j(s) \ u_s(s)]^T \) crossed the limit \( \epsilon_{\text{max}} \) (i.e., \( v(t) - v_s(t) \geq \epsilon_{\text{max}} \)), and the generated event at time \( t_k \) is then sent to the controller in order to update the plant states. Consequently, this strategy reduces the utilization of the available computation and communication resources. Consider the class of large-scale interconnected linear systems being described as follows:

\[
\begin{align*}
    x_i(k+1) &= A_i x_i(k) + B_i u_i(k) \\
    y_j(k) &= C_j(k)x_j(k)
\end{align*}
\]

Where \( x_i \in \mathbb{R}^{n_i} \) denotes the state vector, \( u_i \in \mathbb{R}^{m_i} \) the input vector, \( y_j \in \mathbb{R}^{p_j} \) the output vector for subsystem \( i \) and \( x = [x_1 x_2 \ldots x_M]^T \) represents the state vector of the overall system. It is also assumed that \( (A_i, B_i) \) is a controllable pair and \( (C_j, A_j) \) is an observable pair for all \( i \in \{1, \ldots, M\} \). The global system can then be rewritten as:

\[
x(k+1) = Ax(k) + Bu(k)
\]

Where

\[
\begin{align*}
    A &= \text{diag}(A_1, A_2, \ldots, A_M), \\
    B &= \text{diag}(B_1, B_2, \ldots, B_M), \\
    C &= \text{diag}(C_1, C_2, \ldots, C_M), \\
    u &= [u_1 u_2 \ldots u_M]^T, \\
    y &= [y_1 y_2 \ldots y_M]^T.
\end{align*}
\]

The plant is considered to be faced with the following constraints:

\[
\begin{align*}
    \Delta u_{\text{min}} &\leq \Delta u(k) \leq \Delta u_{\text{max}} \\
    u_{\text{min}} &\leq u(k) \leq u_{\text{max}} \\
    y_{\text{min}} &\leq y(k) \leq y_{\text{max}}
\end{align*}
\]

Where \( \Delta u = u(k) - u(k-1) \) denotes the control increment.

![Figure 1. An event-based control system schematic](image)

An MPC method has been incorporated into the controller such that at each event sample time the infinite horizon quadratic optimization problem is minimized while guaranteeing the closed loop stability of system. This quadratic cost function may be written as:

\[
\begin{align*}
    J(k) &= \sum_{m=1}^{N_p} e(k+m)^T \cdot Q \cdot e(k+m) \\
    &\quad + \sum_{m=1}^{N_u} \Delta u^T(k+m-1) \cdot R \cdot \Delta u(k+m-1)
\end{align*}
\]

\( \hat{x} \) and \( x_s \) denote the predicted controlled state and desired state trajectory, respectively, and \( \Delta u \) indicates the predicted control increments. The matrices \( Q \geq 0 \) and \( R \geq 0 \) represent the weighting matrices which are assumed to be constant over the prediction horizon. It is also assumed that \( N_p \) and \( N_u \) represent the prediction and control horizons, respectively. Compared to the centralized MPC, the decentralized MPC control scheme applies a controller to each subsystem and each controller then solves its online optimization problem locally. The cost function for subsystem \( i \) may be written as:

\[
\begin{align*}
    J_i(k) &= \sum_{m=1}^{N_{pi}} e_i(k+m)^T \cdot Q_i \cdot e_i(k+m) \\
    &\quad + \sum_{m=1}^{N_{ui}} \Delta u_i^T(k+m-1) \cdot R_i \cdot \Delta u_i(k+m-1)
\end{align*}
\]
The paper introduces the decentralized event-based control configuration which is composed of $M$ subsystems where the outputs of subsystem $i \in \{1, \ldots, M\}$ are only sent at the transmission instants $k^i_l, i \in M$. Hence, subsystem $i$ generates its respective signal input $u_i$, while the other signal inputs remain the same at the transmission instant $k^i_l$. The measurement error is defined as follows:

$$er(k|k_l) = \nu(k|k_l) - \nu_s(k|k_l)$$ (10)

Where

$$er(k|k_l) = \left[ er(k|k^i_1) \ldots er(k|k^i_M) \right]$$ (11)

$$\nu(k|k_l) = \left[ \nu(k|k^i_1) \ldots \nu(k|k^i_M) \right]$$ (12)

$$\nu_s(k|k_l) = \left[ \nu_s(k|k^i_1) \ldots \nu_s(k|k^i_M) \right]$$ (13)

$\nu_s$ includes the steady-state values of the plant and controller outputs. In the case where $k^i_l = k^j_l$ or some $i, j \in \{1, \ldots, M\}$, it is assumed that the transmissions of the plant outputs or the controller outputs occur simultaneously.

In a sampled-data implementation of period $h$ for each subsystem, i.e., $t^i_{l+1} = t^i_l + h$, the transmission samples are assumed to be the same, i.e., $k^i = k^j$ for all $i, j \in \{1, \ldots, M\}$. A typical event of each subsystem is generated when the difference between the current values and their steady-state values gets too large. In this case, the subsystem outputs are sent to the corresponding controller in order to update the model state at sample $k^i_l$, satisfying:

$$k^i_{l+1} = \inf \left\{ k > k^i_l \mid \sigma_i e_i^2 + \epsilon_i \right\}, \quad k^i_0 = 0, \sigma_i, \epsilon_i \geq 0$$ (14)

It is assumed that in the event time of subsystem $i \in \{1, \ldots, M\}$, the control sequence $U_i(k|k_l)$, calculating from MPC optimization solution, is sent to the actuator. The control sequence of each subsystem $U_i(k|k_l)$ can be indicated by the following nonlinear operator:

$$U_i(k|k_l) = \left\{ u_i(k|k^i_1) \ldots u_i(k + N - 1|k_l) \right\}$$ (15)

$$U_i(k|k_l) = F_i(X_i(k|k_l)), k > k^i_l$$ (16)

The sequence of predicted states $X_i(k|k_l)$ for the event time of subsystem $i \in \{1, \ldots, M\}, k^i_l$, can be denoted by:

$$X_i(k|k_l) = \left\{ x_i(k|k^i_1) \ldots x_i(k + N - 1|k_l) \right\}$$ (17)

The stability of each subsystem $i \in \{1, \ldots, M\}$ can then be investigated by considering the following assumptions:

1. The optimization problem that fulfills the linear inequality constraints is feasible.

2. There is $K_i$ such that

$$\lim_{k \to \infty} F_i(X_i(k|k_l)) - K_i X_i(k|k_l) = 0$$ and

$$\lim_{k \to \infty} A_i + B_i K_i = 0.$$

Where $K_i$ can be obtained for each subsystem as described in [19] and [14], as a result of

$$\lim_{k \to \infty} X_i(k|k_l) = 0$$ and

$$\lim_{k \to \infty} A + B K = 0$$

and hence the stability of system is then guaranteed.

The proposed decentralized event-based mechanisms based on local information only. Therefore, if the norm of the error, corresponding to subsystem $i$ becomes too large, it solves a local optimization problem, i.e., the local controller $u_i$ uses only the local output signal $y_i$. While the number of transmissions due to the plant and controller outputs are reduced, and the global system is stabilized without any communication between the subsystems. The introduced decentralized event-based MPC scheme consisting of $M$ subsystems has been illustrated by Figure 2.

![Figure 2. Decentralized event-based MPC](image)

3. Application to an Alkylation of Benzene Process

![Figure 3. Flow Diagram of Alkylation of Benzene Process Plant](image)
Performance of the proposed decentralized event-based MPC is evaluated using a simulated industrial process plant. In this study, a widespread benchmark plant in the petrochemical industry has been considered as a challenging industrial plant in which alkylation of benzene with ethylene is simulated to produce ethyl benzene. Over the last two decades, several methods and simulation studies of alkylation of benzene with catalysts have been reported in the literature in [16,17,18]. The process of alkylation of benzene with ethylene essentially consists of four continuously stirred tank reactors (CSTRs) and a flash tank separator, as shown in Figure 3.

The manipulated inputs of the process include the heat injected to or removed from the five vessels $Q_1, Q_2, Q_3, Q_4, Q_5$. The states of the process consist of the concentrations of A, B, C, D in each of the individual five vessels and the temperatures of the vessels. The state vector of the process is assumed to be accessible whose data are shown in Table 1 with the corresponding steady-state outputs. Further details on the plant description and its modeling have been provided in [16,17,18]. The process is inherently nonlinear, and hence a linearized model must be derived in order to apply the proposed decentralized event-based MPC using the sampling interval $h = 10s$.

### Table 1. Steady-state values for inputs and outputs

| $Q_{3s}$  | $-4.4 \times 10^6$ (J/s) | $T_{3s}$  | 477.24(k)  |
| $Q_{2s}$  | $-4.6 \times 10^6$ (J/s) | $T_{2s}$  | 476.97(k)  |
| $Q_{3s}$  | $-4.7 \times 10^6$ (J/s) | $T_{3s}$  | 473.47(k)  |
| $Q_{4s}$  | $9.2 \times 10^6$ (J/s)  | $T_{4s}$  | 470.6(k)   |
| $Q_{5s}$  | $5.9 \times 10^6$ (J/s)  | $T_{5s}$  | 478.28(k)  |

### Table 2. Initial values for outputs

| $T_1$   | 443(k) |
| $T_2$   | 437.1(k) |
| $T_3$   | 428.4(k) |
| $T_4$   | 433.1(k) |
| $T_5$   | 457.6(k) |

### Table 3. Manipulated input constraints

- $|Q_1| \leq 7.5 \times 10^6 \frac{J}{s}$
- $|Q_2| \leq 6 \times 10^6 \frac{J}{s}$
- $|Q_3| \leq 6 \times 10^6 \frac{J}{s}$
- $|Q_4| \leq 5 \times 10^6 \frac{J}{s}$
- $|Q_5| \leq 5 \times 10^6 \frac{J}{s}$

The control objective is to guide the system from an initial state to the steady state and keep it there regulated. The initial output values are shown in Table 2. Five local MPC controllers are designed for the alkylation process plant in order to compute $Q_1, Q_2, Q_3, Q_4$ and $Q_5$, respectively, as shown in Figure 3. The inputs, designated by $Q_1, Q_2, Q_3, Q_4, Q_5$, are responsible to independently control the plant outputs, i.e. $T_1, T_2, T_3, T_4, T_5$ in order to achieve an optimal closed loop performance while reducing the computation and communication effort. The manipulated inputs which are subject to the constraints have been also tabulated in Table 3.

It is assumed that no disturbance acts on the plant and the system is equipped with an event-based mechanism at the sensor-to-controller. Hence, the proposed decentralized event-based control (EBC) system will consist of 5 subsystems. Practical stability of the configured EBC system with $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 0.5$ can be guaranteed. The free design parameters have been set at $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = 10^{-3}$. The simulation results are shown in Figure 4 for the two alternative control techniques being applied to the plant.

Mean square error (MSE) of each plant output and number of event (i.e., control action) are used as the two main evaluating criterionsto comparatively explore performances of the implemented control approaches with respect to regulatory and communication and computation burdens, respectively. The proposed method shows a reduced computational and communication effort compared to the conventional decentralized MPC, which has been summarized by the number of events via each controller in Table 4. The proposed approach has led to a considerable reduction of 98.5%, 99%, 98.25%, 95.75% and 98.5% in the number of control tasks (i.e., communication and computation efforts) with respect to the alternative decentralized MPC approach to track the corresponding temperature set-points in the five vessels. Noting that the control tasks are constantly performed at a fixed sampling interval of $h = 10s$, i.e. control updates should be calculated 400 times at each sampling time (see Table 4). The system closed-loop performance has been comparatively investigated by each output MSE for the proposed method and the conventional decentralized MPC. The summarized results in Table 5 verify the achievement of an acceptable control performance for the proposed control scheme. It is clearly shown that the controller manages to guide the plant outputs to track the desired set points with an extremely reduction in the computation load. It is, however, noted that in practice the number of control action can effectively affect the quality of the control performance. Consequently, a beneficial trade-off can be maintained between the event number reduction and the satisfactory system performance by using free design parameters such as $\sigma$ and $\varepsilon$.

### Table 4. Number of event for each controller

<table>
<thead>
<tr>
<th>Controller</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPC1</td>
<td>6</td>
</tr>
<tr>
<td>MPC2</td>
<td>4</td>
</tr>
<tr>
<td>MPC3</td>
<td>7</td>
</tr>
<tr>
<td>MPC4</td>
<td>17</td>
</tr>
<tr>
<td>MPC5</td>
<td>5</td>
</tr>
</tbody>
</table>

### Table 5. Mean squares error of each output

<table>
<thead>
<tr>
<th>Controller</th>
<th>Decentralized conventional MPC</th>
<th>Decentralized event-based MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPC1</td>
<td>0.05535</td>
<td>0.31717</td>
</tr>
<tr>
<td>MPC2</td>
<td>0.23377</td>
<td>0.46213</td>
</tr>
<tr>
<td>MPC3</td>
<td>0.31030</td>
<td>0.57192</td>
</tr>
<tr>
<td>MPC4</td>
<td>0.14897</td>
<td>0.66861</td>
</tr>
<tr>
<td>MPC5</td>
<td>0.05180</td>
<td>0.34290</td>
</tr>
</tbody>
</table>
The plant linearized model is assumed to be known in the standard form, being introduced in equation (2), where $A$ indicates a $25 \times 25$ matrix, $B$ denotes a $25 \times 5$ matrix, $C$ is a $5 \times 25$ matrix, and $D$ represents a $5 \times 5$ matrix. $K$ will correspondingly be a $25 \times 25$ matrix that can also be calculated to maintain the stability of the individual subsystems [14,19]. Five subsystems can be generated using equation (1) for $i = 1:5$; where $A_i$ is a $5 \times 5$ matrix, $B_i$ is a $5 \times 1$ matrix, $C_i$ is a $1 \times 5$ matrix, and $D_i$ is a $1 \times 1$ matrix. By proper choosing of the MPC parameters, $K_i$ can be determined for $i = 1:5$ (having a form of $1 \times 5$ matrix) as follows:

\[
\begin{bmatrix}
3.9338e+11 & 5.1878e+11 & 1.2527e+11 & 6.4448e+11 & 2.2736e+06 \\
5.4653e+11 & 2.3238e+11 & 3.1425e+11 & 8.1527e+10 & 1.6499e+06 \\
9.5539e+11 & 9.7364e+11 & 1.7923e+13 & 9.9262e+11 & 1.9652e+06 \\
1.5879e+08 & 2.5010e+08 & 1.0966e+11 & 5.4081e+08 & 1.2068e+06
\end{bmatrix}
\]
The design procedure has been carried out for the following design parameters:

\[ Q = 1000 \times \text{diag}([1 1 1 1]) ; R = 0.001 ; N_p = 10 ; N_U = 6 ; \]

It has been assumed that there is no effective disturbance acting on the process and no output constraint is included. Infeasibility can be raised due to possible disturbances, too short prediction horizon and hard output constraints. However, it has been practically observed that disintegrating the large system with 25 state variables into five subsystems, having optimization problem with five state variables each, are feasible at all sampling times and the feasibility of the algorithm can further be enhanced by proper choosing of free parameters. Moreover, it is already known that MPC design parameters can affect the stability. As typical examples, weighting matrices and prediction horizon must properly be adjusted to maintain the stability condition. Too short horizons may cause instability and too long horizons can lead to unnecessary optimization problem to be solved at each sample. For the proposed decentralized event-based model predictive controller feasibility and stability of the process is guaranteed by solving an online optimization problem and properly choosing of MPC parameters. It is noted that the optimization performance is improved due to the decentralized method with a smaller number of control variables. Nevertheless, there is still a lack of theoretical result to guarantee for the case when the system is nonlinear or hard output constraints are imposed.

4. Conclusions

In this paper, a new event-based design method has been proposed in the decentralized MPC configuration for linear systems with stability assurance. The strategy includes solving an online optimal control problem which enables the controller to systematically deal with MIMO plants, having imposed constraints on state, actuator, and computation. The experimental results clearly demonstrate superiority of the proposed decentralized event-based MPC compared to the conventional decentralized MPC approach for the considerable reduction in the number of control task executions, while retaining its satisfactory closed-loop performance. Further research works should be initiated to compromise between the computation and communication reduction and control performance, and networked based control systems using a finite bandwidth communication channel, being faced with robustness difficulties due to varying transmission delays and packet dropouts.

References